**Vegaal Education** 

# NET-JRF JUNE 2021 [SOLUTION]

# PART A

Q1. An inverted cone is filled with water at a constant rate. The volume of water inside the cone as a function of time is represented by the curve



Ans.: (b)

Q2. A spacecraft flies at a constant height R above a planet of radius R. At the instant the spacecraft is over the north pole, the lowest latitude visible from the spacecraft is:

(a)  $0^{\degree}$  (equator) (b)  $30^{\degree} N$  (c)  $45^{\degree} N$  (d)  $60^{\degree} N$ 

Ans. : (b)

Q3. An experiment consists of tossing a coin 20 times. Such an experiment is performed 50 times. The number of heads and the number of tails in each experiment are noted. What is the correlation coefficient between the two?

Ans. : (a)

Q4. Which of these groups of numbers has the smallest mean?



Ans.  $: (c)$ 

Q5. Identical balls are tightly arranged in the shape of an equilateral triangle with each side containing  $n$  balls. How many balls are there in the arrangement?

(a) 
$$
\frac{n^2}{2}
$$
 (b)  $\frac{n(n+1)}{2}$  (c)  $\frac{n(n-1)}{2}$  (d)  $\frac{(n+1)^2}{2}$ 

Ans. : (b)

CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics Q6. A shopkeeper has a faulty pan balance with a zero offset. When an object is placed in the left pan it is balanced by a standard  $100 g$  weight. When it is placed in the right pan it is balanced by a standard  $80 g$  weight. What is the actual weight of the object? (a)  $90 g$  (b)  $88.88 g$  (c)  $95 g$  (d)  $85 g$ Ans. : (a) Q7. A and B start from the same point in opposite directions along a circular track simultaneously. Speed of B is  $2/3^{rd}$  that of A. How many times will A and B cross each other before meeting at the starting point? (a) 2 (b) 3 (c) 5 (d) 4 Ans. : (d) Q8. Consider a solid cube of side 5 units. After painting, it is cut into cubes of 1 unit. Find the probability that a randomly chosen unit cube has only one side painted. (a)  $\frac{56}{1}$ 125 (b)  $\frac{36}{1}$ 125 (c)  $\frac{44}{1}$ 125 (d)  $\frac{54}{1}$ 125 Ans. : (d) Q9. How many integers in the set  $\{1,2,3,...,100\}$  have exactly 3 divisors? (a) 4 (b) 12 (c) 5 (d) 9

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Ans.  $:(a)$ 

- Q10. The arithmetic and geometric means of two numbers are 65 and 25, respectively. What are these two numbers?
	- (a)  $110, 20$  (b)  $115, 15$  (c)  $120, 10$  (d)  $125, 5$

Ans. (d)

Q11. Shyam spent half of his money and was left with as many as he had rupees before, but with half as many rupees as he had paise before. Which of the following is a possible amount of money he is left with?



Ans. : (b)

- Q12. A cylindrical road roller having a diameter of  $1.5 m$  moves at a speed of 3  $km/h$  while levelling a road. How much length of the road will be levelled in 45 minutes?
	- (a) 2.25 km (b) 0.375  $\pi$  km (c) 0.75  $\pi$  km (d) 1.5 km

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Ans. : (a)

- Q13. An intravenous fluid is given to a child of  $7.5\ kg$  at the rate of 20 drop/minute. The prescribed dose of the fluid is  $40 \text{ ml}$  per kg of body weight. If the volume of a drop is  $0.05 \text{ ml}$ , how many hours are needed to complete the dose?
	- (a) 2 (b) 3 (c) 4 (d) 5

Ans. : (d)

- Q14. A cousin is a non-sibling with a common ancestor. If there is exactly one pair of siblings in a group of 5 persons then the maximum possible number of pairs of cousins in the group is
	- (a) 3 (b) 6 (c) 9 (d) 10

Ans. : (c)

Q15. In a tournament with 8 teams, a win fetches 3 points and a draw, 1. After all terms have played three matches each, total number of points earned by all teams put together must lie between

(a) 24 and 36 (b) 24 and 32 (c) 12 and 24 (d) 32 and 48

Ans.  $:(a)$ 

Q16. An appropriate diagram to represent the relations between the categories KEYBOARD, HARDWARE, OPERATING SYSTEM and CPU is



Ans. : (c)

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Q17. Trade figures and populations in appropriate units in a certain year are given for 7countries.



If countries are ranked according to the difference in their per capita exports over import, then the best and worst ranking countries are respectively

(a) C and A (b) A and E (c) C and B (d) A and F

Ans. : (a)

