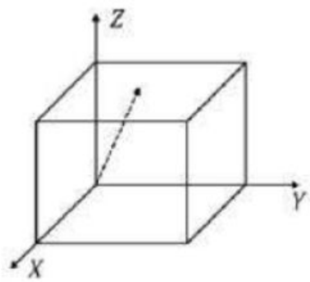
IIT JAM 2023 Solution

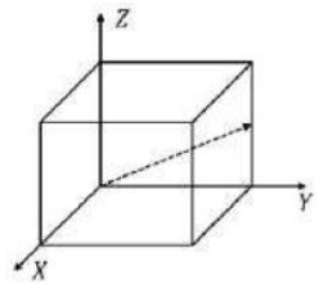
Section A: Q.1-Q.10 Carry ONE mark each.

Q.1 For a cubic unit cell, the dashed arrow in which of the following figures represents the direction [220]?

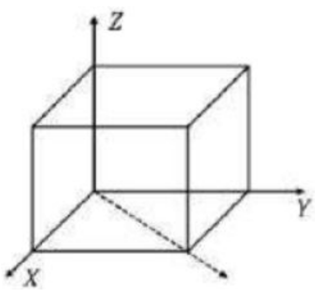
(A)



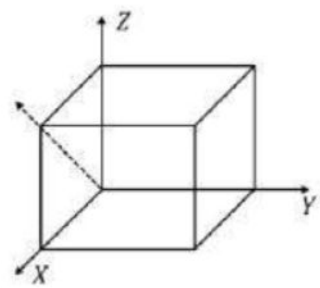
(B)



(C)



(D)



Topic- Solid State Physics

Sub Topic- Crystallography

Ans. : (C)

Solution: From the concept Miller and direction indices

Q.2 Which of the following fields has non-zero curl?

(A) $x\hat{i}+y\hat{j}+z\hat{k}$

(B) $(y+z)\hat{i}+(x+z)\hat{j}+(x+y)\hat{k}$

(C) $y^2\hat{i}+(2xy+z^2)\hat{j}+2yz\hat{k}$

(D) $xy\hat{i}+2yz\hat{j}+3xz\hat{k}$

Topic- Mathematical Physics

Sub Topic- Vector Analysis

Ans. : (D)

Solution: (A) $\nabla \times E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = i(0-0) - j(0-0) + \hat{k}(0-0) = 0$

(B) $\nabla \times E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y+z & x+z & x+z \end{vmatrix} = i(1-1) - \hat{j}(1-1) + \hat{k}(1-1) = 0$

(C) $\nabla \times E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & \left(2xy+z^2\right) & 2yz \end{vmatrix} = \hat{i}(2z-2z) - \hat{j}(0-0) + \hat{k}(2y-2y) = 0$

(D) $\nabla \times E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix} = i(0-2y) - j(3z-0) + \hat{k}(0-x) \neq 0$

Q.3 Which of the following statements about the viscosity of a dilute ideal gas is correct?

(A) It is independent of pressure at fixed temperature

(B) It increases with increasing pressure at fixed temperature

(C) It is independent of temperature

(D) It decreases with increasing temperature

Topic- Thermodynamics and Statistical Mechanics

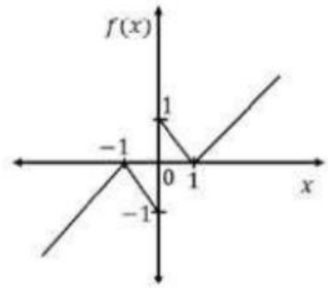
Sub Topic- Kinetic Theory of Gas

Ans. : (A)

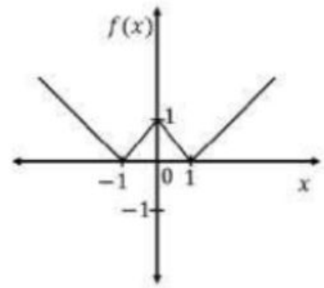
Solution: Viscosity is defined as $\eta = \frac{m\bar{v}}{3\sqrt{2}\pi d^2} \Rightarrow \eta \propto \sqrt{T}$ .Thus, η is independent of pressure at a fix temperature.

Q.4 The plot of the function $f(x)=||x|-1|$ is

(A)



(B)



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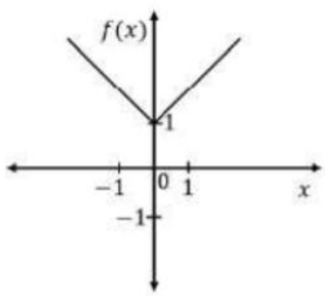

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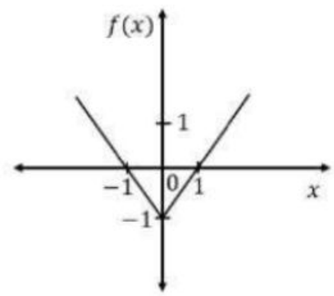


Topic- Mathematical Physics

Sub Topic- Graph Plotting

Ans. : (B)

Solution: $At\, x=1,\ |x|=1 \Rightarrow \ f(x)=|1-1|=0\ \&\, at\, x=0,\ |x|=o \Rightarrow \ f(x)=|0-1|=1$

We can see only option (B) is satisfying the condition.

Q.5 A system has N spins, where each spin is capable of existing in 4 possible states. The difference in entropy of disordered states (where all possible spin configurations are equally probable) and ordered states is

(A) $2(N-1)k_B \ln 2$ (B) $(N-1)k_B \ln 2$

(C) $4k_B \ln N$ (D) $k_B \ln 2$

Topic- Thermodynamics and Statistical Mechanics

Sub Topic- Statistical Mechanics

Ans. : (A)

Solution: 4 possible states & disorder $w=4^N$

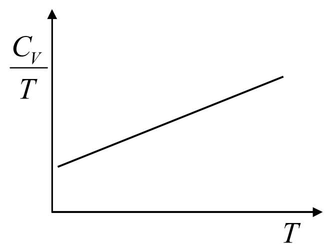
Entropy $S_1 = k_B \ln(4)^N = 2Nk_B \ln 2$

For disorder $w=4$, $S_2 = k_B \ln 4$

$S_1 - S_2 = Nk_B \ln 4 - k_B \ln 4 \Rightarrow = (N-1)k_B \ln 4 = 2(N-1)k_B \ln 2$

Q.6 Temperature (T) dependence of the total specific heat (C_V) for a two dimensional metallic solid at low temperatures is

(A)



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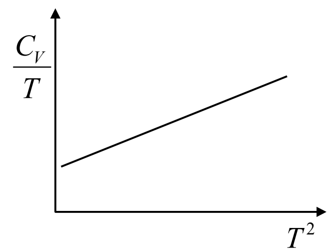

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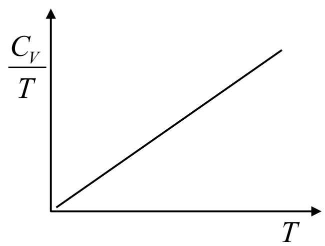
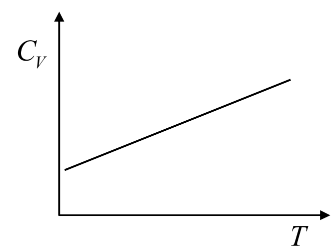


(D)



Topic- Solid State Physics

Sub Topic- Heat Capacity

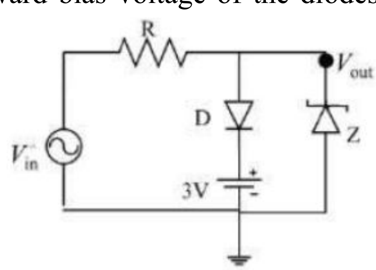
Ans. : (A)

Solution: Electronic part of heat capacity is $C_e = AT$

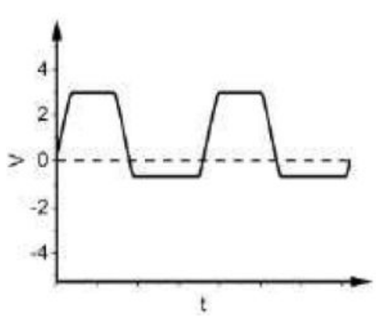
Phonon heat capacity $c_{ph} = BT^d$, $d = Dimension \Rightarrow In\ \ 2-D,\ d=2 \Rightarrow C_{ph} = BT^2$

Total heat capacity $C_v = AT + BT^2 \Rightarrow \frac{c_v}{T} = A + BT$. Thus, correct option will be (A)

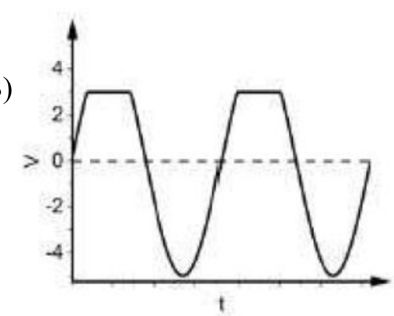
Q.7 For the following circuit, choose the correct waveform corresponding to the output signal (V_{out}). Given $V_{in} = 5\sin(200\pi t)V$, forward bias voltage of the diodes (D and Z) = 0.7 V and reverse Zener voltage $= 3V$.

