Chapter 10

Identical Particles

6. Bose-Einstein Gas at High Temperature

At a high temperature, most of the particles in a gas are in the excited states. The number of particles in the ground states.

Thermal length is $\lambda = h/\sqrt{2\pi m k_B T}$ are negligibly small. In a Bose-Einstein gas, total number N of particles and total energy U are

$$N = \int_0^\infty f(E) g(E) dE \qquad U = \int_0^\infty E f(E) g(E) dE$$

where Bose-Einstein energy distribution $f(E)$ is with $\beta = \frac{1}{K_B T}$ and the number of energy states $g(E)dE$ in the energy range from E to $E + dE$ in three dimensional $g(E)dE = 2g\pi V\left(\frac{2m}{h^2}\right)^{3/2} E^{1/2} dE$ where $g = 2s+1$ is the spin degeneracy (for the spin quantum number s for spin less particles $g = 1$). Thus, we have

$N = 2g\pi V\left(\frac{2m}{h^2}\right)^{3/2}\int_0^\infty \frac{E^{1/2}}{Ae^{\beta E}-1}dE$ and $U = 2g\pi V\left(\frac{2m}{h^2}\right)^{3/2}\int_0^\infty \frac{E^{3/2}}{Ae^{\beta E}-1}dE$

Let $\beta E = u$ then $\beta dE = du$ and we have and put $\beta = \frac{1}{K_B T}$

$N = 2g\pi V\left(\frac{2mk_BT}{h^2}\right)^{3/2}\int_0^\infty \frac{u^{1/2}}{Ae^{u}-1}du$ and $U = 2g\pi VkT_B\left(\frac{2mk_BT}{h^2}\right)^{3/2}\int_0^\infty \frac{u^{3/2}}{Ae^{u}-1}du$

$$N = 2g\pi V\left(\frac{2mk_BT}{h^2}\right)^{3/2}\int_0^\infty \frac{u^{1/2}}{Ae^{u}-1}du = \frac{gV}{\lambda^3}g_{3/2}(A)$$

$$U = 2g\pi Vk_BT\left(\frac{2mk_BT}{h^2}\right)^{3/2}\int_0^\infty \frac{u^{3/2}}{Ae^{u}-1}du = \frac{3}{2}k_BT\frac{gV}{\lambda^3}g_{5/2}(A)$$

where $\lambda = \left(\frac{h^2}{2\pi mk_BT}\right)^{1/2}$ is the thermal wave length and $g_n(A) = \frac{1}{\Gamma(n)}\int_0^\infty \frac{u^{n-1}}{Ae^{u}-1}du$

where $g_n(A) = \sum_{r=1}^{\infty}\frac{1}{A^n}\frac{1}{r^n}$

If temperature T is very large then $U = \frac{3Nk_BT}{2}$ and Bose gas behave as ideal gas.

$$N = \frac{gV}{\lambda^3}\left[\frac{1}{A}+\frac{1}{2\sqrt{2}A^2}\right] \qquad U = \frac{3gVk_BT}{2\lambda^3}\left[\frac{1}{A}+\frac{1}{4\sqrt{2}A^2}\right]$$

$$U = \frac{3Nk_BT}{2}\left[1-\frac{N\lambda^3}{4\sqrt{2}gV}\right]$$

* If temperature T is very large then $U = \frac{3Nk_BT}{2}$ and Bose gas behave as ideal gas.