Chapter 6

Linear Momentum And Energy

4. Work Energy Theorem

The kinetic energy of a particle of mass m, moving with a speed v, is defined as $T=\frac{1}{2}mv^2$.

Let us consider a particle that moves from point 1 to point 2 under the action of a force $\vec{F}$. The total work done on the particle by the force as the particle moves from 1 to 2 is, by definition, the line integral $W_{12}=\int_1^2 \vec{F}\cdot d\vec{s}$, where $d\vec{s}=\vec{v}dt$ is the displacement vector along the particle's trajectory. If the particle under-goes an infinitesimal displacement $d\vec{s}$ under the action of force $\vec{F}$, the scalar product $dW=\vec{F}\cdot d\vec{s}$ is the infinitesimal work done by the force $\vec{F}$ as the particle undergoes the displacement $d\vec{s}$ along the particle's trajectory. We use the Newton's second law of motion, $\vec{F}=\frac{d\left(m\vec{v}\right)}{dt}$ in the equation to obtain an expression for the infinitesimal work

$$dW = \frac{d\left(m\vec{v}\right)}{dt}\cdot\vec{v}dt = \frac{d}{dt}\left(\frac{1}{2}m\vec{v}\cdot\vec{v}\right)dt = d\left(\frac{1}{2}mv^2\right).$$

The scalar quantity $\frac{1}{2}mv^2$ is the kinetic energy of the particle, it follows that $dW = dT$. This Equation $dW = dT$ in the differential form of the work-energy theorem.

It states that the differential work of the resultant of forces acting on a particle is equal, at any time, to the differential change in the kinetic energy of the particle.

Integrating equation between point 1 and point 2, corresponding to the velocities v_1 and v_2 of the particle, we get $W_{12} = \int_1^2 dW = \int_1^2 dT = T_2 - T_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

This is the work-energy theorem, which states that the work done by the resultant force $\vec{F}$ acting on a particle as it move from point 1 to point 2 along its trajectory is equal to the change in the kinetic energy $\left(T_2 - T_1\right)$ of the particle during the given displacement. When the body is accelerated by the resultant force, the work done on the body can be considered a transfer of energy to the body, where it is stored as kinetic energy.