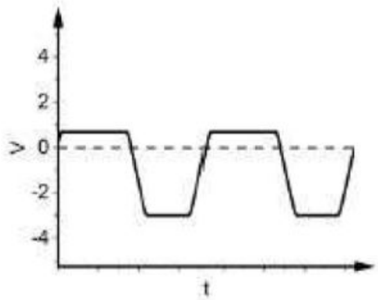


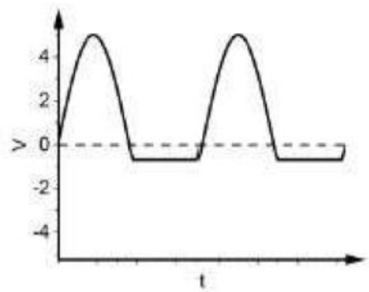
(A)



(B)



(C) 

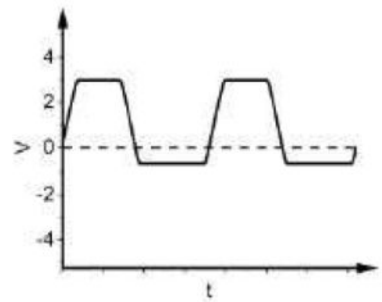
(D) 

Topic- Electronics

Sub Topic- Zener Diode

Ans. : (A)

Solution: Applying up to the voltage of 3 volt, both diode and Zener-diode is open (Conducting). Above 3 volt input, the Zener will not con duct so the voltage will be fixed. Again below 3 volt input, The output feature will be similar as input.



In the negative cycle of input, the diode is reverse and Zener is forward bias. In this negative cycle, the out will be varies as input. However, above 0.7 volt the Zener will act like a Battery. Therefore, above 0.7-volt input, the output will be constant.

Q.8 If the ground state energy of a particle in an infinite potential well of width L_1 is equal to the energy of the second excited state in another infinite potential well of width L_2, then the ratio $\frac{L_1}{L_2}$ is equal to

(A) 1 (B) 1/3 (C) $1/\sqrt{3}$ (D) $1/9$

Topic- Modern Physics

Sub Topic- Particle in a box

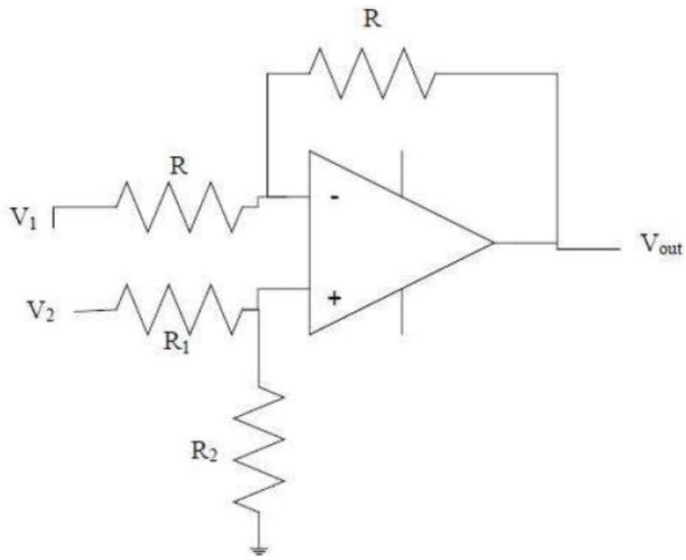
Ans. : (B)

Solution: The energy of 1-D Infinite square well box is $E = \frac{n^2\pi^2\hbar^2}{2ml^2}$

In ground state $n=1,\ E_1 = \frac{1^2\pi^2\hbar^2}{2mL_1^2}$ & in second excited state $n=3,\ E_3 = \frac{3^2\pi^2\hbar^2}{2mL_2^2}$

According to given condition $\frac{1^2\pi^2\hbar^2}{2mL_1^2} = \frac{3^2\pi^2\hbar^2}{2mL_2^2} \Rightarrow \frac{L_1}{L_2} = \frac{1}{3}$

Q.9 In the given circuit, with an ideal op-amp for what value of $\frac{R_1}{R_2}$ the output of the amplifier $V_{out} = V_2 - V_1$?



(A) 1 (B) ½ (C) 2 (D) 3/2

Topic- Electronics

Sub Topic- OPAMP

Ans. : (A)

Solution: For given configuration $v_{out} = v_1 - v_2$, and $v_+ = \frac{v_2 R_2}{R_1 + R_2}$

From Kirchhoff's law $i_1 = i_2 \Rightarrow \frac{v_1 - v_+}{R} = \frac{v_+ - v_{out}}{R} \Rightarrow v_1 - 2v_+ = -v_{out} = v - v_2$

$v_+ = \frac{v_2}{2} = \frac{v_2 R_2}{R_1 + R_2} \Rightarrow \frac{R_1}{R_2} = 1$

Q.10 A projectile of mass m is moving in the vertical x-y plane with the origin on the ground and y-axis pointing vertically up. Taking the gravitational potential energy to be zero on the ground, the total energy of the particle written in planar polar coordinates (r, θ) is (here g is the acceleration due to gravity)

(A) $\frac{m}{2}\dot{r}^2 + mgr\sin\theta$

(B) $\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + mgr\cos\theta$

(C) $\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + mgr\sin\theta$

(D) $\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - mgr\cos\theta$

Topic- Mechanics

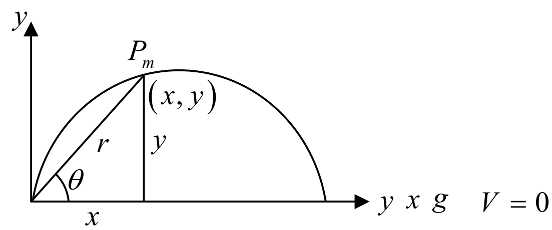
Sub Topic-Projectile Motion

Ans. : (B)

Solution: The total energy can be written as follows $E = T + V$, $T = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right)$

In polar coordinate $x = r\sin(\theta) \Rightarrow \dot{x} = \dot{r}\sin(\theta) + r\dot{\theta}\cos(\theta)$ and

$y = r\cos(\theta) \Rightarrow \dot{x} = \dot{r}\cos(\theta) - r\dot{\theta}\sin(\theta)$



Thus, $\dot{x}^2 + \dot{y}^2 = \dot{r}^2 + r^2\dot{\theta}^2$

Potential energy $V = mgy = mgr\sin(\theta)$

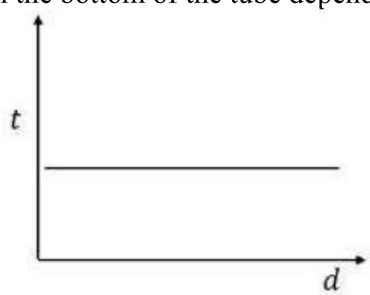
Now $E = T + V = T = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) + mgr\sin(\theta)$

$$E = T + V = T = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + mgr\sin(\theta)$$

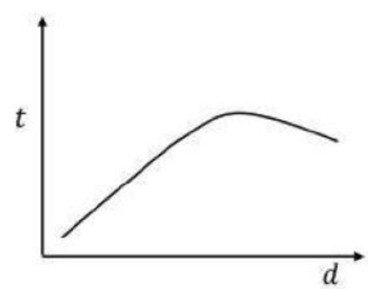
Section A: Q.11-Q.30 Carry TWO marks each.

Q.11 A small bar magnet is dropped through different hollow copper tubes with same length and inner diameter but with different outer diameter. The variation in the time (t) taken for the magnet to reach the bottom of the tube depends on its wall thickness (d) as

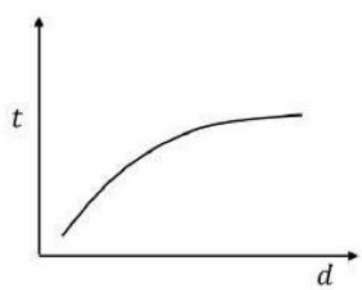
(A)



(B)



(C)



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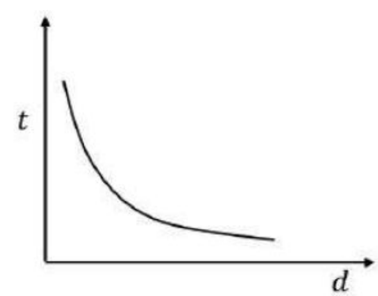
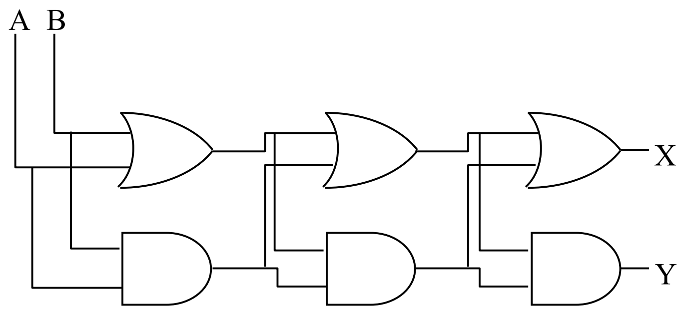


Topic- Electricity and Magnetism

Sub Topic- Magnetostatic

Ans. : (C)

Q.12 Two digital inputs A and B are given to the following circuit. For $A = 1$, $B = 0$, the values of X and Y are:



(A) $X = 1, Y = 0$ (B) $X = 0, Y = 1$ (C) $X = 0, Y = 0$ (D) $X = 1, Y = 1$

Topic-Electronics

Sub Topic- Digital Electronics

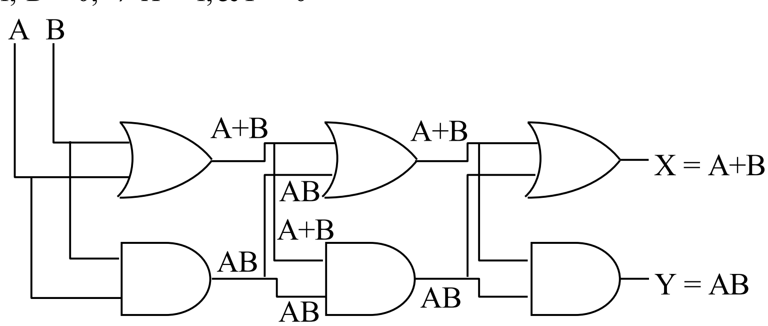
Ans. : (B)

Solution: Rule of binary algebra

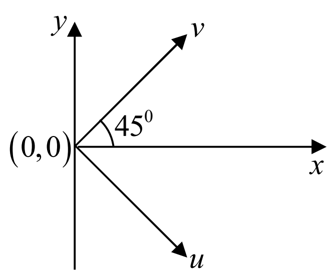
(a) $A + B + AB = A + B(1 + A) = A + B$ & $(A + B)AB = AB + AB = AB$

