Chapter 6

Linear Momentum And Energy

6. Gravitation Force

Let us consider a conservative force $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$, then we have $\vec{F} = \nabla\phi = -\nabla U$

Therefore, we have the following relations:

$$F_x = \frac{\partial\phi}{\partial x} = -\frac{\partial U}{\partial x}, \qquad F_y = \frac{\partial\phi}{\partial y} = -\frac{\partial U}{\partial y}, \qquad F_z = \frac{\partial\phi}{\partial z} = -\frac{\partial U}{\partial z}$$

This shows that the partial derivative of force potential in a given direction gives the force in that direction. An example of a force that derives from a potential is gravitational force $\vec{F}_g = -\nabla U$, which leads to the following equations

$$mg_x = -\frac{\partial U}{\partial x}, \quad mg_y = -\frac{\partial U}{\partial y}, \quad mg_z = -\frac{\partial U}{\partial z}$$

where the gravitational acceleration vector $\vec{g} = (g_x, g_y, g_z)$. It follows that the negative of partial derivative of potential energy in a given direction gives the gravitational force in that direction.

If gravitational acceleration vector is given by $\vec{g} = g(0,0,-1)$

then we have $0 = -\frac{\partial U}{\partial x}$ $\quad 0 = \frac{\partial U}{\partial y}$ $\quad -mg = -\frac{\partial U}{\partial z}$

Integrating the last of the above equation to obtain $U = mgz + f(x,y)$

Setting $f(x,y) = 0$, the potential energy of the particle in a gravitational field is given by $U = mgz$ where $\vec{g}$ acts in the negative z direction. The total mechanical energy E is conserved when a particle moves under the action of the gravitational field.