- Q18. At least two among three persons  $A, B$  and C are truthful. If  $A$  calls  $B$  a liar and if  $B$  calls C a liar, then which of the following is FALSE? It counters are lainted according to the miletime million the periodic exports over import, then<br>the best and worst ranking countries are respectively<br>(a) C and A (b) A and E (c) C and B (d) A and F<br>Ans.: (a)<br>018. At leas
	- (a)  $A$  is truthful (b)  $B$  is truthful
	- (c)  $C$  is truthful  $\qquad \qquad$  (d) At least one is a liar

```
Ans. : (b)
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- Q19. The maximum area of a right-angled triangle inscribed in a circle of radius  $r$  is
	- (a)  $2r^2$  (b)  $\frac{r^2}{r}$ 2 (c)  $\sqrt{2} r^2$  (d)  $r^2$

Ans. : (d)

table:



Ans. : (d)

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### PART B

- CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
Q1. A particle in one dimension executes oscillatory motion in a potential  $V(x) = A|x|$ , where  $A > 0$ <br>
is a constant of appropriate dimension. If the time period is a constant of appropriate dimension. If the time period  $T$  of its oscillation depends on the total energy  $E$  as  $E^{\alpha}$ , then the value of  $\alpha$  is **CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br> **CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br> **PART B**<br> **Q1.** A particle in one dimension executes oscillatory motion in a potential  $V(x) = A|x|$ 
	- (a)  $\frac{1}{1}$ 3 (b)  $\frac{1}{1}$ 2 (c)  $\frac{2}{1}$ 3 (d)  $\frac{3}{2}$

 Topic: Classical Mechanics Sub Topic: Action Angle

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Ans. : (b)

**EXAMPLEJRE, GATE, BTE, GATE, HTE, BAM, JEST, TIER and GRE for Physics  
\n**PAR B**  
\n**Q1.** A particle in one dimension executes oscillatory motion in a potential 
$$
V(x) = A|x|
$$
, where  $A > 0$   
\nis a constant of appropriate dimension. If the time period *T* of its oscillation depends on the  
\ntotaleberg *E* as *E*, then the value of *G* is  
\n(a)  $\frac{1}{3}$  (b)  $\frac{1}{2}$  (c)  $\frac{2}{3}$  (d)  $\frac{3}{4}$   
\nTopic: Classical Mechanics  
\nAns. : (b)  
\n $J = \oint \sqrt{E - V(x)} dx = 4 \int_{0}^{E/d} \sqrt{2m(E - Ax)} dx$   
\n $H = \frac{P^3}{2m} + A|x|$   
\n $J = \oint \sqrt{2m(E - A|x)} dx = 4 \int_{0}^{E/d} \sqrt{2m\sqrt{E - Ax}} dx$   
\nLet,  $u = E - Ax \Rightarrow du = -A dx$   
\n $J = 4 \int_{0}^{R} \sqrt{2m} u^2 du = \frac{8\sqrt{2m}}{3A} E^{3/2} \Rightarrow T = \frac{\partial J}{\partial E} \Rightarrow \frac{4\sqrt{2mE}}{A} = \frac{\sqrt{32mE}}{A} T \propto E^{1/2}$   
\n**Q2.** The equation of motion of a one-dimensional forced harmonic oscillator in the presence of a  
\ndissipative force is described by  $\frac{d^2x}{dt^2} + 10 \frac{dx}{dt} + 16x = 6te^{-kt} + 4te^2e^2$ . The general form of the  
\nparticular solution, in terms of constants *A*, *B* etc.,  
\n(a)  $t(At^2 + Bt + C)e^{2t} + (Dt + E)e^{8t}$  (b)  $(At^2 + Bt + C)e^{2t} + (Dt + E)e^{8t}$   
\n(c)  $t(At^2 + Bt + C)e^{2t} + (Dt + E)e^{8t}$  (d)  $(At^2 + Bt + C)e^{2t} + (Dt + E)e^{8t}$   
\nTo be the formula  $\frac{d^2x}{dt^2} + 10 \frac{dx}{dt} + 16x = 6te^{-8t} + 4te^2e^{2t}$** 

dissipative force is described by  $\frac{d^2x}{dt^2}+10\frac{dx}{dt}+16x=6te^{-8t}+4t^2e^{-2t}$ . The general form of the particular solution, in terms of constants  $A, B$  etc, is  $=4\int_{R}^{0} \sqrt{2mu}^{1/2} du = \frac{8\sqrt{2m}}{3A} E^{3/2} \Rightarrow T = \frac{\partial J}{\partial E} \Rightarrow \frac{4\sqrt{2mE}}{A} = \frac{\sqrt{32mE}}{A} T \propto E^{1/2}$ <br>
equation of motion of a one-dimensional forced harmonic oscillator in the p<br>
sipative force is described by  $\frac{d^3x}{dt^2$  $J = 4 \int_{\alpha}^{0} \sqrt{2m} u^{1/2} du = \frac{8\sqrt{2m}}{3A} E^{3/2} \Rightarrow T = \frac{\partial J}{\partial E} \Rightarrow \frac{4\sqrt{2mE}}{A} = \frac{\sqrt{32mE}}{A} T \propto E^{3/2}$ <br>
The equation of motion of a one-dimensional forced harmonic oscillator in the pre-<br>
dissipative force is described b

(a) 
$$
t(At^2 + Bt + C)e^{-2t} + (Dt + E)e^{-8t}
$$
  
\n(b)  $(At^2 + Bt + C)e^{-2t} + (Dt + E)e^{-8t}$   
\n(c)  $t(At^2 + Bt + C)e^{-2t} + t(Dt + E)e^{-8t}$   
\n(d)  $(At^2 + Bt + C)e^{-2t} + t(Dt + E)e^{-8t}$ 

 Topic: Mathematical Physics Sub Topic: Differential Equation

Ans.  $:(c)$ Solution:  $\frac{1}{12} + 10\frac{1}{11} + 16x$  $2x$  10 dx 16x - 6to<sup>-Bt</sup> 14t<sup>2</sup> o<sup>-2t</sup>  $D^2 + 10D + 16 = 0$ 

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 $D^2 + 2D + 8D + 16 = 0$  $D(D+2)+8(D+2)=0,2,-8$  $C.F. = C_1 e^{-2t} + C_2 e^{-8t}$ **Pravel Education**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $P^2 + 2D + 8D + 16 = 0$ <br>  $P(D+2) + 8(D+2) = 0, 2, -8$ <br>  $E.F. = C_1e^{-2t} + C_2e^{-8t}$ <br>  $E.F. = C_1e^{-2t} + C_2e^{-8t}$ <br>  $E.F. = C_1e^{-2t} + C_2e^{-8t}$ <br>  $= \frac{1}{(D+2)(D+8)} = \$  $\sqrt{2(D+8)}$  $P.I. \rightarrow \frac{1}{(R_1 - R)(R_2 - R)} (6te^{-8t} + 4te^{-2t})$  $\overline{D+2(D+1)}$  $\rightarrow \frac{1}{(R_1 - R)(R_2 - R)} (6te^{-8t} + 4te^{-2t})$  $\overline{+2(D+8)}$ **Pravidle Education**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $D^2 + 2D + 8D + 16 = 0$ <br>  $D(D+2) + 8(D+2) = 0, 2, -8$ <br>  $C.F. = C_1e^{-2t} + C_2e^{-8t}$ <br>  $P.I. \rightarrow \frac{1}{(D+2)(D+8)}(6te^{-8t} + 4te^{-2t})$ <br>  $\frac{1}{(D+2)(D+8)} = \frac{A}{D+2} + \frac$ 1  $A \qquad B \qquad A(D+8) + B(D+2)$ **CONSTRAINER SET TIFR and GRE for Physics**<br>
IIT-JAM, JEST, TIFR and GRE for Physics<br>  $(D+8)+B(D+2)$ <br>  $(D+2)(D+8)$  $\frac{2}{2}(D+8)$  -  $\frac{D+2}{D+2}$  +  $\frac{D+8}{D+8}$  -  $\frac{(D+2)(D+8)}{(D+2)(D+8)}$  $A$   $B$   $A(D+8)+B(D+$  $\frac{D+2(D+8)}{D+2}$  -  $\frac{D+2}{D+8}$  -  $\frac{D+2(D+2)}{(D+2)(D+2)}$  $=\frac{A}{R} + \frac{B}{R} = \frac{A(D+8) + B(D+2)}{(D-2)(D-2)}$  $\frac{1}{2+2(2+8)} = \frac{1}{D+2} + \frac{1}{D+8} = \frac{1}{(D+2)(D+8)}$  $1 = (A + B)D + 8A + 2B$  $A + B = 0$ ,  $8A + 2B = 1$  $A = -B$ ,  $8A - 2A = 1$ 1 6  $D(D+2)+8(D+2)=0, 2, -8$ <br>  $C.F. = C_1e^{-2t} + C_2e^{-8t}$ <br>  $PL \rightarrow \frac{1}{(D+2)(D+8)}$   $(6te^{-8t} + 4te^{-2t})$ <br>  $\frac{1}{(D+2)(D+8)} = \frac{A}{D+2} + \frac{B}{D+8} = \frac{A(D+8) + B(D+2)}{(D+2)(D+8)}$ <br>  $= (A+B)D+8A+2B$ <br>  $A+B=0, 8A+2B=1$ <br>  $A=-B, 8A-2A=1$ <br>  $A=\frac{1}{6}$ <br>  $\frac{1/6}{D+2} +$  $\frac{1}{2}$   $\frac{1}{D+8}$  $te^{-8t} + 4t^2e^{-2t}$  $\left[\frac{1/6}{D+2}+\frac{-1/6}{D+8}\right]$  (6te<sup>-8t</sup> + 4t<sup>2</sup>e<sup>-2t</sup>)  $=\frac{1}{t}e^{-8t}+\frac{2}{t}-\frac{1}{t^2}e^{-2t}$  $\frac{1}{D+2}te^{-8t} + \frac{2}{3}\frac{1}{D+2}t^2e^{-2t}$  $1 \t_{10}^{-8t}$  2 1  $t_{20}^{-2t}$  $\frac{1}{D+8}te^{-8t}-\frac{2}{3}\frac{1}{D+8}t^2e^{-2t}$  $-\frac{1}{D+8}te^{-8t}-\frac{2}{3}\frac{1}{D+8}t^2e^{-2t}$  $8t$  1  $2e^{-2t}$  1  $t^2$  $\frac{1}{6}$  +  $\frac{1}{3}$  e  $e^{-8t}$   $\frac{1}{2}$   $t + \frac{2}{3}e^{-2t}$   $\frac{1}{2}t^2$  - $\overline{D-6}^{t+\frac{1}{3}}e^{-t}$  $= e^{-8t} \frac{1}{D-6}t + \frac{2}{3}e^{-2t}$  $_{8t}$  1  $_{t}$  2 $e^{-2t}$  1  $_{t}$  2  $\frac{1}{3}$   $\frac{1}{D+6}$  $e^{-8t}$   $\frac{1}{R}t - \frac{2e^{-2t}}{2} - \frac{1}{R}t^2$  $\overline{D}$ <sup>t</sup>  $\overline{3}$   $\overline{D}$  +  $-e^{-8t}$   $\frac{1}{2}t-\frac{2e^{-t}}{2}$  $^{+}$  $\left[1-\frac{D}{t}\right]^{-1}t+\frac{2}{t}e^{-2t}\frac{t^3}{2}$  $\frac{1}{6}$   $\begin{bmatrix} 1 - \frac{1}{6} \end{bmatrix}$   $\begin{bmatrix} 1 + \frac{1}{3}e & \frac{1}{3} \end{bmatrix}$  $=-\frac{e^{-8t}}{6}\left[1-\frac{D}{6}\right]^{-1}t+\frac{2}{3}e^{-2t}\frac{t^3}{3}-e^{-8t}\frac{t^2}{2}-\frac{e^{-2t}}{9}\left[1+\frac{D}{6}\right]^{-1}t^2$  $\frac{2}{2}$   $\frac{1}{9}$   $\left[\frac{1}{6}\right]$  $-e^{-8t} \frac{t^2}{2} - \frac{e^{-2t}}{9} \left[1 + \frac{D}{6}\right]^{-1} t^2$  $8t \begin{bmatrix} 1 & 2 \end{bmatrix}$   $2t^3e^{-2t}$  $\frac{1}{6}$  $\binom{1}{1}$  $\frac{1}{6}$  $\frac{1}{9}$  $\binom{1}{1}$  $=-\frac{e^{-8t}}{6}\left[t+\frac{1}{6}\right]+\frac{2}{9}t^3e^{-2t}-\frac{1}{2}t^2e^{-8t}-\frac{e^{-2t}}{9}\left[1-\frac{D}{6}+\frac{D^2}{36}\right]t^2$  $\frac{2}{2}$  e  $-\frac{1}{9}$   $1-\frac{1}{6}+\frac{1}{36}$  $-\frac{1}{2}t^2e^{-8t}-\frac{e^{-2t}}{9}\left[1-\frac{D}{6}+\frac{D^2}{36}\right]t^2$  $\begin{bmatrix} 8t & 1 \ 1 & 1 & 2 \end{bmatrix}$ ,  $\begin{bmatrix} 2 & 3 & -2t \ 1 & 2 & 3 \end{bmatrix}$ ,  $\begin{bmatrix} 1 & 1 & 1 \ 2 & 1 & 1 \end{bmatrix}$  $\frac{6}{6}$   $\left[\frac{1}{6}\right]$  +  $\frac{6}{9}$  e  $\frac{6}{2}$  +  $\frac{2}{2}$  e  $\frac{6}{9}$  -  $\frac{6}{27}$  +  $\frac{1}{12}$  $=-\frac{e^{-8t}}{6}\left[t+\frac{1}{6}\right]+\frac{2}{9}t^3e^{-2t}-\frac{1}{2}t^2e^{-8t}-e^{2t}\left(\frac{t^2}{9}-\frac{1}{27}t+\frac{1}{12}\right)$  $x = C.F + P.I.$ =  $2x + C e^{-8x}$   $1 e^{-8t}t^2$   $1 e^{-8t}t^2$   $2 t^3e^{-2t}$   $1 t^2e^{-2t} t e^{-2t}$  $1^e$  +  $C_2e$  $1 \tfrac{3}{2} \tfrac{8t}{4^2} \tfrac{1}{2} \tfrac{8t}{6} \tfrac{2}{3} \tfrac{3}{2} t \tfrac{1}{4^2}$  $2^e$   $\frac{1}{6}$   $-\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{27}$   $\frac{1}{27}$  $C_1e^{-2x} + C_2e^{-8x} - \frac{1}{2}e^{-8t}t^2 - \frac{1}{6}e^{-8t}t + \frac{2}{3}t^3e^{-2t} - \frac{1}{2}t^2e^{-2t} + \frac{te^{-2t}}{27}$  $P.I. = t\left(At^2 + Bt + C\right)e^{-2t} + t\left(Dt + E\right)e^{-8t}$  $e^{-e^{-x}t}$   $\frac{1}{D-6}t + \frac{2}{3}e^{-x} \frac{1}{D}t^2 - e^{-x} \frac{1}{D}t - \frac{2e^{-x}t}{3} \frac{1}{D+6}t^2$ <br>  $= -\frac{e^{-x}t}{6}\left[1-\frac{D}{6}\right]^{3}t + \frac{2}{3}e^{-x} \frac{t^3}{3} - e^{-x} \frac{t^2}{2} - \frac{e^{-2t}}{9}\left[1+\frac{D}{6}\right]^{3}t^2$ <br>  $= -\frac{e^{-x}t}{6}\left[t + \frac{1}{6}\right] + \frac{2}{9}t^3e$  $y_f = \frac{1}{f(x)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V$  $=\frac{1}{c}e^{ax}V=e^{ax}$  $^{+}$ 

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Q3. The vector potential for an almost point like magnetic dipole located at the origin is  $A = \frac{\mu \sin \theta}{4 \pi r^2}$ sin  $A = \frac{\mu \sin \theta}{4\pi r}$  $=\frac{\mu\sin\theta}{4\pi r^2}\hat{\phi}$ , where  $(r, \theta, \phi)$  denote the spherical polar coordinates and  $\hat{\phi}$  is the unit vector along  $\phi$ . A particle of mass *M* and charge *q*, moving in the equatorial plane of the dipole, starts at time  $t = 0$  with an initial speed  $V_0$  and an impact parameter b. Its instantaneous speed at the point of closest approach is

(a) 
$$
V_0
$$
 (b)  $0/0$  (c)  $v_0 + \frac{\mu q}{4\pi m b^2}$  (d)  $\sqrt{v_0^2 + \left(\frac{\mu q}{4\pi m b^2}\right)^2}$ 

 Topic: Electromagnetic Theory Sub Topic: Magnetic dipole

Ans. :  $(a)$ 

Solution: Initial speed is  $V_0$ ,  $A = \frac{\mu \sin(\theta)}{4\pi r^2} \hat{\phi}$  $\sin(\theta)$   $\hat{\lambda}$ 4 A r  $=\frac{\mu \sin(\theta)}{4\pi r^2}\hat{\phi}$ 

Let us consider the speed at closet approach is  $v_c$ 

According to work energy principle

Work done =change in kinetic energy =  $\frac{1}{2} m v_c^2 - \frac{1}{2} m v_0^2 = \int \vec{F} \cdot d\vec{r}$ 

Since, magnetic force do not perform any work

$$
\int \vec{F} \cdot d\vec{r} = 0
$$
  

$$
\frac{1}{2} m v_c^2 - \frac{1}{2} m v_0^2 = \int \vec{F} \cdot d\vec{r} = 0 \implies \frac{1}{2} m v_c^2 - \frac{1}{2} m v_0^2 = 0 \implies v_0 = v_c
$$

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(d)  $\frac{1}{1}$ 

Q4. A particle, thrown with a sped V from the earth's surface, attains a maximum height  $h$  (measured from the surface of the earth). If V is half the escape velocity and  $R$  denotes the radius of earth,

then 
$$
\frac{h}{R}
$$
 is  
\n(a)  $\frac{2}{3}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{4}$ 

 Topic: Classical Mechanics Sub Topic: Gravitation

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Ans. : (b)

Solution: 
$$
\frac{1}{2}mv^2 - \frac{GmM}{R} = -\frac{GmM}{R+h}
$$
 (i)  
\n $v_e = \sqrt{\frac{2GM}{R}}$  and given that  $r = \frac{1}{2}v_e$   
\nFrom (i),  $\frac{1}{2}m \times \frac{1}{4} \times \frac{2GmM}{R} - \frac{GmM}{R} = -\frac{GmM}{R+h}$   
\n $\Rightarrow -\frac{3}{4}\frac{GmM}{R} = -\frac{GmM}{R+h} \Rightarrow 3R + 3h = 4R \Rightarrow \frac{h}{R} = \frac{1}{3}$ 

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Q5. A particle of mass  $\frac{10e}{c^2}$  $1 GeV$  $\vec{c}$ and its antiparticle, both moving with the same speed  $V$ , produce a new particle  $X$  of mass  $\frac{10 \text{ G}}{c^2}$  $10 \, GeV$  $c^{\mathbf{c}}$ in a head-on collision. The minimum value of  $\nu$  required for this process is closest to (a)  $0.83c$  (b)  $0.93c$  (c)  $0.98c$  (d)  $0.88c$ 

 Topic: Classical Mechanics Sub Topic: STR

Ans. : (c)

Solution: 0.98c topic classical mechanics STR (relativistic mass)

From conservation of momentum particle will rest after collision.

$$
\frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = Mc^2
$$
\n
$$
\frac{2m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = Mc^2 \Rightarrow \frac{2 \frac{1 GeV}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{10 GeV}{c^2} \Rightarrow \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{5}