By applying this, we will get $X = A + B$ & $Y = AB$

When $A = 1,\ B = 0, \Rightarrow X = 1, \& Y = 0$



Q.13 The Jacobian matrix for transforming from (x, y) to another orthogonal coordinates system (u, v) as shown in the figure is



(A) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (B) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ (C) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ (D) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

Topic- Mathematical Physics

Sub Topic- Matrices

Ans. : (C)

Solution: $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} u/\sqrt{2}+v/\sqrt{2} \\ -u/\sqrt{2}+v/\sqrt{2} \end{bmatrix} = \begin{bmatrix} \frac{au+av}{\sqrt{2}}+\frac{-bu+bv}{\sqrt{2}} \\ \frac{cu+cv}{\sqrt{2}}+\frac{-du+dv}{\sqrt{2}} \end{bmatrix}$

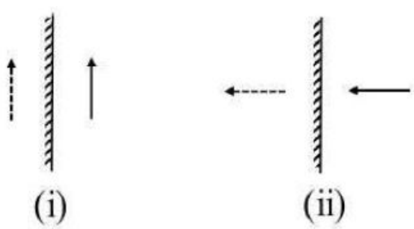
$$u = \frac{(a-b)u}{\sqrt{2}} + \left(\frac{a+b}{\sqrt{2}}\right)v \;\&\; v = \frac{(c-d)u}{\sqrt{2}} + \frac{(c+d)v}{\sqrt{2}}$$

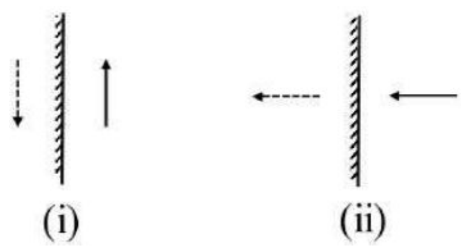
$$\frac{a-b}{\sqrt{2}} = 1,\ \frac{a+b}{\sqrt{2}} = 0,\ \frac{c-d}{\sqrt{2}} = 0,\ \frac{c+d}{\sqrt{2}} = 1$$

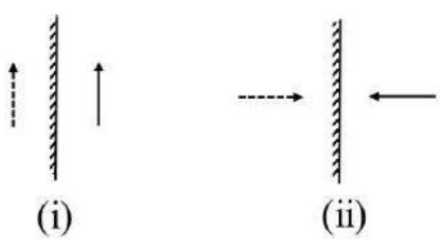
$$\frac{2a}{\sqrt{2}} = 1,\ a = \frac{1}{\sqrt{2}}, \frac{-2b}{\sqrt{2}} = 1, b = -\frac{1}{\sqrt{2}}, c = \frac{1}{\sqrt{2}}, \frac{-2d}{\sqrt{2}} = -1$$

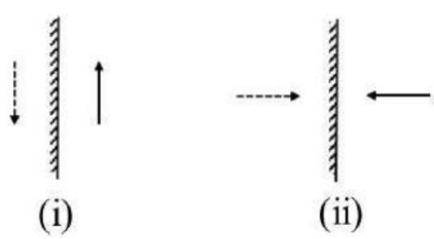
Thus, $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

Q.14 A rotating disc is held in front of a plane mirror in two different orientations which are (i) angular momentum parallel to the mirror and (ii) angular momentum perpendicular to the mirror. Which of the following schematic figures correctly describes the angular momentum (solid arrow) and its mirror image (shown by dashed arrows) in the two orientations?

(A) 

(B) 

(C) 

(D) 

Topic- Oscillation, Waves and Optics

Sub Topic- Ray Optics

Ans. : (B)

Solution: From the concept of mirror image and also we have to apply velocity direction of image with respect to mirror $U_O - U_M = V_I - V_O$

Q.15 Inverse of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(A) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & -1 & 0 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & -2 & -3 \\ -2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Topic- Mathematical Physics

Sub Topic- Matrices

Ans. : (B)

Solution: We know $A^{-1} = \frac{AdjA}{DetA}$. Given $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

For this matrix $DetA = 1$ and $AdjA = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$

Thus, $A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$

Q.16 Suppose the divergence of magnetic field $\vec{B}$ is nonzero and is given as $\vec{\nabla} \cdot \vec{B} = \mu_0 \rho_m$, where μ_0 is the permeability of vacuum and ρ_m is the magnetic charge density. If the corresponding magnetic current density is $\vec{J}_m$, then the curl $\vec{\nabla} \times \vec{E}$ of the electric field $\vec{E}$ is

(A) $\vec{J}_m - \frac{\partial \vec{B}}{\partial t}$ (B) $\mu_0 \vec{J}_m - \frac{\partial \vec{B}}{\partial t}$ (C) $-\vec{J}_m - \frac{\partial \vec{B}}{\partial t}$ (D) $-\mu_0 \vec{J}_m - \frac{\partial \vec{B}}{\partial t}$

Topic- Electricity and Magnetism

Sub Topic- Maxwell's Equation

Ans. : (D)

Solution: As we know $\nabla \cdot (\nabla \times E) = 0$ Given $\vec{\nabla} \cdot \vec{B} = \mu_0 \rho_m$

We know $\nabla \cdot E = \frac{\rho}{\epsilon_0} \& \nabla \times E = -\frac{\partial B}{\partial t}$

We know $\nabla \times E = -\mu_0 j_m - \frac{\partial B}{\partial t}$

$$\nabla\cdot(\nabla\times E)=-\mu_0\nabla\cdot j_m-\frac{\partial}{\partial t}\nabla\cdot B=\nabla.\left(-\mu_0 j_m-\frac{\partial B}{\partial t}\right)$$

Thus, $(\nabla\times E)=\left(-\mu_0 j_m-\frac{\partial B}{\partial t}\right)$

Q.17 For a thermodynamic system, the coefficient of volume expansion $\beta=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$ and compressibility $K=-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$, where V, T, and P are respectively the volume, temperature, and pressure. Considering that $\frac{dV}{V}$ is a perfect differential, we get

(A) $\left(\frac{\partial \beta}{\partial P}\right)_T=\left(\frac{\partial K}{\partial T}\right)_P$ (B) $\left(\frac{\partial \beta}{\partial T}\right)_P=-\left(\frac{\partial K}{\partial P}\right)_T$

(C) $\left(\frac{\partial \beta}{\partial P}\right)_T=-\left(\frac{\partial K}{\partial T}\right)_P$ (D) $\left(\frac{\partial \beta}{\partial T}\right)_P=\left(\frac{\partial K}{\partial P}\right)_T$

Topic- Thermodynamics and Statistical Mechanics

Sub Topic- Thermodynamic Process

Ans. : (C)

Solution: $\beta=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P=\left(\frac{\partial \beta}{\partial P}\right)_P=-\frac{1}{V^2}\left(\frac{\partial V}{\partial T}\right)_P\left(\frac{\partial V}{\partial T}\right)_P$

$$k=-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T,\left(\frac{\partial k}{\partial P}\right)_T=\frac{1}{V^2}\left(\frac{\partial V}{\partial P}\right)_T\left(\frac{\partial V}{\partial P}\right)_T$$

Q.18 A linearly polarized light of wavelength 590 nm is incident normally on the surface of a $20\,\mu m$ thick quartz film. The plane of polarization makes an angle 30^0 with the optic axis. Refractive indices of ordinary and extraordinary waves differ by 0.0091, resulting in a phase difference of $f\pi$ between them after transmission. The value of f (rounded off to two decimal places) and the state of polarization of the transmitted light is

(A) 0.62 and linear (B) 0.62 and elliptical

(C) – 0.38 and elliptical (D) 0.5 and circular

Topic- Oscillation, Waves and Optics

Sub Topic- Polarisation

Ans. : (B)

Solution: From the given information we can say that the amplitude of component of O-Ray $E_y=E_o Sin(30)=\frac{E_o}{2}$. And the amplitude of component of E-Ray $E_x=E_o\cos(30)=\sqrt{3}\frac{E_o}{2}$

So, the both amplitudes are different. Hence, it will be elliptical if the phase differences between emerging rays are not equal to $0, \pi, 2\pi .....$ so on.

The optical path difference $(n_o - n_E)t = 0.0091 \times 20 \times 10^{-6}$.

Phase difference $\delta = \frac{2\pi}{\lambda}(n_o - n_E)t = \frac{2\pi}{590 \times 10^{-9}} 0.0091 \times 20 \times 10^{-6} = 0.62\pi$.

