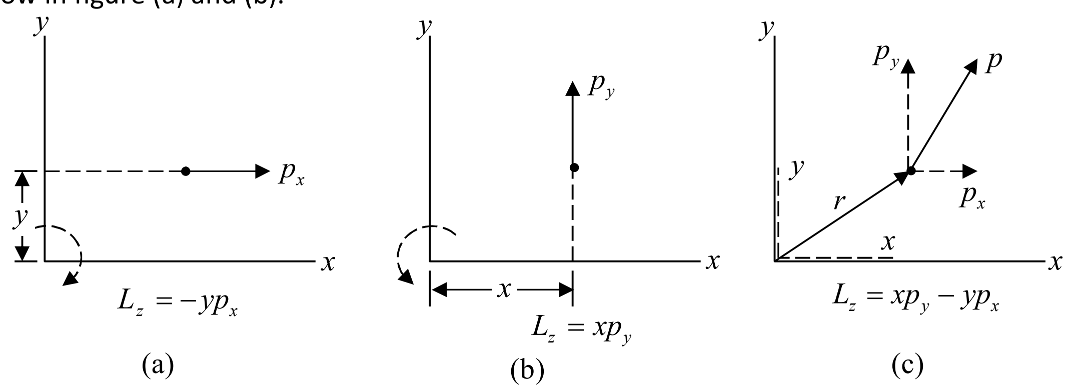
Chapter 7

Rotational Dynamics

1. Angular Momentum Due to Particle

The particle has momentum $\vec{p}$, then angular momentum about point O is given $\vec{L} = \vec{r} \times \vec{p}$, where $\vec{r}$ is position vector of particle with respect to origin.

Consider motion in the x-y plane, first in the x - direction and then in the y - direction, as shown below in figure (a) and (b).



The most general case involves both these motions simultaneously, as shown in above figure.

Hence, $L_z = xp_y - yp_x$

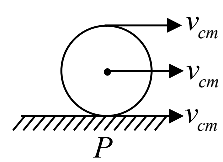
As you can verify by inspection or by evaluating the cross product as follows. Using $r = (x, y, 0)$ and $p = (p_x, p_y, 0)$, we have

$$L = r \times p = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0 \\ p_x & p_y & 0 \end{vmatrix} = (xp_y - yp_x)\hat{k}$$

Motion of a rigid body involving translation and rotation

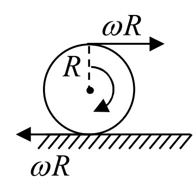
Assume a sphere of mass M and radius R are rolling on a rough surface, where v_{cm} is velocity of center of mass and ω is angular velocity

Pure translation **–**when sphere perform pure translation motion then every point has same speed of speed of center of mass



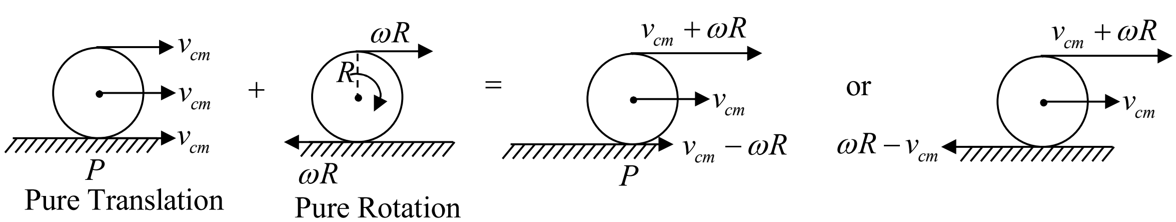
Pure Translation

Pure Rotation **–** when sphere perform pure rotation about center of mass then center of mass have zero velocity and all other point have velocity $\vec{v} = \vec{\omega} \times \vec{r}$. so upper and lower points have $v = \omega R$ but in opposite direction



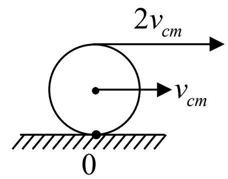
Pure Rotation

Rolling **-** when translation and rotation can be combined then sphere will rolling



The condition of Rolling without slipping

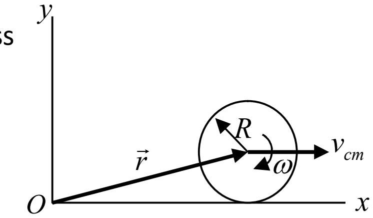
A rigid body such as sphere having translation motion as velocity of center of mass v_{cm} and angular velocity about center of mass is ω. The condition of rolling without slipping will achieve if $v_{cm} = \omega R$ where R is radius of rolling body.



Similarly, if acceleration of center of mass is a_{cm} and α is angular velocity of about center of mass then $a_{cm} = \alpha R$, where R is radius of rolling body.

Angular Momentum of Rigid body involve Rolling

The angular momentum About any point O of rigid body of mass M and radius R



$$\vec{L} = M\left(\vec{r} \times \vec{v}_{cm}\right) + I_{cm}\vec{\omega}$$

Where $\vec{v}_{cm}$ is velocity vector of center of mass

$\vec{r}$ is position vector of center of mass from origin O

$\vec{\omega}$ is angular velocity about center of mass

$I_{c.m}$ is moment of inertia about center of mass

Example: A sphere of mass M and radius R having velocity of center of mass v_0. As friction of surface is enough so that sphere Achieving condition of rolling without slipping.

(a) Find the angular momentum about Center of mass C

(b) Find the Angular momentum about point of contact P

(c) Find Angular momentum about Point O

Solution: (a) $\vec{L} = M\left(\vec{r} \times \vec{v}_{c.m}\right) + I_{c.m}\vec{\omega}$

$\vec{r} = 0$ $\vec{L} = I_{c.m}\vec{\omega} = \frac{2}{5}MR^2\omega\,\hat{z}$ for rolling without slipping, $v_{c.m} = \omega R$ $\omega = \frac{v_0}{R}$

$$\vec{L} = \frac{2}{5}Mv_0R\,\hat{z}$$

(b) $\vec{L} = M\left(\vec{r} \times \vec{v}_{c.m}\right) + I_{c.m}\vec{\omega}$

$|r| = R$ and angle between $\vec{r}$ and $v_{c.m}$ is $\frac{\pi}{2}$, $\vec{L} = MRv_o + I_{c.m}\vec{\omega} = Mv_0R\,\hat{z} + \frac{2}{5}MR^2\omega\,\hat{z}$

for rolling without slipping, $v_{c.m} = \omega R \quad \omega = \frac{v_0}{R} \quad \vec{L} = \frac{7}{5} M v_0 R \, \hat{z}$

(c) $\vec{L} = M\left(\vec{r} \times \vec{v}_{c.m}\right) + I_{c.m}\vec{\omega}$

$|r| = r$ and angle between $\vec{r}$ and $v_{c.m}$ is θ $\quad \vec{L} = Mr\sin\theta v_o \hat{z} + I_{c.m}\vec{\omega}$

$r\sin\theta = R$ so $\vec{L} = Mv_0 R\,\hat{z} + \frac{2}{5}MR^2\omega\,\hat{z}$

For rolling without slipping, $v_{c.m} = \omega R \quad \omega = \frac{v_0}{R} \quad \vec{L} = \frac{7}{5} MRv_0 \, \hat{z}$