Introduction To Statistical Mechanics

4. Probability Calculations

Consider a system in equilibrium which is isolated so that its total energy is known to have a constant value in some range between E and $E+\delta E$. Let $\Omega(E)$ denote the total number of states of the system in this range. Suppose that there are among these states a certain number $\Omega(E;y_k)$ of states for which some parameter y of the system assumes the value y_k. The parameter might be the magnetic moment of the system, or the pressure exerted by the system, etc. (We label the possible values which y may assume by the index k; if the possible values of k as corresponding to values of y which differ by infinitesimal amounts.)

The probability $P(y_k)$ that the parameter y of the system assumes the value y_k

$$\boxed{P(y_k) = \frac{\Omega(E; y_k)}{\Omega(E)}}$$

The mean value of the parameter y for this system

$$\boxed{\overline{y}_k = \frac{\sum_k \Omega(E; y_k) y_k}{\Omega(E)}}$$

Here the summation over k denotes a sum over all possible values which the parameter y can assume.

Example: Consider a system consisting of three spins in equilibrium in a magnetic field H. If the total energy of this system is known to be $-\mu H$.

(a) Find the possible accessible microstates?

(b) Focus attention on one of these spins, say the first. What is the probability P_+ that this spin points up?

(c) What is the mean magnetic moment $\overline{\mu}_z$ (in the $+z$ direction) of such a spin?

Solution: **(a)** The system is equally likely to be in any of the three states

$$(++-) \qquad (+-+) \qquad (-++)$$

(b) Since there are two cases where it points up, one has

$$P_+ = \frac{2}{3}$$

(c) Since the probability of occurrence of each state of the entire system is $\frac{1}{3}$, one has simply

$$\overline{\mu}_e = \frac{1}{3}\mu + \frac{1}{3}\mu + \frac{1}{3}(-\mu) = \frac{1}{3}\mu$$