$$
\n
$$
v = \sqrt{\frac{24}{25}}c = 0.98c
$$

Q6. The volume of the region common to the interiors of two infinitely long cylinders defined by  $x^2 + y^2 = 25$  and  $x^2 + 4z^2 = 25$  is best approximated by  $v = \sqrt{\frac{24}{25}}c = 0.98c$ <br>
The volume of the region common to the interiors of two infinitely long cylinders defined by<br>  $x^2 + y^2 = 25$  and  $x^2 + 4z^2 = 25$  is best approximated by<br>
(a) 225 (b) 333 (c) 423 (d) 625<br>
Topic: Mat

(a) 225 (b) 333 (c) 423 (d) 625

 Topic: Mathematical Physics Sub Topic: Vector Analysis

Ans. : (b)

Solution:  $x^2 + y^2 = 25$ ,  $x^2 + 4z^2 = 25$ 

Common interior volume of two infinitely long cylinder will be 333

Q7. The volume integral

$$
I = \iiint\limits_V A \cdot (\nabla \times A) d^3x
$$

is over a region  $V$  bounded by a surface  $\Sigma$  (an infinitesimal area element being  $\hat{\mathbf{n}}ds$  , where  $\hat{\mathbf{n}}$ is the outward unit normal). If it changes to  $I + \Delta I$  , when the vector  $\bf{A}$  is changed to  $\bf{A} + \nabla \Lambda$  , then  $\Delta l$  can be expressed as **Practice Big COMPLE 10 CONTROVER CONTROVER SET THE AM JEST, TIFR and GRE for Physics**<br>
is over a region V bounded by a surface  $\Sigma$  (an infinitesimal area element being  $\hat{n}ds$ , whis the outward unit normal). If it chang **Education**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
is over a region *V* bounded by a surface  $\Sigma$  (an infinitesimal area element being  $\hat{n}ds$ , with<br>
then  $\Delta l$  can be expressed as<br>
(a)  $\iiint_V \nabla \cdot (\$ 

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.  $\iiint\limits_V \nabla \cdot (\nabla \Lambda \times A) d^3x$  (b)  $-\iiint\limits_V \nabla^2 \Lambda d^3x$  $-\iiint\limits_V \nabla^2 \Lambda \, d^3x$  $-\bigoplus_{\Sigma} (\nabla \Lambda \times \mathbf{A}) \cdot \hat{\mathbf{n}} ds$  (d)  $\bigoplus_{z} \nabla \Lambda$ .  $\hat{\mathbf{n}}$  $\bigoplus_z \nabla \Lambda$ . n̂ds

> Topic: Mathematical Physics Sub Topic: Vector Analysis

Ans. : (c)

Ans. : (c)  
\nSolution: 
$$
I = \iiint A.(\nabla \times A)d^3v
$$
  
\n $I + \Delta I \Rightarrow A + \Delta A$   
\n $I + \Delta I = \iiint (A + \Delta A) \cdot \nabla \times (A + \Delta A)d^3v$   
\n $= \iiint A.(\nabla \times A)d^3v + \iiint \Delta A.(\nabla \times A)d^3v$  [Since,  $\nabla \times \Delta A = 0$ ]  
\n $= I + \iiint \Delta A.(\nabla \times A)d^3v$   
\n $\Delta I = \iiint \Delta A \cdot (\nabla \times A)d^3v = \iiint \Delta A \cdot (\nabla \times A)d^3v = \iiint \nabla \cdot (A \times \Delta A)d^3v$  [Since,  $A \cdot B \times C = B \cdot C \times A$ ]  
\n $= \iiint \nabla \cdot (A \times \Delta A)d^3v = - \iiint \nabla \cdot (\Delta A \times A)d^3v = - \oint (\Delta A \times A)nds$  [Since  $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ ]  
\nQ8. A generic 3×3 real matrix A has eigenvalues 0, 1 and 6 and I is the 3×3 identity matrix. The quantity/quantities that cannot be determined from this information is/are the  
\n(a) eigenvalues  $(I + A)^{-1}$  (b) eigenvalues of  $(I + A^T A)$   
\n(c) determinant of  $A^T A$  (d) rank of A  
\nTopic: Mathematical Physics  
\nAns. : (b)  
\nSolution: (a) eigenvalues  $(I + A)^{-1}$ :  $A + I = B$  hence we know the eigenvalue of B, we can easily find  
\nthe eigenvalue of  $B^{-1}$ .  
\n(b) eigenvalues of  $(I + A^T A)$ : We don't know that what is the matrix  $A^T A$ , so eigenvalue can not

Q8. A generic 3×3 real matrix A has eigenvalues 0, 1 and 6 and I is the 3×3 identity matrix. The quantity/quantities that cannot be determined from this information is/are the

(a) eigenvalues  $(I + A)^{-1}$  $+A$ )<sup>-1</sup> (b) eigenvalues of  $(I + A<sup>T</sup> A)$ 

(c) determinant of  $A^T A$  (d) rank of  $A$ 

 Topic: Mathematical Physics Sub Topic: Matrices

## Ans. : (b)

 $I + A$ <sup>-1</sup>:  $A + I = B$  hence we know the eigenvalue of  $B$  , we can easily find the eigenvalue of  $B^{-1}$  .

(b) eigenvalues of  $(I + A^T A)$ : We don't know that what is the matrix  $A^T A$ , so eigenvalue can not be determined

(c)  $|A^T A| = |A^T |A|| = 0$ , product of eigenvalues of  $A^T$  and  $A$ .

(d) rank of  $A$ , hence eigenvalues are non-degenerate so matrices can be diagonalized and one eigenvalue is zero, so rank will be  $3 - 1 = 2$ .

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Q9. A discrete random variable X takes a value from the set  $\{-1,0,1,2\}$  with the corresponding **Probability**<br> **CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br>
(d) rank of *A*, hence eigenvalues are non-degenerate so matrices can be diagonalized and<br>
eigenvalue is zero, so rank will be  $3-1=2$ .<br>
A disc  $p(X) = \frac{3}{10}, \frac{2}{10}, \frac{2}{10}$  and  $\frac{3}{10}$ , respectively. The probability distribution  $q(Y) = (q(0), q(1), q(4))$  of the random variable  $Y = X^2$  is (a)  $\left(\frac{1}{5}, \frac{3}{5}, \frac{1}{5}\right)$  $\left(\frac{1}{5}, \frac{3}{5}, \frac{1}{5}\right)$  (b)  $\left(\frac{1}{5}, \frac{1}{2}, \frac{3}{10}\right)$  $\left(\frac{1}{5}, \frac{1}{2}, \frac{3}{10}\right)$  (c)  $\left(\frac{2}{5}, \frac{2}{5}, \frac{1}{5}\right)$  $\left(\frac{2}{5}, \frac{2}{5}, \frac{1}{5}\right)$  (d)  $\left(\frac{3}{10}, \frac{3}{10}, \frac{2}{5}\right)$  $\left(\frac{3}{10}, \frac{3}{10}, \frac{2}{5}\right)$  Topic: Mathematical Physics Sub Topic: Probability

Ans. : (b)

Solution:  $X = [-1,0,1,2]$   $p(X) = 3/10, 3/10, 2/10, 3/10$  $q(y) = [q(0), q(1), q(4)]$ ,  $y = x^2$  $q(y) = [q(0), q(1), q(4)]$ ,  $y = x^2$  $P(y=0) = P(x=0) = 2/10$  [sinec,  $y = x^2$ ]  $P(y=1) = P(x=1) + P(x=-1) = 3/10 + 2/10 = 5/10 = 1/2$  [sinec,  $y = x^2$ ]  $P(y = 0) + P(y = 1) + P(y = 4) = 1 \Rightarrow P(y = 4) = 1 - \frac{2}{10} - \frac{1}{2} = 1 - \frac{7}{10} = \frac{3}{10}$ Thus, the correct probability distribution is  $2/10$ ,  $1/2$ ,  $3/10$ 

- Q10. The components of the electric field, in a region of space devoid of any charge or current sources, are given to be  $E_i = a_i + \sum_{j=1,2,3} b_{ij} x_j$  , where  $a_i$  and  $b_{ij}$  are constants independent of the coordinates. The number of independent components of the matrix  $b_{ij}$  is
	- (a) 5 (b) 6 (c) 3 (d) 4

Topic: Electromagnetic Theory

Sub Topic: Relativistic Electrodynamics

# (Field Tensor)

Ans. : (a)

Solution: Given that  $E_i = a_i + \sum_{j=1,2,3} b_{ij} x_j$ 

From the given condition, the  $b_{ij}$  should be symmetric and traceless

The typical feature of  $b_{ij}$  is

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F, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $\begin{bmatrix} u & v & w \\ v & p & s \\ w & s & r \end{bmatrix}$ <br>  $+ p + r = 0$  (It will reduce one independent component) **Pravegale Education**<br>
ET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $b_{ij} = \begin{bmatrix} u & v & w \\ v & p & s \\ w & s & r \end{bmatrix}$ <br>
eans  $u + p + r = 0$  (It will reduce one independent component) **Education**<br>
F, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $\begin{bmatrix} u & v & w \\ v & p & s \\ w & s & r \end{bmatrix}$ <br>  $\begin{bmatrix} v & p & s \\ v & + p + r = 0 \end{bmatrix}$ <br>  $\begin{bmatrix} u & v & w \\ w & w & r \end{bmatrix}$ <br>  $\begin{bmatrix} u & v & w \\ w & w & r \end{bmatrix}$ <br>  $\begin{bmatrix} u & v & w \\ w & w & r \end{bmatrix}$ <br>  $\begin{bmatrix} u & v & w \\$ **avegara** Education<br>
RF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $\begin{bmatrix} u & v & w \\ v & p & s \\ w & s & r \end{bmatrix}$ **TaVegale Education**<br>
JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $=\begin{bmatrix} u & v & w \\ v & p & s \\ w & s & r \end{bmatrix}$ <br>
as  $u + p + r = 0$  (It will reduce one independent component) **Education**<br>
RF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $\begin{bmatrix} u & v & w \\ v & p & s \\ w & s & r \end{bmatrix}$ <br>  $u + p + r = 0$  (It will reduce one independent component)<br>
II reduce three independent components

$$
b_{ij} = \begin{bmatrix} u & v & w \\ v & p & s \\ w & s & r \end{bmatrix}
$$

The trace less of  $b_{ij}$  means  $u + p + r = 0$  (It will reduce one independent component)

The symmetric property will reduce three independent components

Thus,  $b_{ij}$  will have 9-3-1 =5 Independent component

**Practice CONTE, IT-JAM, JEST, TIFR and GRE for Physics**<br>  $b_y = \begin{bmatrix} u & v & w \\ v & p & s \\ w & s & r \end{bmatrix}$ <br>
The trace less of  $b_y$  means  $u + p + r = 0$  (It will reduce one independent component)<br>
The symmetric property will reduce three in bar magnet moves with a constant velocity towards the wire along the  $z$ -axis (as shown in the figure below)



We take  $t = 0$  to be the instant at which the midpoint of the magnet is at the centre of the wire loop and the induced current to be positive when it is counter-clockwise as viewed by the observer facing the loop and the incoming magnet. In these conventions, the best schematic The symmetric property will reduce three independent components<br>Thus,  $b_y$  will have 9-3-1 -5 Independent components<br>Thus,  $b_y$  will have 9-3-1 -5 Independent components<br>A conducting wire in the shape of a circle lies on



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Initially, at  $t = 0$ , the flux will be zero. Once, the bar magnet move towards circular loop, the associated flux will increase. When bar magnet reaches the circular loop, the flux attain constant. But, once bar magnet cross circular loop and move away from loop, then the associated flux will decrease.

We know that,



Q12. In an experiment to measure the charge to mass ratio  $\frac{e}{e}$ m of the electron by Thomson's method, the values of the deflecting electric field and the accelerating potential are  $6 \times 10^6$  N / C (newton per coulomb) and  $150V$ , respectively. The magnitude of the magnetic field that leads to zero deflection of the electron beam is closest to an experiment to measure the charge to mass ratio  $\frac{e}{m}$  of the electron by Thom<br>values of the deflecting electric field and the accelerating potential are  $6 \times 10^6$ <br>coulomb) and  $150V$ , respectively. The magnitude of an experiment to measure the charge to mass ratio  $\frac{e}{m}$  of the electron by Thomson<br>
e values of the deflecting electric field and the accelerating potential are 6×10° N<br>
r coulomb) and 150V, respectively. The magnitud to measure the charge to mass ratio  $\frac{e}{m}$  of the electron by Thomson's method,<br>
deflecting electric field and the accelerating potential are  $6 \times 10^6$  N/C (newton<br>
d 150*F*, respectively. The magnitude of the magneti The charge to mass ratio  $\frac{e}{m}$  of the electron by Thomson's method,<br>ectric field and the accelerating potential are  $6 \times 10^6$  N/C (newton<br>m is closest to<br>r (c) 0.4 T (d) 0.8 T<br>Topic: Electromagnetic Theory<br>Sub Topic: Ant to measure the charge to mass ratio  $\frac{e}{m}$  of the electron by Thomson's method,<br>
the deflecting electric field and the accelerating potential are  $6 \times 10^6$  N/C (newton<br>
and 150*V*, respectively. The magnitude of t In an experiment to measure the charge to mass ratio  $\frac{e}{m}$  of the electron by Thomson's met<br>the values of the deflecting electric field and the accelerating potential are  $6 \times 10^6$  N/C (ne<br>per coulomb) and 150V, resp n experiment to measure the charge to mass ratio  $\frac{e}{m}$  of the electron by Tho<br>values of the deflecting electric field and the accelerating potential are 6×10<br>coulomb) and 150*V*, respectively. The magnitude of the mag experiment to measure the charge to mass ratio  $\frac{e}{m}$  of the electron by Thoms<br>alues of the deflecting electric field and the accelerating potential are 6×10<sup>6</sup><br>coulomb) and 150*V*, respectively. The magnitude of the m charge to mass ratio  $\frac{e}{m}$  of the electron by Thomson's method,<br>ic field and the accelerating potential are  $6 \times 10^6$  N/C (newton<br>viely. The magnitude of the magnetic field that leads to zero<br>closest to<br>(c) 0.4 T (d) tric field and the accelerating potential are  $6 \times 10^6$  N / C (newton<br>
teric field and the accelerating potential are  $6 \times 10^6$  N / C (newton<br>
retively. The magnitude of the magnetic field that leads to zero<br>
is closest Example 1<br>
reasure the charge to mass ratio  $\frac{e}{m}$  of the electron by Thomson's method,<br>
tring electric field and the accelerating potential are 6×10<sup>6</sup> N/C (newton<br>
V, respectively. The magnitude of the magnetic field Final and the accelerating potential are  $6 \times 10^6$  N / C (newton<br>field and the accelerating potential are  $6 \times 10^6$  N / C (newton<br>ely. The magnitude of the magnetic field that leads to zero<br>losest to<br>(c) 0.4 T (d) 0.8 T

(a) 
$$
0.6 T
$$
 (b)  $1.2 T$  (c)  $0.4 T$  (d)  $0.8 T$ 

Topic: Electromagnetic Theory