Obviously, the phase difference is not integer multiple of π thus it will be elliptical with $\delta = 0.62\pi$

Q.19 The phase velocity v_p of transverse waves on a one-dimensional crystal of atomic separation d is related to the wavevector k as

$$v_p = C\frac{\sin(kd/2)}{(kd/2)}$$

The group velocity of these waves is

(A) $C\left[\cos(kd/2) - \frac{\sin(kd/2)}{(kd/2)}\right]$ (B) $C\cos(kd/2)$

(C) $C\left[\cos(kd/2) + \frac{\sin(kd/2)}{(kd/2)}\right]$ (D) $C\frac{\sin(kd/2)}{(kd/2)}$

Topic- Oscillation, Waves and Optics

Sub Topic- Wave Packet

Ans. : (B)

Solution: We know that $v_g = \frac{d\omega}{dk}$, Where $\omega = v_p k$

Given that $v_p = C\frac{\sin(kd/2)}{kd/2}$. Thus, $\omega = \frac{k}{(kd/2)} C\sin(kd/2)$

$$v_g = \frac{d\omega}{dk} = \frac{kd/2}{(kd/2)} C\cos(kd/2) = C\cos(kd/2)$$

Q.20 In a dielectric medium of relative permittivity 5, the amplitudes of the displacement current and conduction current are equal for an applied sinusoidal voltage of frequency $f = 1\,MHz$. The value of conductivity (in $\Omega^{-1}m^{-1}$) of the medium at this frequency is

(A) 2.78×10^{-4} (B) 2.44×10^{-4} (C) 2.78×10^{-3} (D) 2.44×10^{-3}

Topic- Electricity and Magnetism

Sub Topic- Concept of Displacement Current

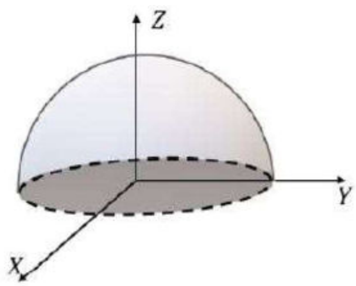
Ans. : (A)

Solution: $V = V_0 \sin 2\pi \times 10^6 t$

$$J_d = 5t_0 \frac{\partial E}{\partial t} = \frac{5t_0}{d} V_0 \times 2\pi \times 10^6 \cos(\omega t) = \sigma \frac{V_0}{d} \sin 2\pi \times 10^6 t = 5 \times 8.85 \times 10^{-12} \times 2 \times 5 \times 10^6$$

$$= 277.89 \times 10^{-6} = 2.7789 \times 10^{-4}$$

Q.21 For a given vector $F = -y\hat{i} + z\hat{j} + x^2\hat{k}$, the surface integral $\int_S \left(\vec{\nabla} \times \vec{F}\right) \cdot \hat{r} dS$ over the surface S of a hemisphere of radius R with the centre of the base at the origin is



(A) πR^2 (B) $\frac{2\pi R^2}{3}$ (C) $-\pi R^2$ (D) $-\frac{2\pi R^2}{3}$

Topic- Mathematical Physics

Sub Topic- Vector Algebra

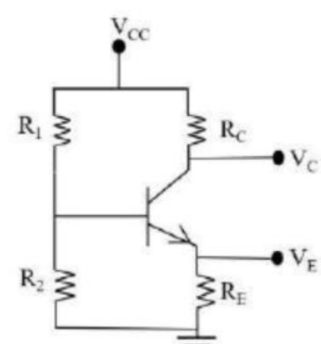
Ans. : (A)

Solution: Given that, $\vec{F} = -y\hat{\imath} + z\hat{\jmath} + x^2\hat{k}$ $\int_s \left(\nabla \times F\right) \cdot \hat{r} ds = \int_l \vec{F} \cdot d\vec{l}$

$$= \int \left(-y\hat{i} + z\hat{j} + x^2\hat{k}\right) \cdot \left(dx\hat{i} + dy\hat{j}\right) == -\int ydx\Big|_{z=0} = -\int R\sin\theta R\left(-\sin\theta\right) d\omega == +R^2 \int_0^{2\pi} \sin^2\theta d\omega = \pi R^2$$

Q.22 In the circuit shown, assuming the current gain $\beta = 100$ and $V_{BE} = 0.7V$, what will be the collector voltage V_C in V ?

Given: $V_{CC} = 15V$, $R_1 = 100\,k\Omega, R_2 = 50\,k\Omega, R_C = 4.7\,k\Omega$ and $R_E = 3.3\,k\Omega$



(A) 8.9 (B) 5.1

(C) 4.3 (D) 3.2

Topic- Electronics

Sub Topic- Transistor

Ans. : **(Answer not matched)**

Solution: Given $\beta = 100$ and $V_{BE} = 0.7$ V $V_{CC} = 15$ V, $R_1 = 100$kΩ, $R_2 = 50$kΩ, $R_C = 4.7$kΩ, and $R_E = 3.3$kΩ

$$V_{BE} = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{50 \times 10^3}{150 \times 10^3} 15 = 5 Volt, \; V_E = V_{BE} - V_B = 5 - 0.7 = 4.3 \; Volt$$

$$i_c \approx i_E = \frac{V_E}{R_E} = \frac{4.3}{3.3 \times 10^3} = 1.3 mA \; \& \; V_c = V_{cc} - I_c R_c = 15 - 1.3 \times 4.7 = 8.9 Volt$$

Q.23 A uniform stick of length l and mass m pivoted at its top end is oscillating with an angular frequency ω_r. Assuming small oscillations, the ratio ω_r / ω_s, where ω_s is the angular frequency of a simple pendulum of the same length, will be

(A) $\sqrt{3}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{3}}$

Topic- Oscillation, Waves and Optics

Sub Topic- Oscillation

Ans. : (B)

Solution: We know that the angular frequency of stick $\omega_r = \sqrt{\frac{mgl'}{I}}$

Where l' is the distance from the centre of gravity to point of suspension (end point here) and I is the moment of Inertia about the point of suspension.

From the given information we can say $l' = \frac{l}{2}$ and $I = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$

$$\omega_r = \sqrt{\frac{mg\frac{l}{2}}{\frac{ml^2}{3}}} = \sqrt{\frac{3g}{2l}} = \sqrt{\frac{3}{2}}\omega_s \Rightarrow \frac{\omega_r}{\omega_s} = \sqrt{\frac{3}{2}}$$

Q.24 An oil film in air of thickness 255 nm is illuminated by white light at normal incidence. As a consequence of interference, which colour will be predominantly visible in the reflected light? Given the refractive index of oil = 1.47

(A) Red (~650 nm) (B) Blue (~450 nm)

(C) Green (~500 nm) (D) Yellow (~560 nm)

Topic- Oscillation, Waves and Optics

Sub Topic- Thin film interference

Ans. : (C)

Solution: Condition for constructive interference in thin film

$$2\mu t \cos(r) = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} .....$$

For normal incident $r = 0^0$ And we have to compare here with path difference $\frac{3\lambda}{2}$.

Thus, $2\mu t = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{4\mu t}{3} = \frac{4\times 255\times 1.47}{3} \approx 500\, nm \to Green$

Q.25 Water from a tank is flowing down through a hole at its bottom with velocity $5\,ms^{-1}$. If this water falls on a flat surface kept below the hole at a distance of $0.1m$ and spreads horizontally, the pressure (in kNm^{-2}) exerted on the flat surface is closest to

Given: acceleration due to gravity $= 9.8\,ms^{-2}$ and density of water $= 1000kgm^{-3}$

(A) 13.5 (B) 27.0 (C) 17.6 (D) 6.8

Topic- Mechanics

Sub Topic- Fluid Mechanics

Ans. : (B)

Solution: Mass flow rate = Density x Velocity x Area

Momentum flow rate = Force = Density x Velocity^2 x Area

Pressure $= Density \times Velocity^2$

Now, $Velocity^2 = 5^2 + 2gh = 5^2 + 2\times 10\times 0.1 = 27$

Pressure $= Density \times Velocity^2 = 1000\times 27$

Q.26 At the planar interface of two dielectrics, which of the following statements related to the electric field ($\vec{E}$), electric displacement ($\vec{D}$) and polarization ($\vec{P}$) is true?

(A) Normal component of both $\vec{D}$ and $\vec{P}$ are continuous

(B) Normal component of both $\vec{D}$ and $\vec{E}$ are discontinuous

(C) Normal component of $\vec{D}$ is continuous and that of $\vec{P}$ is discontinuous

(D) Normal component of both $\vec{E}$ and $\vec{P}$ are continuous

Topic- Electricity and Magnetism

Sub Topic- Boundary Condition

Ans. : (C)

Solution: $E_1^a - E_1^b = \frac{\sigma}{\epsilon_0}$ and Boundary condition $E_{11}^a - E_{11}^b = 0$, $D_1^a - D_1^b = \sigma_f$, $D_a^{11} - D_b^{11} = P_a^{11} - P_b^{11}$,

$D_a^1 - D_b^1 = 0$, $P_a^1 - P_b^1 \neq 0$

Q.27 Consider a system of large number of particles that can be in three energy states with energies $0\ meV$, $1\ meV$, and $2\ meV$. At temperature $T = 300\ K$, the mean energy of the system (in meV) is closest to

Given: Boltzmann constant $k_B = 0.086\ meVK^{-1}$

(A) 0.12 (B) 0.97 (C) 1.32 (D) 1.82

Topic- Kinetic Theory and Thermodynamics

Sub Topic- Partition Function

Ans.: (B)

Solution: $z = e^{-\beta\cdot 0} + e^{-\beta\cdot 1} + e^{-\beta\cdot 2} = 1 + e^{-\beta} + e^{-2\beta}$

Mean energy $U = -\frac{1}{z}\frac{\partial z}{\partial \beta} = -\frac{1}{1+e^{-\beta}+e^{-2\beta}}\left[0 - 1e^{-\beta} - 2e^{-2\beta}\right] = \frac{e^{-\beta}+2e^{-2\beta}}{1+e^{-\beta}+e^{-2\beta}} = 0.97$

Q.28 For the Maxwell-Boltzmann speed distribution, the ratio of the root-mean-square speed (v_{rms}) and the most probable speed ($v_{\max}$) is

Given: Maxwell-Boltzmann speed distribution function for a collection of particles of mass m is

$$f(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2k_B T}\right)$$

where, v is the speed and $k_B T$ is the thermal energy.

