Chapter 10

Identical Particles

4. Bose - Einstein Distribution

In quantum statistics, **Bose–Einstein statistics** (or more colloquially **B–E statistics**) is one of two possible ways in which a collection of indistinguishable particles may occupy a set of available discrete energy state. The aggregation of particles in the same state, which is a characteristic of particles obeying Bose–Einstein statistics, who recognized that a collection of identical and **indistinguishable particles** can be distributed in this way.

The Bose–Einstein statistics apply only to those particles not limited to single occupancy of the same state—that is, particles that **do not obey the Pauli exclusion restrictions**. Such particles **have integer values of spin** and are named boson, after the statistics that correctly describe their behavior.

The wave function of particle will overlap to each other because mean separation of particles is less than the thermal wavelength, which is identified by λ. (where $\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$ is defined as the thermal wavelength)

Suppose there are l states with energies, $E_1, E_2, E_3 ....E_l$ and degeneracy $g_1, g_2, g_3 ....g_l$. Respectively, in which the particles are distributed. If there is N numbers of indistinguishable boson particles out of these $n_1, n_2, n_3 ....n_l$ particles is adjusted in energy level $E_1, E_2, E_3 ....E_l$ respectively. It is given $\sum_{i=1}^{i=l} n_i = N$, $\sum_{i=1}^{i=l} E_i n_i = U$.

The total no of arrangements of the particles in the given distributions is given by

$$W = \frac{\lfloor n_i + g_i - 1}{\lfloor n_i \lfloor (g_i - 1)} , \quad W = \prod_{i=1}^{i=l} \frac{\lfloor n_i + g_i - 1}{\lfloor n_i \lfloor (g_i - 1)}$$

If n_i and g_i are large numbers, we can omit 1 in comparison to them , so we have

$$W = \prod_{i=1}^{i=l} \frac{\lfloor n_i + g_i}{\lfloor n_i \lfloor g_i}$$

Example: Two indistinguishable boson particles have to be adjusted in a state whose degeneracy is three.

(a) How many ways the particles can be adjusted?

(b) Show all arrangement.

Solution: (a) $n_i = 2, g_i = 3$, $W_i = \frac{\lfloor n_i + g_i - 1}{\lfloor n_i \lfloor g_i - 1}$ = 6 ways

(b) Total number of arrangement for 2 indistinguishable boson particles in state, whose degeneracy is 3.

First level	**Second level**	**Third level**
AA	0	0
0	AA	0
0	0	AA
A	A	0
0	A	A
A	0	A