Sub Topic: Magnetostat (Thomson Experiment)

Ans. : (d)

Solution: 
$$
\frac{e}{m} = \frac{1}{2} \frac{E^2}{B^2 V} \Rightarrow 1.77 \times 10^{11} = \frac{1}{2} \times \frac{36 \times 10^{12}}{B^2 \times 150}
$$

$$
\frac{1.77 \times 10^{11} \times 300}{36 \times 10^{12}} = \frac{1}{B^2} \Rightarrow B^2 = \frac{36 \times 10^{12}}{1.77 \times 3 \times 10^{13}} = \frac{3.6}{1.77 \times 3}
$$

$$
B = 0.8 T
$$



Q13. A monochromatic source emitting radiation with a certain frequency moves with a velocity  $V$ away from a stationary observer A . It is moving towards another observer B (also at rest) along a line joining the two. The frequencies of the radiation recorded by A and B are  $V_A$  and  $V_B$ ,

respectively. If the ratio  $\frac{v_B}{r} = 7$ A  $v_i$  $\mathcal{V}_1$  $=$  7, then the value of  $\frac{v}{v}$ c is

(a)  $\frac{1}{ }$ 2 (b)  $\frac{1}{2}$ 4 (c)  $\frac{3}{2}$ 4 (d) 3 2

Topic: Classical Mechanics

Sub Topic: STR Doppler effect

Ans. : (c)

Solution: Using concept of doppler effect of light. For observer  $B$ 1  $\int_B$  =  $\int$   $\sqrt{1}$ . v  $\overline{c}$ v c  $V_R = V$  $\ddot{}$  $=$  $\overline{\phantom{a}}$ and for observer A

$$
V_A = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}
$$
 it is given  $\frac{V_B}{V_A} = \frac{\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}}{\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}} = 7 \Rightarrow \frac{1+\frac{v}{c}}{1-\frac{v}{c}} = 7 \Rightarrow \frac{v}{c} = \frac{3}{4}$ 

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Q14. The Hamiltonian of a particle of mass  $m$  in one-dimension is  $H = \frac{1}{2} p^2 + \lambda |x|^3$  $H = \frac{1}{2m} p^2 + \lambda |x|^3$ , where  $\lambda > 0$  is constant. If  $E_1$  and  $E_2$ , respectively, denote the ground state energies of the particle for  $\lambda = 1$ and  $\lambda = 2$  (in appropriate units) the ratio  $\frac{E_2}{E_1}$ 1  $E_{2}$  . The contract of the  $E_1$  is best approximated by is best approximated by (a)  $1.260$  (b)  $1.414$  (c)  $1.516$  (d)  $1.320$ **EXECUTE:** CATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
The Hamiltonian of a particle of mass *m* in one-dimension is  $H = \frac{1}{2m} p^2 + \lambda |x|^2$ , where  $\lambda > 0$  is<br>
constant. If  $E_1$  and  $E_2$ , respectively, denote the grou **CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br>
mian of a particle of mass *M* in one-dimension is  $H = \frac{1}{2m} p^2 + \lambda |x|^2$ , where  $\lambda > E_1$  and  $E_2$ , respectively, denote the ground state energies of the par **CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physic<br>
Hamiltonian of a particle of mass** *m* **in one-dimension is**  $H = \frac{1}{2m}p^3 + \lambda |x|^3$ **<br>
ttant. If**  $E_1$  **and**  $E_2$ **, respectively, denote the ground state energies of t** CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
The Hamiltonian of a particle of mass *m* in one-dimension is  $H = \frac{1}{2m} p^2 + \lambda |x|^3$ , where  $\lambda > 0$  is<br>
constant. If  $E_1$  and  $E_2$ , respectively, denote the g **EXECUTE DESPATE.** IT JAM, JEST, TIFR and GRE for Physics<br>
e Hamiltonian of a particle of mass *m* in one-dimension is  $H = \frac{1}{2m}p^2 + \lambda |x|^2$ , where  $\lambda > 0$  is<br>
nstant. If  $E_1$  and  $E_2$ , respectively, denote the ground be Hamiltonian of a particle of mass *m* in one-dimension is  $H = \frac{1}{2m} p^2 + \lambda |x|^3$ , where  $\lambda > 0$  is<br>
mstant. If  $E_1$  and  $E_2$ , respectively, denote the ground state energies of the particle for  $\lambda = 1$ <br>
d  $\lambda = 2$  (in miltonian of a particle of mass *m* in one-dimension is  $H = \frac{1}{2m} p^2 + \lambda |x|^2$ , where  $\lambda > 0$  is<br>
t. If  $E_1$  and  $E_2$ , respectively, denote the ground state energies of the particle for  $\lambda = 1$ <br>  $= 2$  (in appropriate un

Topic: Quantum Mechanics

Sub Topic: WKB Approximation

# Ans. : (d)

Solution: Quantum mechanics semiclassical method

constant. If 
$$
E_1
$$
 and  $E_2$ , respectively, denote the ground state energies of the particle for  $\lambda$   
and  $\lambda = 2$  (in appropriate units) the ratio  $\frac{E_2}{E_1}$  is best approximated by  
(a) 1.260 (b) 1.414 (c) 1.516 (d) 1.320  
Topic: **Quantum Mechanics**  
Sub Topic: **WKB Approximation**  
(d) 1.320  
Topic: **WKB Approximation**  
(e) 1.516 (f) 1.320  
Topic: **WKB Approximation**  
 $V(x)$   
 $V(x)$   
 $V(x)$   
 $J = 4 \int_0^{\frac{E}{2}} \sqrt{2m(E - \lambda x^3)} dx = h$  for ground state  $n = 1$   
 $\sqrt{2mE} \left(\frac{E}{\lambda}\right)^{1/3} \approx h \Rightarrow E^{\frac{1}{2} + \frac{1}{3}} \propto \lambda^{\frac{1}{3}} \Rightarrow E^{\frac{5}{6}} \propto \lambda^{\frac{1}{2}} \Rightarrow E \propto \lambda^{\frac{2}{5}}$   
 $-\left(\frac{E}{\lambda}\right)^{1/3} \sqrt{2mE} \left(\frac{E}{\lambda}\right)^{1/3}$   
 $E(\lambda = 2) = (2)^{2/5} = 1.32$   
A particle of mass *m* is in a one dimensional infinite potential well of length *L*, extending f  
 $x = 0$  to  $x = L$ . When it is in the energy eigenstate labelled by  $n, (n = 1, 2, 3, ...)$  the probat



Q15. A particle of mass m is in a one dimensional infinite potential well of length  $L$ , extending from Topic: Quantum Mechanics<br>
Sub Topic: WKB Approximation<br>  $x$ : Quantum mechanics semiclassical method<br>
Using Bohr Somerfield theorem  $J = \oint_{\mathcal{D}} pdx = nh$ <br>  $J = 4\int_{0}^{E} \sqrt{2m(E - \lambda x^2)} dx = h$  for ground state  $n = 1$ <br>  $\sqrt{2mE} \left(\frac{E}{$ of finding it in the interval  $0$ 8  $\leq x \leq \frac{L}{8}$  is  $\frac{1}{8}$ . The minimum value of  $n$  for which this is possible is (a) 4 (b) 2 (c) 6 (d) 8 Topic: Quantum Mechanics  $\frac{1}{2}$   $\Rightarrow E \propto \lambda^{\frac{2}{3}}$   $- \left(\frac{E}{\lambda}\right)^{1/3}$ <br>
and infinite potential well of length L, extending from<br>
eigenstate labelled by  $n, (n = 1, 2, 3,...)$  the probability<br>
The minimum value of  $n$  for which this is possible is<br> x  $\lambda^3$   $-\left(\frac{E}{\lambda}\right)^{1/3}$   $\sqrt{2mE}\left(\frac{E}{\lambda}\right)^{1/3}$ <br>
ite potential well of length L, extending from<br>
te labelled by  $n, (n = 1, 2, 3,...)$  the probability<br>
inimum value of  $n$  for which this is possible is<br>
(d) 8<br>
Topic: Qu  $\int \sqrt[n]{x} \, dx \, dx$ <br>  $\int \sqrt[n]{2mE} \left(\frac{E}{\lambda}\right)^{1/3}$ <br>  $\int \sqrt{2mE} \left(\frac{E}{\lambda}\right)^{1/3}$ <br>  $\int \sqrt[n]{2mE} \left(\frac{E}{$  $-\left(\frac{E}{\lambda}\right)^{1/3}$   $\sqrt{2mE} \left(\frac{E}{\lambda}\right)^{1/3}$ <br>
Ential well of length L, extending from<br>
Elled by  $n, (n = 1, 2, 3, ...)$  the probability<br>
n value of  $n$  for which this is possible is<br>
(d) 8<br>
Topic: Quantum Mechanics<br>
Sub Topic  $\lambda^{\frac{1}{2}} \Rightarrow E \propto \lambda^{\frac{2}{3}}$ <br>
and infinite potential well of length *L*, extending from<br>
eigenstate labelled by *n*, (*n* = 1, 2, 3, ...) the probability<br>  $\frac{1}{\lambda}$ . The minimum value of *n* for which this is possible i  $\lambda^{\frac{1}{2}} \Rightarrow E \propto \lambda^{\frac{2}{3}}$ <br>
and infinite potential well of length L, extending from<br>
eigenstate labelled by  $n$ ,  $(n = 1, 2, 3,...)$  the probability<br>  $\frac{1}{3}$ . The minimum value of  $n$  for which this is possible is<br>
(c) 6 (  $\frac{n(A-2)}{E(\lambda-1)} = (2)^{2/3} = 1.32$ <br>
A particle of mass m is in a one dimensional infinite potential well of length L, extending from<br>  $x = 0$  to  $x = L$ . When it is in the energy eigenstate labelled by  $n,(n = 1,2,3,...)$  the probab ( $A = 1$ )<br>
article of mass m is in a one dimensional infinite potential well of length L, extending from<br>  $= 0$  to  $x = L$ . When it is in the energy eigenstate labelled by  $n_r(n = 1, 2, 3, ...)$  the probability<br>
finding it in the  $\frac{1}{2} = (2)^{2/3} = 1.32$ <br>
tricle of mass *m* is in a one dimensional infinite potential well of length *L*, extending from<br>
to  $x = L$ . When it is in the energy eigenstate labelled by *n*,  $(n = 1, 2, 3,...)$  the probability<br>
dd *E*( $\angle$ = 1)<br>
A particle of mass *m* is in a one dimensional infinite potential well of length *L*, extending from<br>  $x = 0$  to  $x = L$ . When it is in the energy eigenstate labelled by *n*, (*n* = 1, 2, 3,...) the probability <sup>15</sup> = 1.32<br>
ss *m* is in a one dimensional infinite potential well of length *L*, extending from<br>
When it is in the energy eigenstate labelled by *n*,  $(n = 1, 2, 3,...)$  the probability<br>
le interval  $0 \le x \le \frac{L}{8}$  is  $\frac{1}{$  $\frac{z-2}{z-1}$  = (2)<sup>273</sup> = 1.32<br>
tride of mass m is in a one dimensional infinite potential well of length L, extending from<br>
b to  $x = L$ . When it is in the energy eigenstate labelled by  $n,(n=1,2,3,...)$  the probability<br>
ddin  $x = 0$  to  $x = L$ . When it is in the energy eigenstate labelled by  $n_1(n = 1, 2, 3,...)$  the probability<br>
of finding it in the interval  $0 \le x \le \frac{L}{8}$  is  $\frac{1}{8}$ . The minimum value of *M* for which this is possible is<br>
(a) 4

Sub Topic: Particle in Box

Ans. : (a)

Solution: For particle in one dimensional box  $P\left(0 \le x \le \frac{L}{2}\right) = \int_{0}^{8} \frac{2}{3} \sin^2 x$ 

$$
\frac{2}{L}\frac{1}{2}\int_{0}^{\sqrt{2}}\left(1-\cos\frac{2n\pi x}{L}\right)dx = \frac{1}{8}\Rightarrow\frac{1}{8}-\frac{1}{2n\pi}\sin\frac{2n\pi}{8} = \frac{1}{8}\Rightarrow\sin\frac{n\pi}{4} = 0\Rightarrow n = 4
$$

Q16. A two-state system evolves under the action of the Hamiltonian of finding it in the interval  $0 \le x \le \frac{L}{8}$  is  $\frac{1}{8}$ . The minimum value of *H* for which this is possible is<br>
(a) 4 (b) 2 (c) 6 (d) 8<br>
Topic: Quantum Mechanics<br>
Sub Topic: Particle in Box<br>
a)<br>
(c) 6 (d) 8<br>
Topic: Qu the system is in a state of definite parity  $P=1$ , the earliest time t at which the probability of finding the system in a state of parity  $P = -1$  is one, is

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(a) 
$$
\frac{\pi \hbar}{2\Delta}
$$
 \t\t (b)  $\frac{\pi \hbar}{\Delta}$  \t\t (c)  $\frac{2\pi \hbar}{2\Delta}$  \t\t (d)  $\frac{2\pi \hbar}{\Delta}$ 

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 Topic: Quantum Mechanics Sub Topic: Postulates of Quantum Mechanics

Ans. : (b)

Ans.: (b)  
\n
$$
\Delta \qquad \text{Topic: Quantum Mechanics}
$$
\nShb Topic: Postulates of Quantum Mechanics\nShdution:  
\n
$$
H = E_0 |A\rangle \langle A| + (E_0 + \Delta)|B\rangle \langle B| - P|A\rangle = |B\rangle, P|B\rangle = |A\rangle
$$
\n
$$
H|A\rangle = E_0 A, H|B\rangle = (E_0 + \Delta)|B\rangle
$$
\nSo  $|A\rangle$  and  $|B\rangle$  are eigen vector of Hamiltonian with eigen value  $E_0$  and  $E_0 + \Delta$ \nAt  $t = 0, |\psi\rangle = c_1 |A\rangle + c_2 |B\rangle$ \nIt is given  $P|\psi\rangle = \psi \Rightarrow c_1 |B\rangle + c_2 |A\rangle = c_1 |A\rangle + c_2 |B\rangle \Rightarrow c_1 = c_2 = \frac{1}{\sqrt{2}}$ \nNow  $|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left( |A\rangle \exp\left(-\frac{iE_0 t}{\hbar}\right) + |B\rangle \exp\left(-\frac{i(E_0 + \Delta)t}{\hbar}\right) \right)$ \n
$$
P|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left( |B\rangle \exp\left(-\frac{iE_0 t}{\hbar}\right) + |A\rangle \exp\left(-\frac{i(E_0 + \Delta)t}{\hbar}\right) \right) = -|\psi(t)\rangle
$$
\n
$$
\frac{1}{\sqrt{2}} \left( |B\rangle \exp\left(-\frac{iE_0 t}{\hbar}\right) + |A\rangle \exp\left(-\frac{i(E_0 + \Delta)t}{\hbar}\right) \right) = -\left( \frac{1}{\sqrt{2}} \left( |A\rangle \exp\left(-\frac{iE_0 t}{\hbar}\right) + |B\rangle \exp\left(-\frac{i(E_0 + \Delta)t}{\hbar}\right) \right)
$$
\n
$$
|B\rangle + A \exp\left(\frac{i\Delta t}{\hbar}\right) = -1A\rangle - B \exp\left(\frac{i\Delta t}{\hbar}\right) \Rightarrow \exp\left(\frac{i\Delta t}{\hbar}\right) = -1
$$
\n
$$
\Rightarrow \exp\left(\frac{i\Delta t}{\hbar}\right) = -1 \Rightarrow \cos\left(\frac{\Delta t}{\hbar}\right) = -1 \Rightarrow t = \frac{\pi\hbar}{\Delta}
$$

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Q17. The figures below depict three different wavefunctions of a particle confined to a one-



The wavefunctions that correspond to the maximum expectation values  $\vert \langle x \rangle \vert$  (absolute value of the mean position) and  $\langle x^2 \rangle$ , respectively, are

(a) B and C (b) B and A (c) C and B (d) A and B Topic: Quantum Mechanics

Sub Topic: One Dimensional Box

Ans.  $:(a)$ 

Solution: The probability distribution is symmetric about  $x = 0$  for  $A, C$  so  $\langle X \rangle = 0$  for these wave function in case B the area under of probability density is more in region  $x > 0$  so  $\langle X \rangle > 0$  in the mean position) and  $\langle x^2 \rangle$ , respectively, are<br>
(a) B and C (c) C and B Topic: Outarium Mechanics<br>
Sub Topic: One Dimensional Box<br>
a)<br>