(A) $\sqrt{\frac{3}{2}}$ (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

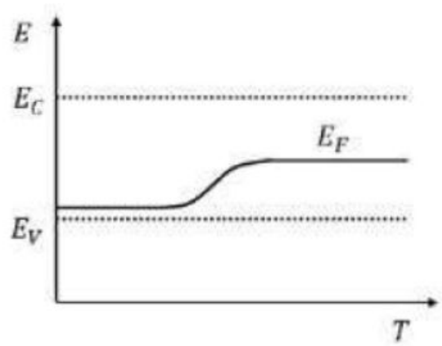
Topic- Kinetic Theory and Thermodynamics

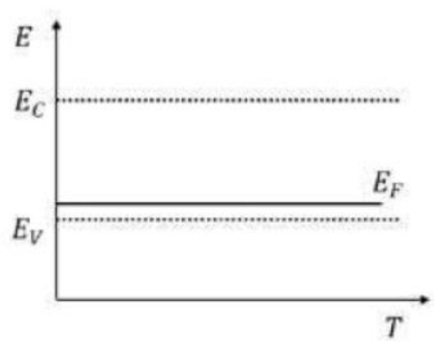
Sub Topic- Maxwell Boltzmann Distribution

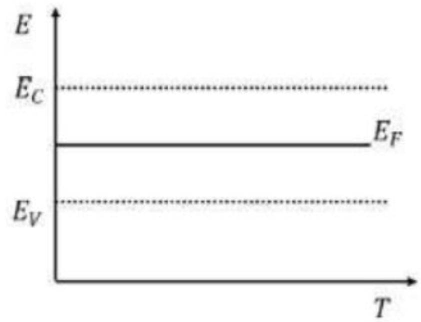
Ans. : (A)

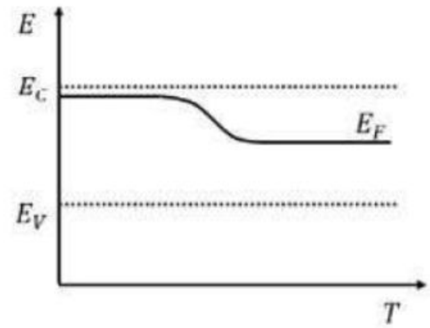
Solution: $V_{rms} = \sqrt{\frac{3k_B T}{m}}, V_{mf} = \sqrt{\frac{2k_B T}{m}} \Rightarrow \frac{V_{rms}}{V_{mf}} = \sqrt{\frac{3}{2}}$

Q.29 In an extrinsic p-type semiconductor, which of the following schematic diagram depicts the variation of the Fermi energy level (E_F) with temperature (T)?

(A)


(B)


(C) 

(D) 

Topic- Electronics

Sub Topic- Semiconductor Physics

Ans. : (A)

Solution: The Fermi level of P-type semiconductor can be written as follows

$$E_{FP} = E_F - k_B T \ln(\frac{N_A}{n_i})$$

As we know that then Fermi level at normal temperature is close to the valance band. However, if we increase the temperature beyond the range, the intrinsic carrier concentration n_i will in crease . As it becomes $n_i = N_A$ then $E_{FP} = E_F$. Thus, correct representation for Fermi level for P-type is shown in figure A.

Q.30 A container is occupied by a fixed number of non-interacting particles. If they are obeying Fermi-Dirac, Bose-Einstein, and Maxwell-Boltzmann statistics, the pressure in the container is P_{FD}, P_{BE} and P_{MB}, respectively. Then

(A) $P_{FD} > P_{MB} > P_{BE}$ (B) $P_{FD} > P_{MB} = P_{BE}$

(C) $P_{FD} > P_{BE} > P_{MB}$ (D) $P_{FD} = P_{MB} = P_{BE}$

Topic- Kinetic Theory and Thermodynamics

Sub Topic- Statistical Distribution

Ans. : (A)

Solution: In Fermi gas $PV = NkT\left[1 + \frac{\pi^{3/2}}{2}\frac{\rho\hbar^3}{(mkT)^{3/2}}\right]$

In Bose gas $PV = NkT\left[1 - \frac{\pi^{3/2}}{2}\frac{\rho\hbar^3}{(mkT)^{3/2}}\right]$

In MB gas $PV = NkT$. Thus, $P_{BE} < P_{MB} < P_{F\cdot D}$

Section A: Q.31-Q.40 Carry TWO marks each.

Q.31 The spectral energy density $u_T(\lambda)$ vs wavelength (λ) curve of a black body shows a peak at $\lambda = \lambda_{\max}$. If the temperature of the black body is doubled, then

(A) the maximum of $u_T(\lambda)$ shifts to $\lambda_{\max}/2$

(B) the maximum of $u_T(\lambda)$ shifts to $2\lambda_{\max}$

(C) the area under the curve becomes 16 times the original area

(D) the area under the curve becomes 8 times the original area

Topic- Kinetic Theory and Thermodynamics

Sub Topic- Black Body Radiation

Ans. : (A and C)

Solution: $u = \sigma T^4$ as $T \to 2T$ $u' = \sigma 16 T^4 \Rightarrow \frac{u'}{u} = 16$

$$\lambda_m T = b \quad \text{as} \quad \lambda'_m 2T = b \Rightarrow \frac{\lambda'_m 2}{\lambda_m} = 1 \Rightarrow \lambda'_m = \frac{\lambda_m}{2}$$

Q.32 A periodic function $f(x) = x^2$ for $-\pi < x < \pi$ is expanded in a Fourier series. Which of the following statement(s) is/are correct?

(A) Coefficients of all the sine terms are zero

(B) The first term in the series is $\frac{\pi^2}{3}$

(C) The second term in the series is $-4\cos x$

(D) Coefficients of all the cosine terms are zero

Topic- Mathematical Physics

Sub Topic- Fourier Series

Ans. : (A, B, C)

Solution: $f(x) = x^2, -\pi < x < \pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos x$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{2}{\pi}\int_0^{\pi} x^2 dx = \frac{1}{\pi} \times \frac{\pi^3}{3}\pi^2 = \frac{2\pi^2}{3} \Rightarrow \frac{a_0}{2} = \frac{\pi^2}{3}$$

Second term

$$a_1 = \frac{2}{\pi}\left[x^2(\lambda x) - 2\times(-ax) + 2(-\lambda x)\right] = \frac{2}{\pi}\times 2\times(\cos\pi\times\pi) = -4$$

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos x dx = \frac{2}{\pi}\left[x^2(\sin x) - 1(-\cos x)\right] = -4$$

Since, the function is Even. Thus, sine term will be zero.

Since, the function is Even. Thus, sine term will be non-zero.

Q.33 The state of a harmonic oscillator is given as $\psi = \frac{1}{\sqrt{3}}\psi_0 - \frac{1}{\sqrt{6}}\psi_1 + \frac{1}{\sqrt{2}}\psi_2$, where ψ_0, ψ_1 and ψ_2 are the normalized wave functions of ground, first excited, and second excited states, respectively. Which of the following statement(s) is/are true?

(A) A measurement of the energy of the system yields $E = \frac{1}{2}\hbar\omega$ with non-zero probability

(B) A measurement of the energy of the system yields $E = \frac{5}{3}\hbar\omega$ with non-zero probability

(C) Expectation value of the energy of the system $\langle E\rangle = \frac{5}{3}\hbar\omega$

(D) Expectation value of the energy of the system $\langle E\rangle = \frac{7}{6}\hbar\omega$

Topic- Modern Physics

Sub Topic- Harmonic Oscillator

Ans. : (A, & C)

Solution:

Given $\psi = \frac{1}{\sqrt{3}}\psi_0 - \frac{1}{\sqrt{6}}\psi_1 + \frac{1}{\sqrt{2}}\psi_2$

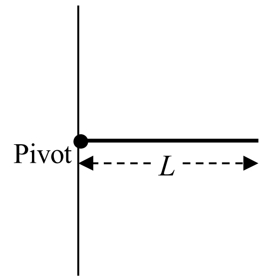
Total probability $\frac{1}{3} + \frac{1}{6} + \frac{1}{2} = \frac{2+1+3}{6} = \frac{6}{6} = 1$

For harmonic oscillator $V(x) = \frac{1m\omega^2 x^2}{2}$ and $E_n = (n + 1/2)h\omega$

Probability of a specific state $P\left(\frac{\hbar\omega}{2}\right) = \frac{1}{3}$ $P\left(\frac{3}{2}\hbar\omega\right) = \frac{1}{6}$ $P\left(\frac{5}{2}\hbar\omega\right) = \frac{1}{2}$

$$\langle E\rangle = \sum E_n P(E_n) = \frac{\hbar\omega}{2}\times\frac{1}{3} + \frac{3}{2}\hbar\omega\times\frac{1}{6} + \frac{5}{2}\hbar\omega\times\frac{1}{2} = \frac{5}{3}\hbar\omega$$

Q.34 A rod of mass M, length L and non-uniform mass per unit length $\lambda(x)=\dfrac{3Mx^2}{L^3}$, is held horizontally by a pivot, as shown in the figure, and is free to move in the plane of the figure. For this rod, which of the following statements are true?



(A) Moment of inertia of the rod about an axis passing through the pivot is $\dfrac{3}{5}ML^2$

(B) Moment of inertia of the rod about an axis passing through the pivot is $\dfrac{1}{3}ML^2$

(C) Torque on the rod about the pivot is $\dfrac{3}{4}MgL$

(D) If the rod is released, the point at a distance $\dfrac{2L}{3}$ from the pivot will fall with acceleration g

Topic- Mechanics

Sub Topic- Moment of Inertia

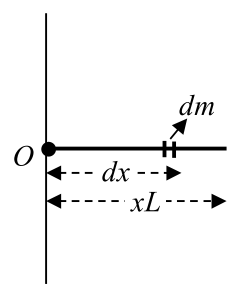
Ans. : (A, & C)

Solution: Given $\lambda = \frac{3Mx^2}{L^2}$

We know $I_0=\int dm\times x^2,\ \dfrac{dm}{dx}=\lambda \Rightarrow dm=\lambda dx \Rightarrow I_0=\dfrac{3m}{L^2}\int_o^L x^4 dx$

$$=\frac{3M}{L^2}\times\frac{L^5}{5}=\frac{3ML^2}{5}$$

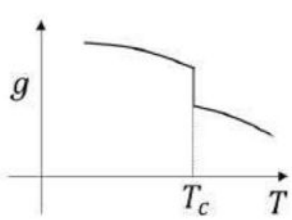
$$x_{cm}=\frac{\int_0^L x\,dm}{\int_0^L dm}=\frac{\frac{3M}{L^2}\int_0^L x\cdot x^2 dx}{\frac{3M}{L^2}\int_0^L x^2 dx}=\frac{3}{4}L$$



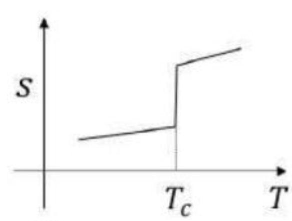
Torque, $\vec{\tau}=\vec{r}\times\vec{F}=\dfrac{3L}{4}\times \text{Mgsin}\,90=\dfrac{3}{4}MgL$

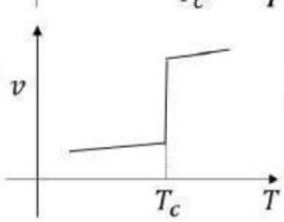
Q.35 Which of the following schematic plots correctly represent(s) a first order phase transition occurring at temperature $T=T_C$? Here g, s, v are specific Gibbs free energy, entropy and volume, respectively.

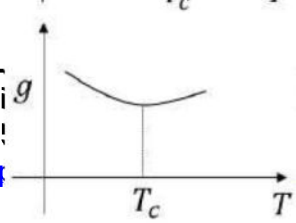
(A)



(B)







(C) (D)

Topic- Kinetic Theory and Thermodynamics

Sub Topic- Gibbs Free Energy

Ans. : (B, and C)

Solution: We know that the Gibbs free energy can be written as $dG = Vdp - Sdt$

In first order, first derivative of Gibbs free energy will be discontinuous.

Thus, correct plot will be described in B, C, and D

Q.36 A particle (p_1) of mass m moving with speed v collides with a stationary identical particle (p_2). The particles bounce off each other elastically with p_1 getting deflected by an angle $\theta = 30^0$ from its original direction. Then, which of the following statement(s) is/are true after the collision?

(A) Speed of p_1 is $\frac{\sqrt{3}}{2}v$

(B) Kinetic energy of p_2 is 25% of the total energy

(C) Angle between the directions of motion of the two particles is 90^0

(D) The kinetic energy of the centre of mass of p_1 and p_2 decreases

Topic- Mechanics

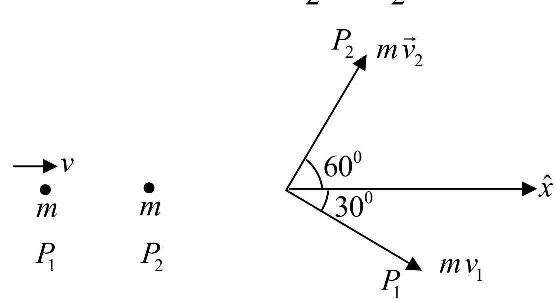
Sub Topic- Collision Kinematics

Ans. : (A,B,C)

Solution: The angle between the direction of motion of two particles are 90^0

The conservation of linear momentum along x (chosen)

$$mv + 0 = mv_1\cos(30) + mv_2\cos(60) \Rightarrow v = v_1\frac{\sqrt{3}}{2} + v_2\frac{1}{2} .....(1)$$



The conservation of linear momentum along y (chosen)

$$0+0=-mv_1\sin(30)+mv_2\sin(60)\Rightarrow v_2=\frac{v_1}{\sqrt{3}}....(2)$$

Now, $v=v_1\frac{\sqrt{3}}{2}+v_2\frac{1}{2}=\frac{v_1}{2}\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)=\frac{2v_1}{\sqrt{3}}\Rightarrow v_1=\frac{1}{2}\sqrt{3}v$

Now, from equation 2 $v_2=\frac{v_1}{\sqrt{3}}\Rightarrow v_2=\frac{1}{\sqrt{3}}\frac{1}{2}\sqrt{3}v=\frac{1}{2}v$

$$T_2=\frac{1}{2}mv^2{}_2=\frac{1}{4}\frac{1}{2}mv^2=\frac{1}{4}T=25\%\,of\,T$$

Q.37 A wave travelling along the x-axis with y representing its displacement is described by (v is the speed of the wave)

(A) $\frac{\partial y}{\partial x}+\frac{1}{v}\frac{\partial y}{\partial t}=0$ (B) $\frac{\partial y}{\partial x}-\frac{1}{v}\frac{\partial y}{\partial t}=0$

(C) $\frac{\partial^2 y}{\partial x^2}+\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}=0$ (D) $\frac{\partial^2 y}{\partial x^2}-\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}=0$

Topic- Oscillation, Waves and Optics

Sub Topic- Wave Propagation

Ans. : (A, B, D)

Solution: The differential form of wave equation $\frac{\partial^2 y}{\partial t^2}=v^2\frac{\partial^2 y}{\partial x^2}$........(1)

Its solution will be $y=A\sin(\omega t-kx)$.......(2) & $y=A\sin(\omega t+kx)$ ....(3)

From first equation we can say,

$$\frac{\partial^2 y}{\partial x^2}-\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}=0$$

From second equation,

$$\frac{dy}{dt}=A\omega\cos(\omega t-kx)\ ,\ \frac{dy}{dx}=-Ak\cos(\omega t-kx)\Rightarrow\frac{dy}{dt}=-\frac{\omega}{k}\times Ak\cos(\omega t-kx)=-\frac{\omega}{k}\frac{dy}{dx}$$

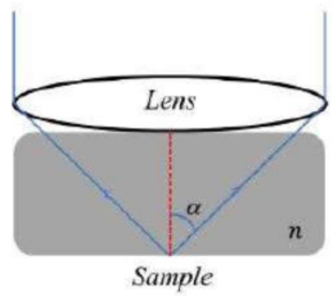
$$\Rightarrow\frac{dy}{dt}+v\frac{dy}{dx}=0\Rightarrow\frac{dy}{vdt}+\frac{dy}{dx}=0$$

From third equation,

$$\frac{dy}{dt}=A\omega\cos(\omega t+kx)\ ,\ \frac{dy}{dx}=Ak\cos(\omega t+kx)\Rightarrow\frac{dy}{dt}=\frac{\omega}{k}\times Ak\cos(\omega t+kx)=\frac{\omega}{k}\frac{dy}{dx}$$

$$\Rightarrow\frac{dy}{dt}-v\frac{dy}{dx}=0\Rightarrow\Rightarrow\frac{dy}{vdt}-\frac{dy}{dx}=0$$

Q.38 An objective lens with half angular aperture α is illuminated with light of wavelength λ. The refractive index of the medium between the sample and the objective is n. The lateral resolving power of the optical system can be increased by



(A) decreasing both λ and α

(B) decreasing λ and increasing α

(C) increasing both α and n

(D) decreasing λ and increasing n

Topic- Oscillation, Waves and Optics

Sub Topic- Microscope

Ans. : (B, C, D)

Solution: R.P. $R.P. = \dfrac{2\mu \sin(\alpha)}{\lambda}$ Thus, R.P. decreasing λ and increasing α

It is increasing both α and n . It is decreasing λ and increasing n

Q.39 Which of the following statement(s) is/are true for a LC circuit with $l = 25\,mH$ and $C = 4\,\mu F$?

(A) Resonance frequency is close to $503\,Hz$

(B) The impedance at $1\,kHz$ is 15Ω

(C) At a frequency of $200\,Hz$, the voltage lags the current in the circuit

(D) At a frequency of $700\,Hz$, the voltage lags the current in the circuit

Topic- Electronics

Sub Topic-LC Circuit

Ans. : (A, C) or (A, D)

Solution: $c = 4\mu\text{F}$

$$\text{Resonance frequency } (\omega) = \frac{1}{\sqrt{LC}} \Rightarrow f_R = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{25\times10^{-3}\times4\times10^{-6}}} = 503$$

Thus, Option A is correct

Since, $f_R = 503$ Hz

So, $f > f_R$ current lags the voltage & $f < f_R$ current leads the voltage.

option C is correct because at 200 Hz voltage lags the current. But, option D is not correct becz at 700H current lags the voltage.

Impedance $(z) = j\left(\omega L - \frac{1}{\omega c}\right)$ and $\omega = 2\pi f = 2000\pi$

$$z = j\left(2000\pi \times 25 \times 10^{-3} - \frac{1}{2000 \times \pi \times 4 \times 10^{-6}}\right) = j(117)$$

Hence, Option, A, C are correct.

Q.40 For a particle moving in a general central force field, which of the following statement(s) is/are true?

(A) The angular momentum is a constant of motion

(B) Kepler's second law is valid

(C) The motion is confined to a plane

(D) Kepler's third law is valid

Topic- Mechanics

Sub Topic-Central Force Problem

Ans. : (A, B, C)

Solution: The angular momentum is a constant of motion -This statement is always true for central force problem.

Kepler's second law is valid - This statement is always true because $\frac{dA}{dt} = \text{constant}$

Motion is confined to a plane: This statement is always true

Kepler's third law is valid-This statement is not always true. In third law we have $T^2 = a^3$ which is true for potential like $v(r) = \frac{-k}{r}$

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 The lattice constant (in $\overset{\circ}{A}$) of copper, which has FCC structure, is _________ (rounded off to two decimal places).

Given: density of copper is $8.91\, g\, cm^{-3}$ and its atomic mass is $63.55\, g\, mol^{-1}$; Avogadro's number $= 6.023 \times 10^{23}\, mol^{-1}$

Topic- Solid State Physics

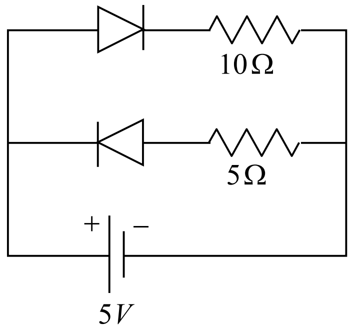
Sub Topic-Crystal Structure

Ans. : 3.6

Solution: $\rho = 8.91\, g/cm^3$, $M_{cu} = 63.5\ g/mol$, $N_A = 6.023 \times 10^{23}$. We know $\rho = \frac{Mass}{volume}$

For FCC $a = \left(\frac{4 \times 63.5}{6.023 \times 10^{23} \times 8.91}\right)^{1/3} = 3.6\, A^0$

Q.42 Two silicon diodes are connected to a battery and two resistors as shown in the figure. The current through the battery is ___________ A (rounded off to two decimal places).