n: The probability distribution is symmetric about  $x = 0$  for  $A$ ,  $C$  so  $\langle X \rangle =$ 

case B.<br>The maximum fluctuation is in C so max  $\langle x^2 \rangle$  in C So answer is B and C

- Q18. Which of the following two physical quantities cannot be measured simultaneously with arbitrary accuracy for the motion of a quantum particle in three dimensions? case *B*.<br>
The maximum fluctuation is in *C* so max  $\langle x^2 \rangle$  in *C*. So answer is *B* and *C*<br>
Q18. Which of the following two physical quantities cannot be measured simultaneously with arbitrary<br>
accuracy for the motion
	-
	-
	- (c) y -component of position and z-component of angular momentum (y and  $L_z$ )
	- (d) squares of the magnitudes of the linear and angular momenta (  $p^2$  and  $L^2$  )

 Topic: Quantum Mechanics Sub Topic: Angular Momentum

Ans. : (c)

Q19. The ratio  $\frac{C_p}{C}$ V  $C_{i}$  $\frac{C_P}{C_V}$  of the specific heats at constant pressure and volume of a monoatomic ideal gas

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in two dimensions is

(a) 
$$
\frac{3}{2}
$$
 (b) 2 (c)  $\frac{5}{3}$ 

$$
\frac{5}{3} \qquad \qquad (d) \frac{5}{2}
$$

 Topic: Thermodynamics & Statistical Mechanics Sub Topic: Kinetic Theory of Gas

Ans. : (b) Solution:  $C_V = \frac{f}{2} R$ ,  $f =$  D.O.F.

For monoatomic in 2D

$$
C_V = \frac{2}{2}R = R
$$

We know that  $C_p - C_V = R$ 

$$
C_p = 2R
$$

$$
\frac{C_p}{C_V} = \frac{2R}{R} = 2
$$

Q20. The volume and temperature of a spherical cavity filled with black body radiation are  $V$  and 300K respectively. If it expands adiabatically to a volume  $2V$  , its temperature will be closest to (a) 150 K (b) 300 K (c) 250 K (d) 240 K We know that  $C_p - C_V = R$ <br>  $C_p = 2R$ <br>  $\frac{C_p}{C_r} = \frac{2R}{R} = 2$ <br>
The volume and temperature of a spherical cavity filled with black body radiation<br>
300 K respectively. If it expands adiabatically to a volume 2V , its temperatur  $C_p = 2R$ <br>  $\frac{C_p}{C_r} = \frac{2R}{R} = 2$ <br>
volume and temperature of a spherical cavity filled with black body radial<br>
respectively. If it expands adiabatically to a volume 2V, its temperature<br>
(0)  $\frac{d}{dx}$  (1)  $\frac{d}{dx}$  (1)  $\frac$ Q20. The volume and temperature of a spherical cavity filled with black body radiation are V and<br>
300K respectively. If it expands adiabatically to a volume 2V , its temperature will be closest to<br>
(a) 150 K (b) 300 K (c)

 Topic: Thermodynamics & Statistical Mechanics Sub Topic: Black body radiation

Ans. : (d)

Solution: For adiabatic in black body

 $VT^3 =$ Constant

$$
V \times (300)^3 = K \qquad \text{(i}
$$

$$
2V \times (T)^3 = K \qquad \text{(ii)}
$$

By dividing (ii) by (i)  $T = \frac{300}{2^{1/3}} = 240$  $T = \frac{300}{2^{1/3}} = 2$ 

 $\int_{0}^{\omega_{D}} g(\omega) d\omega = 3N$  , where  $N$  is

the number of primitive cells,  $Q_j$  is the Debye frequency and density of photon modes is

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<sup>2</sup> g AV (with A <sup>0</sup> a constant). If the density of the solid doubles in a phase transition,

the Debye temperature  $\theta$ <sub>D</sub> will

- (a) increase by a factor of  $2^{2/3}$  (b) increase by a factor of  $2^{1/3}$
- (c) decrease by a factor of  $2^{2/3}$  (d) decrease by a factor of  $2^{1/3}$

Topic: Solid State Physics

Sub Topic: Lattice Specific Heat, Lattice Vibration

Ans. : (b)

**PROOF EXAMPLE 18.1 CP UCATION CSIR NET-JRF, GATE, HIT- JAM, JEST, TIFR and GRE for Physics** 
$$
g(\omega) = AV\omega^2
$$
 (with  $A > 0$  a constant). If the density of the solid doubles in a phase transition, the Debye temperature  $\theta_0$  will\n\n(a) increase by a factor of  $2^{1/3}$ \n(b) increase by a factor of  $2^{1/3}$ \n(c) decrease by a factor of  $2^{1/3}$ \n(d) decrease by a factor of  $2^{1/3}$ \n**Topic: Solid State Physics Sub Topic: Lattice Specific Heat, Lattice Vibration**  $\int_0^{\infty} AV\omega^2 d\omega = N \Rightarrow AV\frac{\omega_0^3}{3} = N \Rightarrow \omega_0^3 \alpha \frac{N}{V} \alpha \rho \Rightarrow \theta_0^3 \alpha \rho$  [Since,  $\omega_0 \alpha \theta_0$ ]\n\nIf  $\rho \rightarrow 2\rho \Rightarrow \theta'_{D} \alpha 2^{\frac{1}{2}} \rho^{\frac{1}{3}}$ \n\nThus, the Debye temperature is increased by  $2^{\frac{1}{2}}$ \n\nQ22. The position of a particle in one dimension changes in discrete steps. With each step it moves to the right, however, the length of the step is drawn from a uniform distribution from the interval  $\left[\lambda - \frac{1}{2}w, \lambda + \frac{1}{2}w\right]$ , where  $\lambda$  and  $W$  are positive constants. If  $X$  denotes the distance from the starting point after  $N$  steps, the standard deviation  $\sqrt{\langle X^2 \rangle - \langle X \rangle^2}$  for large values of  $N$  is\n(a)  $\frac{\lambda}{2} \times \sqrt{N}$ \n(b)  $\frac{\lambda}{2} \times \sqrt{\frac{N}{3}}$ \n\nTopic: Thermodynamics & Statistical Mechanics Sub Topic: Random Walk Problem

Thus, the Debye temperature is increased by  $2^{\frac{1}{3}}$  $2^3$ 

Q22. The position of a particle in one dimension changes in discrete steps. With each step it moves to the right, however, the length of the step is drawn from a uniform distribution from the interval

Physics<br>
e Specific Heat, Lattice Vibration<br>  $\alpha \rho$  [Since,  $\omega_0 \alpha \theta_0$ ]<br>
siscrete steps. With each step it moves to<br>
a uniform distribution from the interval<br>
ants. If X denotes the distance from the<br>  $\sqrt{X^2}$   $-\langle X \rangle^2$  $\Rightarrow \theta_{\rho}^3 \alpha \rho$  [Since,  $\omega_{\rho} \alpha \theta_{\rho}$ ]<br>
sin discrete steps. With each step it moves to<br>
from a uniform distribution from the interval<br>
constants. If X denotes the distance from the<br>
n  $\sqrt{\langle X^2 \rangle - \langle X \rangle^2}$  for large va w N

starting point after  $N$  steps, the standard deviation  $\sqrt{\langle X^2 \rangle - {\langle X \rangle}^2}$  to

(a)  $\frac{\lambda}{\cdot} \times \sqrt{N}$  $\frac{\lambda}{2} \times \sqrt{N}$  $\frac{\lambda}{\lambda} \times \sqrt{\frac{N}{2}}$  (c)  $\frac{W}{X} \times \sqrt{N}$  (d)  $\frac{W}{X} \times \sqrt{\frac{N}{2}}$ 2  $\frac{w}{\sqrt{N}}$  (d)  $\frac{w}{\sqrt{N}} \times \sqrt{\frac{N}{N}}$ 

> Topic: Thermodynamics & Statistical Mechanics Sub Topic: Random Walk Problem

Ans. : (d)

Thus, the Debye temperature is increased by 
$$
2^{\frac{1}{3}}
$$
  
\nQ22. The position of a particle in one dimension changes in discrete steps. With each step it moves to  
\nthe right, however, the length of the step is drawn from a uniform distribution from the interval  
\n
$$
\left[\lambda - \frac{1}{2}w, \lambda + \frac{1}{2}w\right]
$$
, where  $\lambda$  and  $W$  are positive constants. If  $X$  denotes the distance from the  
\nstarting point after  $N$  steps, the standard deviation  $\sqrt{\langle X^2 \rangle - \langle X \rangle^2}$  for large values of  $N$  is  
\n(a)  $\frac{\lambda}{2} \times \sqrt{N}$  (b)  $\frac{\lambda}{2} \times \sqrt{\frac{N}{3}}$  (c)  $\frac{w}{2} \times \sqrt{N}$  (d)  $\frac{w}{2} \times \sqrt{\frac{N}{3}}$   
\nTopic: Thermodynamics & Statistical Mechanics  
\nSub Topic: Random Walk Problem  
\nAns. : (d)  
\nSolution:  $\sqrt{\langle X^2 \rangle - \langle X \rangle^2} = \sqrt{\frac{N^2}{2} \left(\lambda - \frac{w}{2}\right)^2 + \frac{N^2}{2} \left(\lambda + \frac{w}{2}\right)^2 - \left[\frac{N}{2} \left(\lambda - \frac{w}{2} + \lambda + \frac{w}{2}\right)\right]^2}$   
\n $\sqrt{\frac{N^2}{2} \left(\lambda - \frac{w}{2}\right)^2 + \frac{N^2}{2} \left(\lambda + \frac{w}{2}\right)^2 - \left[\frac{N}{2} \left(\lambda - \frac{w}{2} + \lambda + \frac{w}{2}\right)\right]^2} = \sqrt{\frac{N^2w^2}{4}} = \frac{w}{2}\sqrt{N}$   
\nFor one dimension  $N = \frac{N}{3}$ . Thus,  $\sqrt{\langle X^2 \rangle - \langle X \rangle^2} = \frac{w}{2} \sqrt{\frac{N}{3}}$   
\nQ23. In the LCR circuit shown below, the resistance  $R = 0.05 \Omega$ , the inductance  $L = 1 H$  and the  
\ncapacitance  $C = 0.04 F$ .

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If the input  $V_{in}$  is a square wave of angular frequency 1 rad/s the output  $V_{out}$  is best approximated by a

- (a) square wave of angular frequency  $1 rad/s$
- (b) sine wave of angular frequency  $1 rad/s$
- (c) square wave of angular frequency  $5$  rad/s
- (d) sine wave of angular frequency  $5 \text{ rad/s}$

# Topic: Electronics Sub Topic: AC Circuit

## Ans. : (d)

Q24. In an experiment, the velocity of a non-relativistic neutron is determined by measuring the time of the input  $V_{\omega i}$  is a square wave of angular frequency 1 rad/s the output  $V_{\omega d}$  is best<br>approximated by a<br>(a) square wave of angular frequency 1 rad/s<br>(b) sine wave of angular frequency 1 rad/s<br>(c) square wave of the error in the measurement of  $L$  is negligibly small. If we want to estimate the kinetic energy the velocity of a non-relativistic neutron is determined by measuring the time<br>
to travel from the source to the detector kept at a distance L. Assume that<br>
surement of L is negligibly small. If we want to estimate the ki be velocity of a non-relativistic neutron is determined by measuring the time<br>
o travel from the source to the detector kept at a distance L. Assume that<br>
surement of L is negligibly small. If we want to estimate the kine the velocity of a non-relativistic neutron is determined by measuring the time<br>
ses to travel from the source to the detector kept at a distance L. Assume that<br>
measurement of L is negligibly small. If we want to estimate **In an experiment, the velocity of a non-relativistic neutron is determined by measuring the time**  $(-50 \text{ ns})$  **it takes to travel from the source to the detector kept at a distance** *L***. Assume that the error in the measurem** the state of two the source to the detector kept at a distance  $L$ . Assume that<br>
measurement of  $L$  is negligibly small. If we want to estimate the kinetic energy<br>
on to within 5% accuracy i.e.,  $\left| \frac{\partial T}{T} \right| \le 0.05$ , t

 $T$  of the neutron to within 5% accuracy i.e.,  $\left|\frac{\partial T}{\partial x}\right| \leq 0.05$  , the maximum permissible error  $\left| \cdot \right|$ T  $\frac{\delta T}{\delta T}\leq 0.05$  , the maximum permissible error  $|\delta T|$ 

in measuring the time of flight is nearest to

(a)  $1.75 \text{ ns}$  (b)  $0.75 \text{ ns}$  (c)  $2.25 \text{ ns}$  (d)  $1.25 \text{ ns}$ 

 Topic: Electronics Sub Topic: Experimental Technique

Ans. : (d)

Solution:  $t = 50 ns$ 

\n
$$
(2-50 \text{ m})
$$
 it takes to the order from the source to the detection kept at a distance *L*. Assume that the error in the measurement of *L* is negligibly small. If we want to estimate the kinetic energy *T* of the neutron to within 5% accuracy, i.e.,  $\left| \frac{\delta T}{T} \right| \leq 0.05$ , the maximum permissible error  $|\delta T|$  in measuring the time of flight is nearest to\n

\n\n(a) 1.75 *ns* (b) 0.75 *ns* (c) 2.25 *ns* (d) 1.25 *ns* **Topic: Electronic** Sub Topic: **Electronic** Sub Topic: **Experimental Technique**\n

\n\n(d) 0\n

\n\n $m: t = 50 \text{ ns}$ \n

\n\n $\frac{\delta T}{T} = 0 + 0 + 2 \frac{\delta T}{T} = \frac{0.05 \times 50}{2} = \Delta t$ \n

\n\n $\Delta t = 25 \times 0.05 = 1.25 \text{ ns}$ \n

\n\n $\Delta t = 25 \times 0.05 = 1.25 \text{ ns}$ \n

\n\n $\Delta t = 25 \times 0.05 = 1.25 \text{ ns}$ \n

\n\n $\Delta t = 25 \times 0.05 = 1.25 \text{ ns}$ \n

\n\n $\Delta t = 25 \times 0.05 = 1.25 \text{ ns}$ \n

\n\n $\Delta t = 25 \times 0.05 = 1.25 \text{ ns}$ \n

\n\n $\Delta t = 25 \times 0.05 = 1.25 \text{ ns}$ \n

\n\n $\Delta t = 25 \times$ 

Q25. The door of an  $X$ -ray machine room is fitted with a sensor  $D$  (0 is open and 1 is closed). It is

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The  $X$ -ray machine can operate only if the door is closed and at least  $2$  fire sensors are enabled.

The logic circuit to ensure that the machine can be operated is







 Topic: Electronics Sub Topic: Logic Gate

Ans. : (b)

# **PraVegaEl Education** CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics

## PART C

 $\phi(x, y) = \int e^{ikx} \phi_k(y) dk$  to solve the partial differential equation

**EXAMPLE 1.11** CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics  
\nPART C  
\nQ1. If we use the Fourier transform 
$$
\phi(x, y) = \int e^{ikx} \phi_k(y) dk
$$
 to solve the partial differential equation  
\n
$$
-\frac{\partial^2 \phi(x, y)}{\partial y^2} - \frac{1}{y^2} \frac{\partial^2 \phi(x, y)}{\partial x^2} + \frac{m^2}{y^2} \phi(x, y) = 0
$$
 in the half-plane  $\{(x, y) : -\infty < x < \infty, 0 < y < \infty\}$  the Fourier modes  $\phi_k(y)$  depend on y as  $y^{\alpha}$  and  $y^{\beta}$ . The values of  $\alpha$  and  $\beta$  are

Fourier modes  $\phi_k(y)$  depend on  $y$  as  $y^{\alpha}$  and  $y^{\beta}$ . The values of  $\alpha$  and  $\beta$  are