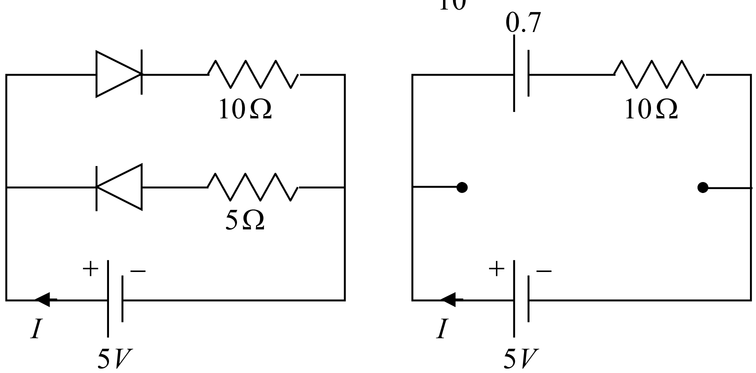
Given: The forward voltage drop across each diode = 0.7 V

Topic- Electronics

Sub Topic-Diode

Ans. : 0.43

Solution: By applying KVL law $-5+0.7+10I=0 \Rightarrow I=\frac{4.3}{10}=0.43\,amp$



Q.43 The absolute error in the value of sinθ if approximated up to two terms in the Taylor's series for $\theta=60^0$ is ____________ (rounded off to three decimal places).

Topic- Mathematical Physics

Sub Topic-Taylor Series

Ans. : 0.011

Solution: $\frac{\sin\pi}{3}=\frac{\sqrt{3}}{2}=0.866$

In Traylor series $\sin\left(\frac{\pi}{3}\right)=\frac{\pi}{3}-\frac{\pi^3}{3\times2\times27}=0.855$

Error $\Delta=0.866-0.855=0.011$

Q.44 A single pendulum hanging vertically in an elevator has a time period T_0 when the elevator is stationary. If the elevator moves upward with an acceleration of $a = 0.2\,g$, the time period of oscillations is T_1. Here g is the acceleration due to gravity. The ratio $\frac{T_0}{T_1}$ is ____________ (rounded off to two decimal places).

Topic- Oscillation, Waves and Optics

Sub Topic- Oscillation

Ans. : 1.09

Solution: $T = 2\pi\sqrt{\frac{l}{g_{eff}}} \Rightarrow T_0 = 2\pi\sqrt{\frac{l}{g}}$

$$T_1 = 2\pi\sqrt{\frac{l}{g+0.2g}} = 2\pi\sqrt{\frac{l}{1.2g}} \Rightarrow \frac{T_0}{T_1} = \sqrt{1.2} = 1.09$$

Q.45 A spacecraft has speed $v_s = fc$ with respect to the earth, where c is the speed of light in vacuum. An observer in the spacecraft measures the time of one complete rotation of the earth to be 48 hours. The value of f is __________ (rounded off to two decimal places).

Topic- Modern Physics

Sub Topic- STR

Ans. : 0.85

Solution: From the concept of time dilation

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 48 = \frac{24}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2} \Rightarrow 1-\frac{v^2}{c^2} = \frac{1}{4} \Rightarrow v = \frac{\sqrt{3}}{2}c = 0.85c$$

Q.46 The sum of the *x*-components of unit vectors $\dot{r}$ and $\dot{\theta}$ for a particle moving with angular speed 2 rad s^{-1} at angle $\theta = 215^0$ is__________ (rounded off to two decimal places)

Topic- Mechanics

Sub Topic- Motion of a Particle in Different Coordinate

Ans. : 2.7

Solution: $r\hat{i} + \theta\hat{i}$. Given that $\theta = 215$ $and\ \dot{\theta} = 2\,rad/s \Rightarrow \hat{r} = \cos(\theta)\hat{i} + \sin(\theta)\hat{j}$

$$\hat{\theta} = -\sin(\theta)\hat{i} + \cos(\theta)\hat{j}$$

$$\frac{d\hat{r}}{dt}\hat{\imath} = \dot{\theta}(-\sin\theta)\ \&\ \frac{d\hat{\theta}}{dt}\hat{\imath} = -\dot{\theta}(\cos\theta)$$

$$\frac{d\hat{r}}{dt}\hat{\imath}+\frac{d\hat{\theta}}{dt}\hat{\imath}=-\dot{\theta}(\sin\,\theta+\text{con}\,\theta)$$

At $\theta = 215\ and\ \dot{\theta} = 2\,rad/s$

$$\frac{d\hat{r}}{dt}\hat{\imath}+\frac{d\hat{\theta}}{dt}\hat{\imath}=-\dot{\theta}(\sin\theta+\text{con}\,\theta)=2.7$$

Q.47 Consider a spring mass system with mass 0.5 kg and spring constant $k = 2Nm^{-1}$ in a viscous medium with drag coefficient $b = 3\,kg\,s^{-1}$. The additional mass required so that the motion becomes critically damped is _________ kg (rounded off to three decimal places).

Topic- Oscillation, Waves and optics

Sub Topic- Oscillation

Ans. : 0.62

Solution: $b^2 = 4mk \Rightarrow m = \frac{b^2}{4k} = \frac{3^2}{4\times 2} = \frac{9}{8} = 1.12\,kg$

Excess mass $m_{excess} = 1.12 - 0.5 = 0.62\,kg$

Q.48 Unit vector normal to the equipotential surface of $V(x, y, z) = 4x_2 + y_2 + z$ at (1,2,1) is given by $\left(a\hat{i} + b\hat{j} + c\hat{k}\right)$. The value of $|b|$ is _____________ (rounded off to two decimal places).

Topic- Mathematical Physics

Sub Topic- Vector Analysis

Ans. : 0.43

Solution: $v = 4x^2 + y^2 + z \Rightarrow \nabla v = 8x\hat{i} + 2y\hat{j} + \hat{k}$

Normal unit vector $\Rightarrow \hat{n} = \frac{\nabla v_{1,2,1}}{|\nabla v|_{1,2,1}} = \frac{8\hat{i} + 4\hat{j} + \hat{k}}{9} \Rightarrow b = 0.43$

Q.49 A rectangular pulse of width 0.5 cm is travelling to the right on a taut string (shown by full line in the figure) that has mass per unit length μ_1. The string is attached to another taut string (shown by dashed line) of mass per unit length μ_2. If the tension in both the strings is the same, and the transmitted pulse has width 0.7 cm, the ratio μ_1/μ_2 is ___________(rounded off to two decimal places).

Topic- Oscillations, Waves & Optics

Sub Topic- Waves

Ans. : 1.96

Solution: Frequency of pulse will not change while on moving. Given that, rectangular pulse of width 0.5 cm, we con see as its wavelength.

$\Rightarrow\ f_1 = f_2 \ \Rightarrow \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \ldots \ldots (1)$

$\lambda_1 = \omega_1 = 0.5$ cm & $\lambda_2 = \omega_2 = 0.7$ cm

Velocity, $v = \sqrt{\frac{T}{\mu}}$

where T is tension & $\mu =$ moss/lergth

From Equation (1) $\frac{1}{\omega_1}\sqrt{\frac{T_1}{\mu_1}} = \frac{1}{\omega_2}\sqrt{\frac{T_2}{\mu_2}} \qquad (T_1 = T_2)$

$\frac{\mu_1}{\mu_2} = \frac{\omega_2^2}{\omega_1^2} = \frac{(0.7)^2}{(0.5)^2} \Rightarrow \frac{\mu_1}{\mu_2} = 1.96$

Q.50 An α particle with energy of $3\,MeV$ is moving towards a nucleus of ^{50}Sn. Its minimum distance of approach to the nucleus is $f \times 10^{-14}\,m$. The value of f is ________ (rounded off to one decimal place).

Topic- Modern Physics

Sub Topic- Rutherford Scattering

Ans. : 4.8

Solution: From the conservation of energy, the distance of closest approach can be written as

$$\frac{1}{2}mv^2 = k_\alpha = \frac{Z_1 \times Z_2 \times e^2}{4\pi\varepsilon_0 x} \Rightarrow x = \frac{e^2}{4\pi\varepsilon_0}\frac{2\times 50}{k_\alpha} = 1.44\,(Mev - fm)\frac{2\times 50}{3\,MeV} = 4.8\times 10^{-14}\,m$$

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 In a X-Ray tube operating at 20 kV, the ratio of the de-Broglie wavelength of the incident electrons to the shortest wavelength of the generated X-rays is ________ (rounded off to two decimal places).

Given: e/m ratio for an electron $= 1.76\times 10^{11}\,C\,kg^{-1}$ and the speed of light in vacuum is $3\times 10^{8}\,ms^{-1}$

Topic- Modern Physics

Sub Topic-X-Ray Production

Ans. : 0.14

Solution: The de-Broglie wavelength of incident electron can be written as $\lambda_D = \frac{h}{p} = \frac{h}{\sqrt{2m_e eV}}$

The shortest wavelength of X-ray $\lambda_{\min} = \frac{hc}{eV}$

The given ratio $\frac{\lambda_D}{\lambda_{\min}} = \frac{1}{c}\sqrt{\frac{V}{2}\frac{e}{m_e}} = \frac{1}{3\times10^8}\sqrt{\frac{20\times10^3}{2}1.76\times10^{11}} = 0.14$

Q.52 A point source emitting photons of 2 eV energy and 1 W of power is kept at a distance of 1 m from a small piece of a photoelectric material of area $10^{-4} m^2$. If the efficiency of generation of photoelectrons is 10%, then the number of photoelectrons generated are $f\times10^{12}$ per second. The value of f is __________ (rounded off to two decimal places).

Given: $1eV = 1.6\times10^{-19} J$

Topic- Modern Physics

Sub Topic-Photoelectric Effect

Ans. : 2.48

Solution: $I = \frac{P}{A} = \frac{1}{4\pi(1)^2} = \frac{1}{4\pi}$

The energy falling on material per unit time $\frac{1}{4\pi}$ x Area of material$=\frac{1}{4\pi}\times10^{-4}$

The number of photoelectrons generated is given by $= \frac{\text{Energy falling on material per unit time}}{\text{Energy of Photons}}$

$$= \frac{10^{-4}}{4\pi\times2\times1.6\times10^{-19}}\times0.1 = 2.48\times10^{12} \Rightarrow f = 2.48$$

Q.53 Consider the α -decay $^{90}Th^{232} \rightarrow ^{88}Ra^{228}$. In an experiment with one gram of $^{90}Th_{232}$, the average count rate (integrated over the entire volume) measured by the α-detector is 3000 counts s^{-1}. If the half life of $^{90}Th_{232}$ is given as $4.4\times10^{17} s$, then the efficiency of the α-detector is __________ (rounded off to two decimal places).

Given: Avogadro's number $= 6.023\times10^{23} mol^{-1}$

Topic- Modern Physics

Sub Topic-Radioactivity

Ans. : 0.73

Solution: The activity or decay rate is defined as $A = N_0\lambda$

The value of N_0 in one gram of Th^{232} is $\dfrac{6.023\times10^{23}}{232}$ & $\lambda = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{4.4\times10^{17}}$

Thus, $A = \dfrac{6.023\times10^{23}}{232}\times\dfrac{0.693}{4.4\times10^{17}} = 4088$

The efficiency is $\dfrac{\text{Total count rate}}{\text{Total decay rate}} = \dfrac{3000}{4088} = 0.73$

Q.54 In the Thomson model of hydrogen atom, the nuclear charge is distributed uniformly over a sphere of radius R. The average potential energy of an electron confined within this atom can be taken as $V = -\dfrac{e^2}{4\pi\epsilon_0 R}$. Taking the uncertainty in position to be the radius of the atom, the minimum value of R for which an electron will be confined within the atom is estimated to be $f\times10^{-11}m$. The value of f is ___________ (rounded off to one decimal place).

Given: The uncertainty product of momentum and position is $\hbar = 1\times10^{-34}Js^{-1}$, $e = 1.6\times10^{-19}C$, and $\dfrac{1}{4\pi\epsilon_0} = 9\times10^{9}\ Nm^2C^{-2}$.

Topic- Electrodynamics

Sub Topic-Thomson Experiment

Ans.: 2.2

Q.55 The sum of the eigenvalues λ_1 and λ_2 of matrix $B = I + A + A^2$, where $A = \begin{bmatrix} 2 & 1 \\ -0.5 & 0.5 \end{bmatrix}$ is____________ (rounded off to two decimal places).

Topic- Mathematical Physics

Sub Topic-Eigen Value of Matrices

Ans. : 7.7

Solution: $B = I + A + A^2$

$$A = \begin{bmatrix} 2 & 1 \\ -0.5 & 0.5 \end{bmatrix} = |A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 2-\lambda & 1 \\ -0.5 & 0.5-\lambda \end{vmatrix} = 0 \Rightarrow (2-\lambda)(0.5-\lambda)+0.5 = 0$$

$$1-2\lambda-0.5\lambda+\lambda^2+0.5 = 0 \Rightarrow \lambda^2-2.5\lambda+1.5 = 0 \Rightarrow \lambda^2-\lambda-1.5\lambda+1.5 = 0$$

$$\lambda(\lambda-1)-1.5(\lambda-1) = 0 \Rightarrow \lambda = 1,\ 1.5$$

$$A^2=\begin{bmatrix}\vec{2} & 1\\ -0.5 & 0.5\end{bmatrix}\begin{bmatrix}2 & 1\\ -0.5 & 0.5\end{bmatrix}=\begin{bmatrix}4-0.5 & 2.5\\ -1.25 & -0.25\end{bmatrix}$$

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$I_r=2$$

$$A^2-2.5A+1.5I=0\Rightarrow A^2+A-A+I-I-2.5A+1.5I=0$$

$$A^2+A+I=B=3.5A-0.5I$$

Thus, sum of eigen values of B $=\Sigma\lambda_i=3.5\Sigma\lambda-0.5\Sigma\lambda_r=3.5[2.5]-0.5\times2.5=7.75$

Q.56 A container of volume V has helium gas in it with N number of He atoms. The mean free path of these atoms is λ_{He}. Another container has argon gas with the same number of Ar atoms in volume $2V$ with their mean free path being λ_{Ar}. Taking the radius of Ar atoms to be 1.5 times the radius of He atoms, the ratio $\lambda_{Ar}/\lambda_{He}$ is__________ (rounded off to two decimal places).

Topic- Kinetic Theory and Thermodynamics

Sub Topic-KTG

Ans. : 0.88

Solution: We know that the mean free path $\lambda=\dfrac{1}{n\pi d^2}$, where, $n=\dfrac{N}{V}$

$$\lambda\,\alpha\,\frac{Volume}{radius^2}.\ \text{Thus, } \frac{\lambda_{Ar}}{\lambda_{He}}\,\alpha\,\frac{2V}{V}\frac{r^2}{(\frac{3}{2}r)^2}\Rightarrow\frac{\lambda_{Ar}}{\lambda_{He}}\,\alpha\,\frac{2V}{V}\frac{r^2}{(\frac{3}{2}r)^2}=8/9=0.88$$

Q.57 Three frames F_0, F_1 and F_2 are in relative motion. The frame F_0 is at rest, F_1 is moving with velocity $v_1\hat{i}$ with respect to F_0 and F_2 is moving with velocity $v_2\hat{i}$ with respect to F_1. A particle is moving with velocity $v3\hat{\imath}$ with respect to F_2. If $v_1=v_2=v_3=c/2$, where c is the speed of light, the speed of the particle with respect to F_0 is fc. The value of f is ____________ (rounded off to two decimal places).

Topic- Modern Physics

Sub Topic- STR

Ans.: 0.93

Solution: Speed of F_2 with respect to the F_0 is $v_{2,0}=\dfrac{v_{1,0}+v_{2,1}}{1+\dfrac{v_{1,0}.v_{2,1}}{c^2}}=\dfrac{\frac{c}{2}+\frac{c}{2}}{1+\frac{c}{2}.\frac{c}{2}.\frac{1}{c^2}}=\dfrac{4}{5}c$

Now speed of particle with respect to F_0 is

$$\frac{v_{p,2}+v_{2,0}}{1+\frac{v_{p,2}.v_{2,0}}{c^2}}=\frac{\frac{c}{2}+\frac{4c}{5}}{1+\frac{c}{2}.\frac{4c}{5}.\frac{1}{c^2}}=\frac{\frac{c(5+8)}{10}}{\frac{14}{10}}=\frac{13}{14}c=0.93c\Rightarrow f=0.93$$

Q.58 A fission device explodes into two pieces of rest masses m and $0.5m$ with no loss of energy into any other form. These masses move apart respectively with speeds $\frac{c}{\sqrt{13}}$ and $\frac{c}{2}$, with respect to the stationary frame. If the rest mass of the device is fm then f is________ (rounded off to two decimal places).

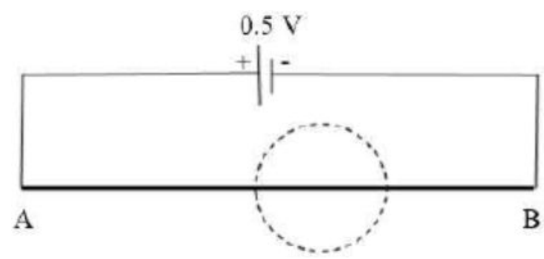
Topic- Modern Physics

Sub Topic-Nuclear Fission

Ans. : 1.60

Solution: From conservation of energy $Mc^2=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}+\frac{0.5m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{mc^2}{\sqrt{1-\frac{c^2}{13c^2}}}+\frac{0.5mc^2}{\sqrt{1-\frac{c^2}{4c^2}}}=1.62\,m$

Q.59 A conducting wire AB of length m has resistance of $.6\,\Omega$. It is connected to a voltage source of 0.5 V with negligible resistance as shown in the figure. The corresponding electric and magnetic fields give Poynting vectors $\vec{S}(\vec{r})$ all around the wire. Surface integral $\int\vec{S}\cdot d\vec{a}$ is calculated over a virtual sphere of diameter 0.2 m with its centre on the wire, as shown. The value of the integral is ______W (rounded off to three decimal places).



Topic- Electricity and Magnetism

Sub Topic-Poynting Vector

Ans. : 0.03

Solution: $P=\int S.da=\int\frac{V}{\mu_0 d}\frac{\mu_0 I}{2\pi\rho}r^2\sin^2(\theta)2\pi d\theta=0.03125\quad[\rho=r\sin\theta]$

Q.60 A metallic sphere of radius R is held at electrostatic potential V. It is enclosed in a concentric thin metallic shell of radius $2R$ at potential $2V$. If the potential at the distance $\frac{3}{2}R$ from the centre of the sphere is fV, then the value of f is _______ (rounded off to two decimal places).

Topic- Electricity and Magnetism

Sub Topic-Electrostatic Potential

Ans. : 1.66

Solution: $V(r,\theta)=\sum_{r=0}^{\infty}\left(A_n r^n+\frac{B_n}{r^{n+1}}\right)P_n(\cos\theta)$

$$V=A_0R^0+\frac{B_0}{R},\ A_0=V-\frac{B_0}{R}=V+2V=3V$$

$$2V=A_0(2R)^0+\frac{B_0}{2R}$$

$$V=\frac{B_0}{2R}-\frac{B_0}{R}$$

$$V=\frac{-B_0}{2R},\ B_0=-2RV$$

$$V\left(\frac{3}{2}R\right)=A_0+\frac{B_0}{\frac{3}{2}R}=3V+\frac{2}{3R}\times -2=fV=\left(3-\frac{4}{3}\right)V=\frac{5}{3}V\Leftrightarrow f=1.66$$