**PROBLEM SET 5.1.1.2 CAUCAUOR**  
\nCSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics  
\nPART C  
\nIf we use the Fourier transform 
$$
\phi(x, y) = \int e^{ikx} \phi_k(y) dk
$$
 to solve the partial differential equation  
\n
$$
\frac{\partial^2 \phi(x, y)}{\partial y^2} - \frac{1}{y^2} \frac{\partial^2 \phi(x, y)}{\partial x^2} + \frac{m^2}{y^2} \phi(x, y) = 0
$$
 in the half-plane  $\{(x, y) : -\infty < x < \infty, 0 < y < \infty\}$  the  
\nFourier modes  $\phi_k(y)$  depend on y as  $y^{\alpha}$  and  $y^{\beta}$ . The values of  $\alpha$  and  $\beta$  are  
\n(a)  $\frac{1}{2} + \sqrt{1 + 4(k^2 + m^2)}$  and  $\frac{1}{2} - \sqrt{1 + 4(k^2 + m^2)}$   
\n(b)  $1 + \sqrt{1 + 4(k^2 + m^2)}$  and  $1 - \sqrt{1 + 4(k^2 + m^2)}$   
\n(c)  $\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4(k^2 + m^2)}$  and  $\frac{1}{2} - \frac{1}{2} \sqrt{1 + 4(k^2 + m^2)}$   
\nTopic: Mathematical Physics  
\nSub Topic: Ordinary Differential Equation

 Topic: Mathematical Physics Sub Topic: Ordinary Differential Equation

Ans. : (c)

Solution:  $\phi(x, y) = \int e^{ik.x} \phi_k(y) dk$ 

$$
\phi(x, y) = \int e^{ik.x} \phi_k(y) dk \Rightarrow \frac{d^2 \phi(x, y)}{dx^2} = -k^2 \phi
$$

Given that,

$$
-\frac{d^2\phi(x, y)}{dy^2} - \frac{1}{y^2}\frac{d^2\phi(x, y)}{dx^2} + \frac{m^2}{y^2}\phi(x, y) = 0
$$
  

$$
y^2\frac{d^2\phi(x, y)}{dy^2} + \frac{d^2\phi(x, y)}{dx^2} - m^2\phi(x, y) = 0
$$
  

$$
y^2\frac{d^2\phi(x, y)}{dy^2} - k^2\phi(x, y) - m^2\phi(x, y) = 0
$$

Let,  $y = e^{\tilde{\epsilon}}$ : Cauchy-Euler Form

Given that,  
\n
$$
-\frac{d^2 \phi(x, y)}{dy^2} - \frac{1}{y^2} \frac{d^2 \phi(x, y)}{dx^2} + \frac{m^2}{y^2} \phi(x, y) = 0
$$
\n
$$
y^2 \frac{d^2 \phi(x, y)}{dy^2} + \frac{d^2 \phi(x, y)}{dx^2} - m^2 \phi(x, y) = 0
$$
\n
$$
y^2 \frac{d^2 \phi(x, y)}{dy^2} - k^2 \phi(x, y) - m^2 \phi(x, y) = 0
$$
\nLet,  $y = \tilde{e}$ : Cauchy-Euler Form  
\n
$$
D(D-1)\phi(x, y) - k^2 \phi(x, y) - m^2 \phi(x, y) = 0, \quad D = \frac{d}{dz}
$$
\n
$$
D^2 \phi(x, y) - D\phi(x, y) - k^2 \phi(x, y) - m^2 \phi(x, y) = 0
$$
\n
$$
D^2 - D - (k^2 + m^2) = 0 \Rightarrow D = \frac{1 \pm \sqrt{1 + 4(k^2 + m^2)}}{2}
$$
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Two roots are

$$
\alpha = \frac{1 + \sqrt{1 + 4\left(k^2 + m^2\right)}}{2}, \ \beta = \frac{1 - \sqrt{1 + 4\left(k^2 + m^2\right)}}{2}
$$

Q2. The Newton-Raphson method is to be used to determine the reciprocal of the number  $x = 4$ . If we start with the initial guess 0.20 then after the first iteration the reciprocal is

(a) 0.23 (b) 0.24 (c) 0.25 (d) 0.26

Topic: Mathematical Physics

Sub Topic: Numerical Techniques (Newton-Raphson Method)

Ans. : (b)

**EXAMPLE 1.1.1** JAM, JEST, TIFR and GRE for Physics  
\nTwo roots are  
\n
$$
\alpha = \frac{1 + \sqrt{1 + 4(k^2 + m^2)}}{2}, \beta = \frac{1 - \sqrt{1 + 4(k^2 + m^2)}}{2}
$$
\nQ2. The Newton-Raphson method is to be used to determine the reciprocal of the number  $x = 4$ . If  
\nwe start with the initial guess 0.20 then after the first iteration the reciprocal is  
\n(a) 0.23 (b) 0.24 (c) 0.25 (d) 0.26  
\nTopic: Mathematical Physics  
\nSub Topic: Numerical Techniques (Newton-Raphson Method)  
\nAns. : (b)  
\nSolution:  $N = 4$ ,  $\frac{1}{x} = N \Rightarrow \frac{1}{N} = x$   
\n $f(x) = \frac{1}{x} - N$ ,  $x_{n-1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - (\frac{1}{x_n} - N)$   
\n $\Rightarrow x_{n+1} = x_n - \frac{(\frac{1}{x_n} - N)}{1 - \frac{1}{x_n}}$  [ $\Rightarrow f'(x) = \frac{-1}{x_n^2}$ ]  
\n $= x_n + x_n - Nx_n$   
\n $x_{n+1} = x_n (2 - Nx_n) = x_n + x_n (\frac{1}{x_n} - N) = x_n + x_n - Nx$   
\nGiven  $X_0 = 0.2$   
\n $\Rightarrow x_1 = x_n [2 - 4 \times x_0] = 0.2 \times [2 - 4 \times 0.2] = 0.24$   
\nQ3. The Legendre polynomials  $P_n(x), n = 0, 1, 2, ...$ , satisfying the orthogonality condition  
\n $\int_{-1}^{1} P_n(x) P_n(x) dx = \frac{2}{2n+1} \delta_{nn}$  on the interval [-1, +1] may be defined by the Rodrigues formula  
\n $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ . The value of the definite integral  $\int_{-1}^{1} (4 + 2x - 3x^2 + 4x^3) P_1(x) dx$  is  
\n(a)  $\frac{3}{5}$  (b)  $\frac{11}{15}$  (c)  $\frac{23}{32}$  (d)  $\frac{16}{35}$ 

Q3. The Legendre polynomials  $P_n(x)$ ,  $n = 0,1,2,...$ , satisfying the orthogonality condition 1 1 2  $P_n(x) P_m(x) dx = \frac{2}{2n+1} \delta_{nm}$  $\int_{-1}^{1} P_n(x) P_m(x) dx = \frac{2}{2n+1} \delta_{nm}$  on the interval  $[-1, +1]$  may be defined by the Rodrigues formula  $\frac{1}{2^n n!} \frac{a}{dx^n} \left( x^2 - 1 \right)$  $\binom{n}{2}$  $n(\lambda)$ <sup>n</sup>  $2^n n! dx^n$  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 \int_{-1}^{1} (4 + 2x - 3x^2 + 4x^3) P_3(x) dx$  is (a)  $\frac{3}{2}$ 5 (b)  $\frac{11}{1}$ 15 (c)  $\frac{23}{11}$ 32 (d)  $\frac{16}{1}$ 35  $x_{s+1} = x_n (2 - Nx_n) = x_n + x_n \left(\frac{1}{x_s} - N\right) = x_n + x_n - Nx$ <br>
Given  $x_0 = 0.2$ <br>  $\Rightarrow x_1 = x_0 [2 - 4 \times x_0] = 0.2 \times [2 - 4 \times 0.2] = 0.24$ <br>
Q3. The Legendre polynomials  $P_n(x), n = 0, 1, 2, ...,$  satisfying the orthogonality condition<br>  $\int_{-1}^{1} P_n(x) P_n(x)$ iven  $x_0 = 0.2$ <br>  $\log x_1 = x_0 \left[ 2 - 4 \times x_0 \right] = 0.2 \times \left[ 2 - 4 \times 0.2 \right] = 0.24$ <br>
i.e. Legendre polynomials  $P_n(x), n = 0, 1, 2, ...$ , satisfying the orthogonality condition<br>  $\int_{-1}^{1} P_n(x) P_m(x) dx = \frac{2}{2n+1} \delta_{nm}$  on the interval  $[-1, +1$ 

Topic: Mathematical Physics

Sub Topic: Polynomials Legendre Polynomial

Ans. : (d)  
\nSolution: 
$$
\int_{-1}^{1} (4 + 2x - 3x^{2} + 4x^{3}) P_{3}(x) dx
$$
\n
$$
\int_{-1}^{1} (4 + 2x - 3x^{2} + 4x^{3}) \frac{1}{2} (5x^{3} - 3x) dx
$$

Only take even function

**Regara Education** 

**PROBLEM SET 10.1 EXAMPLE 10.2 Equation 10.3 EXAMPLE 10.3 EXAMPLE 10.4 EXECUTE:** 
$$
\int_{-1}^{1} (5x^4 - 3x^2 + 10x^6 - 6x^4) dx = x^5 \Big|_{-1}^{1} - x^3 \Big|_{-1}^{1} + \frac{10}{7} x^7 \Big|_{-1}^{1} - \frac{6}{5} x^5 \Big|_{-1}^{1}
$$
\n $= 2 - 2 + \frac{20}{7} - \frac{12}{5} = \frac{100 - 84}{35} = \frac{16}{35}$ \nA particle of mass *m* moves in a potential that is  $V = \frac{1}{2} m \left( \omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2 \right)$  in the coordinates

CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $\int_{-1}^{1} (5x^4 - 3x^2 + 10x^6 - 6x^4) dx = x^5 \Big|_{-1}^{1} - x^3 \Big|_{-1}^{1} + \frac{10}{7} x^7 \Big|_{-1}^{1} - \frac{6}{5} x^5 \Big|_{-1}^{1}$ <br>  $= 2 - 2 + \frac{20}{7} - \frac{12}{5} = \frac{100 - 84}{35} = \frac{16}{35}$  $\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z$ 1  $V = \frac{1}{2} m \left( \omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2 \right)$  in the coordinates of a non-inertial frame  $F$ . The frame  $F$  is rotating with respect to an inertial frame with an angular velocity  $\hat{k}\Omega$ , where  $\hat{k}$  is the unit vector along their common  $z$ -axis. The motion of the particle is unstable for all angular frequencies satisfying **Practicle SEARTLE TO THE CONSTRAINED AND CONTROLLED**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $\int_{-1}^{1} (5x^4 - 3x^2 + 10x^6 - 6x^4) dx = x^3 \Big|_{-1}^{1} - x^3 \Big|_{-1}^{1} + \frac{10}{7} x^7 \Big|_{-1}^{1} - \frac{6}{5} x^5 \Big|_{-1}^{$ **Practical Education**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $x^4 - 3x^2 + 10x^6 - 6x^4 dx$ <br>  $x^4 - 3x^2 + 10x^6 - 6x^4 dx$ <br>  $= 2 + \frac{20}{7} - \frac{12}{5} = \frac{100 - 84}{35} = \frac{16}{35}$ <br>
Tricle of mass m moves in a poten **EST, TIFR and GRE for Physics**<br>  $\frac{1}{2} \left| \frac{1}{1} - \frac{6}{5} x^5 \right|_1^1$ <br>  $\frac{1}{2} m \left( \omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2 \right)$  in the coordinates<br>
ting with respect to an inertial frame with an<br>
along their common  $z$ -axis. The mo **Practicle Example 11**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>  $\int_{-1}^{1} (5x^4 - 3x^2 + 10x^6 - 6x^4) dx = x^5 \Big|_{-1}^{1} - x^5 \Big|_{-1}^{1} + \frac{10}{7} x^7 \Big|_{-1}^{1} - \frac{6}{5} x^5 \Big|_{-1}^{1}$ <br>  $= 2 - 2 + \frac{20}{7} - \frac{12}{5} =$ **CONDUCT:** CALCONDINGLENT IF THE RANGE OF Physics<br>  $x^4 - 3x^2 + 10x^6 - 6x^4$   $|dx = x^2\Big|_{x_1}^4 - x^2\Big|_{x_2}^4 + \frac{10}{7}x^2\Big|_{x_3}^4 - \frac{6}{5}x^5\Big|_{x_4}^4$ <br>  $-2 + \frac{20}{7} - \frac{12}{5} = \frac{100 - 84}{35} - \frac{16}{35}$ <br>
and the sum moves i 1 2 1 2 <sup>0</sup> = 2-2 +  $\frac{2V}{l}$  -  $\frac{2}{3}$  =  $\frac{1000-64}{35}$  =  $\frac{1}{35}$ <br>
Q4. A particle of mass m moves in a potential that is  $V = \frac{1}{2}m(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_2^2 z^2)$  in the coordinates<br>
of a non-inertial frame *F*. The frame A particle of mass m moves in a potential that is  $V = \frac{1}{2} m (\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2)$  in the coordinates<br>
of a non-inertial frame F. The frame F is rotating with respect to an inertial frame with an<br>
angular veloci

(a) 
$$
(\Omega^2 - \alpha_1^2)(\Omega^2 - \alpha_2^2) > 0
$$
  
\n(b)  $(\Omega^2 - \alpha_1^2)(\Omega^2 - \alpha_2^2) < 0$   
\n(c)  $(\Omega^2 - (\alpha_1 + \alpha_2)^2)(\Omega^2 - |\alpha_1 - \alpha_2|^2) > 0$   
\n(d)  $(\Omega^2 - (\alpha_1 + \alpha_2)^2)(\Omega^2 - |\alpha_1 - \alpha_2|^2) < 0$   
\nTopic: Classical Mechanics  
\nSub Topic: Pseudoforce

Ans. : (b)

If  $B(x)$  denotes the magnitude of the magnetic fields at these points, which of the following<br>
(a)  $B(P_1) > B(P_2)$  and  $B(P_1) > B(P_2)$  and  $B(P_1) > B(P_1)$ <br>
In B figure below shows an ideal capacitor consisting of two parallel c (a) B B B B P 3 and B B R 3 and B B R 3 and B B R 3 and B  $B$  and B  $\frac{R}{2}$  are at a transverse distance  $r_i > R$  from the line joining the centres of the plates,<br>while points  $P_3$  and  $P_4$  are at a transverse distance The figure below shows an ideal capacitor consisting of two parallel circular plates of radius R.<br>
Points  $P_1$  and  $P_2$  are at a transverse distance  $r_1 > R$  from the line joining the centres of the plates,<br>
while points





(c) 
$$
B(P_1) = B(P_2)
$$
 and  $B(P_3) < B(P_4)$ 

(d) 
$$
B(P_1) = B(P_2)
$$
 and  $B(P_3) > B(P_4)$ 

 Topic: Electromagnetic Theory Sub Topic: Maxwell Equation (Displacement Current)

Ans. : (c)

**Zega& Education** 

Solution:



The magnetic field at  $P_1$  and  $P_3$  are due to displacement current. On the other hand, the magnetic field at  $P_2$  and  $P_4$  are due to conduction current. **Practice CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br>
The magnetic field at  $P_1$  and  $P_2$  are due to displacement current. On the other hand, the<br>
magnetic field at  $P_2$  and  $P_3$  are due to conductio <sup>D</sup>I J ds . Thus, 3 4 B P B P ( ) ( ) The magnetic field at  $P_1$  and  $P_3$  are due to displacement current. On the other hand, the magnetic field at  $P_2$  and  $P_3$  are due to conduction current.<br>
The current at  $P_1$  and  $P_2$  are equal i.e.  $I_c = I_D$ . Thus, R<br>  $\frac{1}{P_2}$ <br>
Id at  $P_1$  and  $P_3$  are due to displacement current. On the other hand, the<br>  $P_2$  and  $P_4$  are due to conduction current.<br>
and  $P_2$  are equal i.e.  $I_c = I_D$ . Thus,  $B(P_1) = B(P_2)$ <br>
is greater than the  $P_$ The magnetic field at  $P_1$  and  $P_3$  are due to displacement current. On the other hand,<br>
magnetic field at  $P_2$  and  $P_4$  are due to conduction current.<br>
The current at  $P_1$  and  $P_2$  are equal i.e.  $I_c = I_D$ . Thus,  $B(P$  $P_1$ <br>  $P_2$ <br>  $P_3$  and  $P_3$  are due to displacement current. On the other hand, the<br>  $P_2$  are equal i.e.  $I_c = I_D$ . Thus,  $B(P_1) = B(P_2)$ <br>
ater than the  $P_3$  i.e.  $I_c > I_D$ . Since  $I_D = \int J.ds$ .<br>
Unid, of permittivity Eand per

Q6. A perfectly conducting fluid, of permittivity  $\mathcal E$  and permeability  $\mu$ , flows with a uniform velocity V in the presence of time dependent electric and magnetic fields  $E$  and  $B$ , respectively. If there is a finite current density in the fluid, then The current at  $P_4$  is greater than the  $P_3$  i.e.  $I_z > I_D$ . Since  $I_D = \int J.ds$ .<br>
Thus,  $B(P_3) < B(P_4)$ <br>
Q6. A perfectly conducting fluid, of permittivity Eand permeability  $\mu$ , flows with a uniform velocity<br>
Vin the presenc e current at  $P_4$  is greater than the  $P_3$  i.e.  $I_c > I_D$ . Since  $I_D = \int J.ds$ .<br>
us,  $B(P_3) < B(P_4)$ <br>
berfectly conducting fluid, of permittivity  $E$  and permeability  $\mu$ , flows with a uniform velocity<br>
the presence of time d current at  $P_4$  is greater than the  $P_3$  i.e.  $I_c > I_D$ . Since  $I_D = \int J.ds$ .<br>  $I_b(P_3) < B(P_4)$ <br>
frectly conducting fluid, of permittivity  $S$  and permeability  $\mu$ , flows with a uniform velocity<br>
the presence of time dependen

(a) 
$$
\nabla \times (\nu \times B) = \frac{\partial B}{\partial t}
$$
  
\n(b)  $\nabla \times (\nu \times B) = -\frac{\partial B}{\partial t}$   
\n(c)  $\nabla \times (\nu \times B) = \sqrt{\varepsilon \mu} \frac{\partial E}{\partial t}$   
\n(d)  $\nabla \times (\nu \times B) = -\sqrt{\varepsilon \mu} \frac{\partial E}{\partial t}$   
\nTopic: Electroms  
\n(a)  $\nabla \times (\nu \times B) = -\sqrt{\varepsilon \mu} \frac{\partial E}{\partial t}$   
\nTopic: Electroms  
\n(b)  $\nabla \times (\nu \times B) = -\frac{\partial B}{\partial t}$   
\n(d)  $\nabla \times (\nu \times B) = -\sqrt{\varepsilon \mu} \frac{\partial E}{\partial t}$   
\nTopic: Marking curl on both sides  
\n
$$
\nabla \times \vec{E} = (\nabla \times \vec{B} \times \vec{v}) \Rightarrow (\nabla \times \vec{B} \times \vec{v}) = -\frac{\partial B}{\partial t} \Rightarrow (\nabla \times \vec{v} \times \vec{B}) = \frac{\partial B}{\partial t}
$$

 Topic: Electromagnetic Theory Sub Topic: Maxwell's Equation

Ans. : (a)

 $\sigma$  and the set of  $\sigma$ Solution:  $\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) = 0 \implies (\vec{E} + \vec{v} \times \vec{B}) = \frac{\vec{J}}{\sigma} = 0$ 

a)  
\nn: 
$$
\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) = 0 \Rightarrow (\vec{E} + \vec{v} \times \vec{B}) = \frac{\vec{J}}{\sigma} = 0
$$
  
\n $\vec{E} = \vec{B} \times \vec{v}$   
\nTaking curl on both sides  
\n
$$
\nabla \times \vec{E} = (\nabla \times \vec{B} \times \vec{v}) \Rightarrow (\nabla \times \vec{B} \times \vec{v}) = -\frac{\partial B}{\partial t} \Rightarrow (\nabla \times \vec{v} \times \vec{B}) = \frac{\partial B}{\partial t}
$$
\nThis is known as Alfven's theorem

Q7. A laser beam propagates from fiber 1 to fiber 2 in a cavity made up of two optical fibers (as shown in the figure). The loss factor of fiber 2 is  $10$  dB /  $km$ .

**PraVega@ Education** 



**Practice SER NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br>
A laser beam propagates from fiber 1 to fiber 2 in a cavity made up of two optical fibers (as<br>
shown in the figure). The loss factor of fiber 2 is 10 **Practice CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br>A laser beam propagates from fiber 1 to fiber 2 in a cavity made up of two optical fibers (as<br>shown in the figure). The loss factor of fiber 2 is 10 d **PITT-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br>
tes from fiber 1 to fiber 2 in a cavity made up of two optical fibers (as<br>
he loss factor of fiber 2 is 10 *dB* /*km*.<br>
Fiber1 *d* = 0 Fiber2<br> **Willer COVERTY COV**  $\displaystyle{\frac{E_2\left(0\right)}{E_2\left(d\right)}}$  for  $d$  = 10  $km$  , is **Prayer** JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>gates from fiber 1 to fiber 2 in a cavity made up of two optical fibers (a<br>The loss factor of fiber 2 is 10 dB/km.<br>Fiber 1 d = 0 Fiber 2<br>The magnitude of the ele (a)  $10^2$  (b)  $10^3$  (c)  $10^5$  (d)  $10^7$ 2 and the figure). The loss factor of fiber 2 is 10 *dB* / km .<br>
Fiber1  $d = 0$  Fiber 2<br>
Fiber 1  $d = 0$  Fiber 2<br>
Fiber 1  $d = 0$  Fiber 2<br>
Fiber 1  $d = 0$  Fiber 2<br>
Fiber 2 n in the figure). The loss factor of fiber 2 is 10 dB / km .<br>
Fiber 1 d = 0 Fiber 2<br>
Tiber 2<br>
(d) denotes the magnitude of the electric field in fiber 2 at a distance d f<br>
(ace, the ratio  $\frac{E_2(0)}{E_2(d)}$  for  $d = 10$  km ser beam propagates from fiber 1 to fiber 2 in a cavity made up of two optical fibe<br>wn in the figure). The loss factor of fiber 2 is  $10 dB/km$ .<br>Fiber 1  $d = 0$  Fiber 2<br> $\frac{V}{V}$ <br> $V$ <br> $\frac{V}{V}$ <br> $\frac{V}{V}$ <br> $\frac{V}{V}$ <br> $\frac{V}{V}$ <br> $\$ wh in the figure). The loss factor of fiber 2 is 10 dB/km.<br>
Fiber 1 d = 0 Fiber 2<br>  $\frac{F_2}{2}$  (d) denotes the magnitude of the electric field in fiber 2 at a distant<br>
rface, the ratio  $\frac{E_2(0)}{E_2(d)}$  for  $d = 10 km$ , is<br> 10 10 E E E E and the probability of the  $2 \sin \theta$  and  $\theta$  is  $\theta$  is to denote the set of the  $2 \sin \theta$  and  $\theta$  finds  $\theta$  finds  $\theta$  finds  $\theta$  from the  $\frac{1}{2}$  magnitude of the electric field in fiber 2 at a distance *d* from the  $\$ 2. (d) denotes the magnitude of the electric field in fiber 2 at a distant<br>
face, the ratio  $\frac{E_2(0)}{E_2(d)}$  for  $d = 10 km$ , is<br>
0<sup>2</sup> (b) 10<sup>3</sup> (c) 10<sup>3</sup> (d) 10<sup>7</sup><br>
Topic: Electromagnetic T<br>
Sub Topic: Optics<br>
0ss in 2<sup>nd</sup>  $E_2(d)$  denotes the magnitude of the electric field in fiber 2 at a distance d 4<br>
erface, the ratio  $\frac{E_2(0)}{E_2(d)}$  for  $d = 10 km$ , is<br>
10<sup>2</sup> (b) 10<sup>3</sup> (c) 10<sup>5</sup> (d) 10<sup>7</sup><br>
Topic: Electromagnetic Theory<br>
Sub Topic: Optics  $E_2(d)$  denotes the magnitude of the electric field in fiber 2 at a dist<br>terface, the ratio  $\frac{E_2(0)}{E_2(d)}$  for  $d = 10 km$ , is<br>) 10<sup>2</sup> (b) 10<sup>3</sup> (c) 10<sup>5</sup> (d) 10<sup>7</sup><br>Topic: Electromagnetic<br>Sub Topic: Optics<br>Loss in 2<sup>nd</sup> m  $E_2(d)$  denotes the magnitude of the electric field in fiber 2 at a dist<br>
terface, the ratio  $\frac{E_2(0)}{E_2(d)}$  for  $d = 10 km$ , is<br>  $0.10^3$  (c)  $10^3$  (c)  $10^5$  (d)  $10^7$ <br>
Topic: Electromagnetic<br>
Sub Topic: Optics<br>
Loss i Fig. (d) denotes the magnitude of the electric field in fiber 2 at a distance d from the rface, the ratio  $\frac{E_2(0)}{E_2(d)}$  for  $d = 10 km$ , is<br>
(a) 10<sup>2</sup> (b) 10<sup>3</sup> (c) 10<sup>5</sup> (d) 10<sup>3</sup> Topic: Electromagnetic Theory<br>
Sub Topi f  $E_2(d)$  denotes the magnitude of the electric field in fiber 2 at a distance d from the ratio  $\frac{E_2(0)}{E_2(d)}$  for  $d = 10 km$ , is<br>
(a) 10<sup>2</sup> (b) 10<sup>3</sup> (c) 10<sup>3</sup> (d) 10<sup>7</sup> Topic: Electromagnetic Theory<br>
Sub Topic: Optics<br> f  $E_2(d)$  denotes the magnitude of the electric field in fiber 2 at a distance d from the<br>nterface, the ratio  $\frac{E_2(0)}{E_3(d)}$  for  $d = 10 km$ , is<br>(a) 10<sup>3</sup> (b) 10<sup>3</sup> (c) 10<sup>3</sup> (d) 10<sup>3</sup><br>(a) 10<sup>3</sup> (b) 10<sup>3</sup> (c) 10<sup>3</sup> (d) 10 agnitude of the electric field in fiber 2 at a distance d from the<br>
for  $d = 10 km$ , is<br>
(03)<br>
(2)  $10^3$ <br>
(2)  $10^3$ <br>
(3)  $10^3$ <br>
(3) Topic: Electromagnetic Theory<br>
Sub Topic: Optics<br>
(4)  $\frac{10}{10}$ <br>
( $\frac{E_1}{E_2}$ ) $= 10 \Rightarrow$ de of the electric field in fiber 2 at a distance d from the<br>
= 10 km, is<br>
(c) 10<sup>5</sup> (d) 10<sup>7</sup><br>
Topic: Electromagnetic Theory<br>
Sub Topic: Optics<br>
m<br>
m<br>
=  $10 \Rightarrow \frac{E_1}{E_2} = \sqrt{10}$ <br>  $\frac{E_0}{E_2(d)} = (\sqrt{10})^{10} = 10^5$ the magnitude of the electric field in fiber 2 at a distance d from the<br>  $\frac{E_2(0)}{E_2(d)}$  for  $d = 10 km$ , is<br>
(b) 10<sup>3</sup> (c) 10<sup>5</sup> (d) 10<sup>7</sup><br>
Topic: Electromagnetic Theory<br>
Sub Topic: Optics<br>
m = 10 dB/km<br>  $= 10 \Rightarrow \left(\frac{E_1}{E$ 

Topic: Electromagnetic Theory

Sub Topic: Optics

Ans. : (c)

Solution: Loss in  $2<sup>nd</sup>$  medium = 10 dB/km

$$
\log\left(\frac{P_1}{P_2}\right) = 1 \Longrightarrow \frac{P_1}{P_2} = 10 \Longrightarrow \left(\frac{E_1}{E_2}\right)^2 = 10 \Longrightarrow \frac{E_1}{E_2} = \sqrt{10}
$$

The most general formula

$$
\left[\frac{E_1(d)}{E_2(d)} = (\sqrt{10})^d\right] \Rightarrow d = 10 \Rightarrow \frac{E_0}{E_2(d)} = (\sqrt{10})^{10} = 10^5
$$

# **PraVegaEl Education** CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics

Q8. A particle in two dimensions is found to trace an orbit  $r(\theta) = r_0 \theta^2$ . If it is moving under the influence of a central potential  $V(r) = c_1 r^{-a} + c_2 r^{-b}$ , where  $r_0^{\prime}, c_1^{\prime}$  and  $c_2^{\prime}$  are constants of appropriate dimensions, the values of  $a$  and  $b$ , respectively, are (a)  $2$  and  $4$  (b)  $2$  and  $3$ 

(c) 3 and 4 (d) 1 and 3

Topic: Classical Mechanics

Sub Topic: Central Force Problem

Ans. : (b)

Solution:  $r(\theta) = r_0 \theta^2$ 

$$
\text{Let } u = \frac{1}{r} = \frac{1}{r_0} \theta^{-2}
$$

Differential equation of orbit

$$
\left(\frac{d^2u}{d\theta^2} + u\right) = -\frac{m}{l^2u^2} f \Rightarrow \left[\frac{1 \times 6 \times \theta^{-4}}{r_0} + \frac{1}{r_0} \times \theta^{-2}\right] = -\frac{m}{l^2u^2} f
$$
  

$$
\Rightarrow \left[A_1u^2 + A_2u\right] = -\frac{m}{l^2u^2} f \Rightarrow \left[B_1u^4 + B_2u^3\right] = f
$$
  

$$
\left[B_1r^{-4} + B_2r^{-3}\right] = f(r) = -\frac{dV}{dr} \Rightarrow V(r) = c_1r^{-2} + c_3r^{-3} \Rightarrow a = 2, b = 3
$$

# **PraVegam Education**

# CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics

Q9. The fulcrum of a simple pendulum (consisting of a particle of mass  $m$ attached to the support by a massless string of length  $l$ ) oscillates **Practically as sin z** (*t*) = *asin ot* , where  $\omega$  is a constant. The pendulum  $\theta(t)$  and  $\theta(t)$  are  $\theta(t)$  and  $\theta(t)$  are  $\theta(t)$  and  $\theta(t)$  are  $\theta(t)$  is a constant. The pendulum  $\theta(t)$  and  $\theta(t)$  and  $\theta(t)$  denotes i **Prayer COLORE EQUICATION**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
The fulcrum of a simple pendulum (consisting of a particle of mass *m*<br>
attached to the support by a massless string of length *l*) respect to the  $z$ -axis. **Education**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
The fulcrum of a simple pendulum (consisting of a particle of mass *m*<br>
attached to the support by a massless string of length *l*) oscillates<br>
ver **Practice COMPTE.** CALCONDINGLES IN A SET, TIFR and GRE for Physics<br>
The fulcrum of a simple pendulum (consisting of a particle of mass  $m$ <br>
attached to the support by a massless string of length 1) oscillates<br>
vertically **EXECUTE:** CATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
The fulcrum of a simple pendulum (consisting of a particle of mass *m*<br>
attached to the support by a massless string of length *1*) oscillates<br>
vertically as  $\sin z(t$ **Practically as singlet in the CE of the CE for Physics**<br>
The fulcrum of a simple pendulum (consisting of a particle of mass  $m$ <br>
attached to the support by a massless string of a particle of mass  $m$ <br>
attached to the sup attached to the support by a massiess string of length /) oscillates<br>
vertically as  $\sin z(t) = a \sin \alpha t$ , where  $\omega$  is a constant. The pendulum<br>
moves in a vertical plane and  $\theta(t)$  denotes its angular position with<br>
respect t



2  $\ell \frac{d^2\theta}{dt^2} + \sin\theta\left(g - f\left(t\right)\right) = 0$  (where  $g$  is the acceleration due to gravity) describes the equation

Topic: Classical Mechanics

Sub Topic: Lagrangian of a System

# Ans. : (d)

*θ(t)* λ)  
\nmoves in a vertical plane and *θ(t)* denotes its angular position with  
\nrespect to the *z*-axis.  
\nIf 
$$
\frac{d^2\theta}{dt^2} + \sin\theta(g - f(t)) = 0
$$
 (where *g* is the acceleration due to gravity) describes the equation  
\nof motion of the mass, then  $f(t)$  is  
\n(a)  $a\omega^2 \cos \omega t$  (b)  $a\omega^2 \sin \omega t$   
\n(c)  $-a\omega^2 \cos \omega t$  (d)  $-a\omega^2 \sin \omega t$   
\n(d)  $-a\omega^2 \sin \omega t$   
\nTopic: Classical Mechanics  
\nSub Topic: Lagrangian of a System  
\n
$$
f(t) = \frac{1}{2}m(t^2\theta^2 + z^2 - I\theta z \sin \theta) + mgl \cos \theta - z
$$
\n(2*h* =  $-\frac{aL}{d\theta} = ml^2\theta - mlz \sin \theta$ ,  $\frac{d}{dt}(\frac{\partial L}{\partial \theta}) = ml^2\theta - mlz \sin \theta - mlz \cos \theta \dot{\theta}$   
\n $\frac{\partial L}{\partial \theta} = -ml\theta z \cos \theta - mgl \sin \theta$   
\n $\frac{d}{dt}(\frac{\partial L}{\partial \theta}) - \frac{\partial L}{\partial \theta} = 0 \Rightarrow ml^2\theta - mlz \sin \theta - mlz \dot{\theta} \cos \theta + mlz \dot{\theta} \cos \theta + mgl \sin \theta = 0$   
\n $ml^2\theta - mlz \sin \theta + mgl \sin \theta = 0 \Rightarrow l\theta - z \sin \theta + g \sin \theta = 0$   
\nPut  $z = a \sin \omega t \Rightarrow \bar{z} = -a\omega^2 \sin \omega t$   
\n $l\ddot{\theta} + a\omega^2 \sin \omega t \sin \theta + g \sin \theta = 0 \Rightarrow l\dot{\theta} + \sin \theta(a\omega^2 \sin \omega t + g) = 0$   
\nIn problem it is given by  
\n $l\ddot{\theta} + \sin^2(g - f(t)) \cos f(t) = -a\omega^2 \sin \omega t$   
\nAs atellite of mass *m* orbits around earth in an elliptic trajectory of semi-major axis *α*. At a  
\nradial distance *t* = *t*

Q10. A satellite of mass *M* orbits around earth in an elliptic trajectory of semi-major axis  $\alpha$ . At a

the magnitude of the total energy. If  $M$  denotes the mass of the earth and the total energy is

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2  $GMm$ a  $-\frac{GMm}{2}$ , the value of  $\frac{r_0}{2}$ a is nearest to

(a)  $1.33$  (b)  $1.48$  (c)  $1.25$  (d)  $1.67$ 

 Topic: Classical Mechanics Sub Topic: Central Force Problem

Ans. : (a)

Solution:  $\tilde{c}_0$  2*a* 4*a*  $r_0$ 1 1 1  $\frac{1}{2a} - \frac{1}{4a} - \frac{1}{r_0} \rightarrow -\frac{1}{2a} - \frac{1}{4a}$  $E = T + V \Rightarrow -\frac{GMm}{2} = \frac{GMm}{4} - \frac{GMm}{4}$  $\frac{a}{a}$  -  $\frac{a}{4a}$  -  $\frac{a}{r_0}$   $\rightarrow$  -  $\frac{a}{2a}$  -  $\frac{a}{4a}$  -  $\frac{b}{r_0}$  $=T+V \Rightarrow -\frac{GMm}{2} = \frac{GMm}{2} - \frac{GMm}{2} \Rightarrow -\frac{1}{2} - \frac{1}{2} = -\frac{1}{2}$ 0 0  $\frac{3}{1} = -\frac{1}{2} \Rightarrow \frac{r_0}{9} = \frac{4}{2} = 1.33$  $\frac{a}{4a}$  -  $\frac{a}{r_0}$   $\rightarrow$   $\frac{a}{a}$  -  $\frac{a}{3}$   $r_{0}$  $\frac{a}{a}$  -  $\frac{a}{r_0}$   $\rightarrow$   $\frac{a}{a}$  - $-\frac{3}{4}=-\frac{1}{4}\Rightarrow \frac{r_0}{2}=\frac{4}{3}=1.$ 

Q11. A particle of mass  $m$  in one dimension is in the ground state of a simple harmonic oscillator described by a Hamiltonian  $H = \frac{p^2}{r^2} + \frac{1}{r} m \omega^2 x^2$  $H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2$  in the standard notation. An impulsive force at time  $t = 0$  suddenly imparts a momentum  $p_0 = \sqrt{\hbar m \omega}$  to it. The probability that the particle remains in the original ground state is

(a)  $e^{-2}$ (b)  $e^{-3/2}$ (c)  $e^{-1}$ (d)  $e^{-1/2}$  Topic: Quantum Mechanics Sub Topic: Harmonic Oscillator

### Ans. : (d)

Solution: The ground state wave function of Harmonic oscillator

$$
|\psi_{gs}\rangle = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{1/2} e^{-\frac{\alpha^2 x^2}{2}}
$$

$$
H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{P_0^2}{2m}
$$

New wave function is due combination of harmonic oscillator and free particle to impulsive force Since, the wave function is multiplicative in nature. Therefore, we can write

$$
|\psi_{new}\rangle = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{1/2} e^{-\frac{\alpha^2 x^2}{2}} e^{\frac{ip_0 x}{\hbar}}
$$

Probability of finding in the original ground state is

$$
\big\langle \psi_{_{\mathit{new}}}\big|\big|\psi_{_{\mathit{S} S}}\big\rangle \!=\! \big\langle \! \big|\psi_{_{\mathit{new}}}\big|\big|\psi_{_{\mathit{S} S}}\big\rangle \!\big|^2
$$

Now



$$
\langle \psi_{\text{new}} | \psi_{\text{gs}} \rangle = \int_{-\infty}^{\infty} \left( \frac{\alpha}{\pi} \right)^{1/2} e^{-\frac{\alpha^2 x^2}{2}} e^{-\frac{i p_0 x}{\hbar}} \left( \frac{\alpha}{\pi} \right)^{1/2} e^{-\frac{\alpha^2 x^2}{2}} dx = \left( \frac{\alpha}{\sqrt{\pi}} \right) \int_{-\infty}^{\infty} e^{-(\alpha^2 x^2 + \frac{i p_0 x}{\hbar})} dx
$$

$$
= \left( \frac{\alpha}{\sqrt{\pi}} \right) e^{-\frac{\left( \frac{p_0}{\hbar} \right)^2}{4\alpha^2}} \left( \frac{\sqrt{\pi}}{\sqrt{\pi}} \right) \qquad \left[ \text{Sine, } \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(\alpha x^2 + \beta x)} dx = e^{\frac{\beta^2}{4\alpha}} \sqrt{\frac{\pi}{\pi}} \right]
$$

$$
= \left(\frac{\alpha}{\sqrt{\pi}}\right) e^{-\frac{\left(\frac{\beta}{\alpha}\right)^2}{4\alpha^2}} \left(\frac{\sqrt{\pi}}{\alpha}\right) \qquad \left[\text{Sine, } \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(\alpha x^2 + \beta x)} dx = e^{\frac{\beta^2}{4\alpha}} \sqrt{\frac{\pi}{\alpha}}\right]
$$

Now Substitute the value of  $p_0^2 = m\omega\hbar$ ,  $\alpha^2 = \frac{m\omega}{\hbar}$ ħ

$$
\langle \psi_{new} || \psi_{gs} \rangle = e^{-\frac{m\omega}{4m\omega}} = e^{-\frac{1}{4}}
$$
 Thus, the probability  $\langle \psi_{new} || \psi_{gs} \rangle^2 = e^{-\frac{1}{2}}$ 

Q12. The energies of a two-state quantum system are  $E_0$  and  $E_0 + \alpha \hbar$ , (where  $\alpha$  > 0 is a constant) and the corresponding normalized state vectors are  $|0\rangle$  and  $|1\rangle$ , respectively. At time  $t = 0$ , when the system is in the state  $|0\rangle$ , the potential is altered by a time independent term  $V$  such that  $\langle 1 | V | 0 \rangle = \frac{\hbar \alpha}{10}$ . The transition probability to the state  $\big| 1 \big\rangle$  at times  $t << \frac{1}{\alpha}$ , is (a)  $\frac{\alpha^2 t^2}{\alpha}$ 25  $\frac{\alpha^2 t^2}{\alpha}$  (b)  $2 + 2$ 50  $\frac{\alpha^2 t^2}{\alpha}$  (c)  $2 + 2$ 100  $\frac{\alpha^2 t^2}{\alpha^2 t^2}$  (d)  $\frac{\alpha^2 t^2}{\alpha^2 t^2}$ 200  $\alpha^2$ t  $\frac{d}{d\theta} = e^{-\frac{1}{4}}$  Thus, the probability  $\langle \psi_{\text{av}} | \psi_{\text{gs}} \rangle^2 = e^{-\frac{1}{2}}$ <br>two-state quantum system are  $E_0$  and  $E_0 + \alpha h$ , (where  $\alpha > 0$  is a constant)<br>nding normalized state vectors are  $|0\rangle$  and  $|1\rangle$ , respect

Topic: Quantum Mechanics

Sub Topic: Time Dependent Perturbation Theory

Ans. : (c)

Solution:  $\langle 0 | V | 1 \rangle = \frac{\alpha h}{100}$  $V|1\rangle = \frac{\alpha \hbar}{\Delta}$ 2  $\frac{|v|^{2}/|v|}{r^{2} \omega^{2}} \sin^{2} \frac{\omega_{0}^{2}}{2}$ 0  $4 \langle 0 |V|1 \rangle |^2$  $(0 \rightarrow 1) = \frac{\pi |U|^{V} |I|}{2} \sin^2$ 2  $P(0 \rightarrow 1) = \frac{4|\langle 0|V|1\rangle|^2}{r^2} \sin^2 \frac{\omega_0 t}{r^2}$  $\rightarrow 1) = \frac{\mathcal{F}[\nabla \cdot \mathbf{r}]}{\hbar^2 \omega_0^2}$  $\hbar^4$ for small time  $t \rightarrow 0$  $\int_{0}^{2} \omega^{2} t^{2} |dV| |V|^{2}$   $\omega^{2} t^{2} \hbar^{2}$   $\omega^{2} t^{2}$  $\frac{d_0 t}{d_2}$ .  $\frac{d_0 t}{d} = \frac{|\nabla |t| + |t|}{\hbar^2} t^2 = \frac{d_0 t}{100 \hbar^2}$ 0  $4|\langle 0|V|1\rangle|^2 \quad \omega_0^2 t^2 \quad |\langle 0|V|1\rangle|^2$  $\frac{a_0 t}{t} = \frac{|\nabla |\cdot|^{1/2}}{t^2} t^2 =$  $\frac{4}{4}$  -  $\frac{h^2}{\hbar^2}$  -  $\frac{1}{100\hbar^2}$  -  $\frac{1}{100}$  $\frac{|V|1\rangle|^2}{\sqrt{2}} \cdot \frac{\omega_0^2 t^2}{\omega_0^2 t^2} = \frac{|v(0|V|1)\rangle|^2}{\sqrt{2}} \cdot t^2 = \frac{\alpha^2 t^2 \hbar^2}{\omega_0^2} = \frac{\alpha^2 t^2}{\omega_0^2}$  $\frac{|\mathcal{V}|^2|}{\omega_0^2} \cdot \frac{\omega_0^2 t^2}{4} = \frac{|v|^2 |v|}{\hbar^2} t^2 = \frac{\alpha^2 t^2 \hbar^2}{100 \hbar^2} = \frac{\alpha^2 t^2}{1}$  $\hbar^2 \omega_0^2$  4  $\hbar^2$  100 $\hbar^2$ Q13. In an elastic scattering process at an energy  $E$  , the phase shift  $\delta_0 \! \approx \! \! 30^{\!0}, \! \delta_1 \! \approx \! \! 10^{\!0}$  , while the other phase shifts are zero. The polar angle at which the differential cross section peaks is closest to (a)  $20^{\circ}$  (b)  $10^{\circ}$  (c)  $0^{\circ}$  (d)  $30^{\circ}$  Topic: Quantum Mechanics Sub Topic: Scattering

Ans. : (c)

# **PraVegam Education** CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics

Q14. The unnormalized wavefunction of a particle in one dimension in an infinite square well with walls at  $x = 0$  and  $x = a$ , is  $\psi(x) = x(a-x)$ . If  $\psi(x)$  is expanded as a linear combination of the **Education**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
The unnormalized wavefunction of a particle in one dimension in an infinite square well with<br>
walls at  $x = 0$  and  $x=a$ , is  $\psi(x) = x(a-x)$ . If  $\psi(x)$  i 0  $\int_0^a \left| \psi \left( x\right) \right|^2 dx$  is proportional to the infinite series (you may use  $\int_0^a t \sin t \, dt = -a \cos a + \sin a$  and  $\int_0^a t^2 \sin t \, dt = -2 - (a^2 - 2)(\cos a + 2a \sin a)$ ) **EQUEST**<br>
JAM, JEST, TIFR and GRE for Physics<br>
ticle in one dimension in an infinite square well with<br>
x). If  $\psi(x)$  is expanded as a linear combination of the<br>
oportional to the infinite series<br>
and  $\int_0^a t^2 \sin t \, dt = -2 - ($ **PRODURE Education**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
The unnormalized wavefunction of a particle in one dimension in an infinite square woulls at  $x = 0$  and  $x=a$ , is  $\psi(x) = x(a-x)$ . If  $\psi(x)$  is  $\sum_{n=1}^{\infty} (2n-1)^{-6}$ **Practice COMPLE EXECUTE:**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
unnormalized wavefunction of a particle in one dimension in an infinite square well with<br>
s at  $x = 0$  and  $x = a$ , is  $\psi(x) = x(a-x)$ . If  $\sum_{n=1}^{\infty} (2n-1)^{-4}$ **Education**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
The unnormalized wavefunction of a particle in one dimension in an infinite square w<br>
walls at  $x = 0$  and  $x=a$ , is  $\psi(x) = x(a-x)$ . If  $\psi(x)$  is expand  $\sum_{n=1}^{\infty} (2n-1)^{-2}$ **Practice Education**<br>
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unnormalized wavefunction of a particle in one dimension in an infinite square well with<br>
ls at  $x = 0$  and  $x = a$ , is  $\psi(x) = x(a-x)$ . If  $\psi(x)$  $\sum_{n=1}^{\infty} (2n-1)^{-8}$ rgy eigenfunctions,  $\int_0^{\infty} |w(x)|^2 dx$  is proportional to the infinite series<br>  $\int_0^{\infty} r \sin t dt = -a \cos a + \sin a$  and  $\int_0^{\infty} t^2 \sin t dt = -2 - (a^2 - 2)(\cos a + 2a \sin a)$ <br>  $\sum_{n=1}^{\infty} (2n-1)^6$  (b)  $\sum_{n=1}^{\infty} (2n-1)^4$ <br>  $\sum_{n=1}^{\infty} (2n-1)^$ 

Topic: Quantum Mechanics

Sub Topic: Particle in Box

# Ans. : (a)

Solution: The normalize wavefunction can be written as

$$
\psi = Ax(a-x)
$$

Where A is normalization constant.

$$
\int_{0}^{a} \psi^{2} dx = 1, \int_{0}^{a} \left[ Ax(a - x) \right]^{2} dx = 1 \implies A = \sqrt{\frac{30}{a^{5}}}
$$

This wavefunction can be written in basis form

$$
\psi = \sum_n c_n \phi_n,
$$

The nth coefficient for infinite square well can be written as follows

Where A is normalization constant.  
\n
$$
\int_{0}^{a} \psi^{2} dx = 1, \int_{0}^{a} [Ax(a - x)]^{2} dx = 1 \implies A = \sqrt{\frac{30}{a^{5}}}
$$
\nThis wavefunction can be written in basis form  
\n
$$
\psi = \sum_{n} c_{n} \phi_{n},
$$
\nThe nth coefficient for infinite square well can be written as follows  
\n
$$
c_{n} = \sqrt{\frac{2}{a}} \int_{0}^{a} \sin(\frac{n\pi x}{a}) \sqrt{\frac{30}{a^{5}}} x(a - x) dx = \frac{2\sqrt{15}}{a^{3}} \left[ a \int_{0}^{a} x \sin(\frac{n\pi x}{a}) dx - \int_{0}^{a} x^{2} \sin(\frac{n\pi x}{a}) dx \right]
$$
\n
$$
= \frac{2\sqrt{15}}{a^{3}} \left\{ a \left[ \left( \frac{a}{n\pi} \right)^{2} \sin(\frac{n\pi x}{a}) - \frac{ax}{n\pi} \cos(\frac{n\pi x}{a}) \right] \right\}_{0}^{a} - \left[ 2 \left( \frac{a}{n\pi} \right)^{2} x \sin(\frac{n\pi x}{a}) - \frac{\left( \frac{n\pi x}{a} \right)^{2} - 2}{\left( \frac{n\pi}{a} \right)^{3}} \cos(\frac{n\pi x}{a}) \right] \right]_{0}^{a}
$$
\n
$$
c_{n} = \frac{4\sqrt{15}}{(n\pi)^{3}} \left[ \cos(0) - \cos(n\pi) \right] \implies c_{n} = \begin{cases} 0, & \text{for even } n \\ \frac{8\sqrt{15}}{(n\pi)^{3}}, & \text{for odd } n \end{cases}
$$
\nThus,  $c_{n} a \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{3}}$ 

$$
c_n = \frac{4\sqrt{15}}{(n\pi)^3} \left[\cos(0) - \cos(n\pi)\right] \Rightarrow c_n = \begin{cases} 0, & \text{if } n \in \mathbb{N} \\ \frac{8\sqrt{15}}{(n\pi)^3}, & \text{for odd } n \end{cases}
$$

Thus, 
$$
c_n \alpha \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}
$$
  
Now,  $\int |\psi|^2 dx \alpha |c_n|^2 \alpha \sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}$ 

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Q15. The pressure of a gas in a vessel needs be maintained between 1.5 bar to 2.5 bar in an experiment. The vessel is fitted with a pressure transducer that generates  $4mA$  to  $20mA$ current for pressure in the range 1 bar to 5 bar. The current output of the transducer has a linear dependence on the pressure.



The reference voltages  $V_1$  and  $V_2$  in the comparators in the circuit (shown in figure above) suitable for the desired operating conditions, are, respectively

(a)  $2 V$  and  $10 V$  (b)  $2 V$  and  $5 V$  (c)  $3 V$  and  $10 V$  (d)  $3 V$  and  $5 V$ 

Topic: Electronics

Sub Topic: OP-AMP

Solution: Slope 2 1 2 1 5 1 4 x x mA/bar 1.5 4 2.5 6 bar mA 2.5 4 2.5 10 bar mA <sup>1</sup>V mA V 6 500 3 <sup>2</sup> V mA V 10 500 5 <sup>23</sup> 6.023 10 I 20mA 4mA 1br 5br P bar

Q16. The nuclei of  $^{137}Cs$  decay by the emission of  $\beta$  -particles with a half of 30.08 years. The activity (in units of disintegrations per second or Bq) of a 1 mg source of  $^{137}Cs$ , prepared on January 1, 1980 as measured on January 1, 2021 is closest to

(a) 
$$
1.79 \times 10^{16}
$$
 (b)  $1.79 \times 10^{9}$  (c)  $1.24 \times 10^{16}$  (d)  $1.24 \times 10^{9}$   
Topic: Nuclear & Particle Physics  
Sub Topic: Radioactive Decay

Ans. : (d)

Ans. : (d)

Solution: 137 gm Cs contains  $6.023 \times 10^{23}$  atoms

 $1 \text{ gm Cs contains } \frac{3.28 \times 10^{-6}}{137} \text{ atoms}$  $\times 10^{23}$ atoms

> H.N. 28 A/1, Jia Sarai, Near IIT-Delhi, Hauz Khas, New Delhi-110016 #: +91-89207-59559

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**PROBLEM SET 101**  
\nCSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics  
\n1 mg Cs contains 
$$
\frac{6.023 \times 10^{23} \times 10^{-3}}{137} = N_0 \text{ atoms}
$$
\nActivity:  $N_0 \lambda e^{-\lambda t} = \frac{6.023 \times 0.693 \times 10^{23} \times 10^{-3}}{137} e^{-\frac{0.693 \times 41}{30.08}}$   
\n
$$
= \frac{6.023 \times 0.693 \times 10^{20} \times e^{-\frac{0.693 \times 41}{30.08}}}{137 \times 30.08 \times 365 \times 24 \times 3600} = \frac{6.023 \times 0.693 \times 10^{20-9} \times e^{-\frac{0.693 \times 41}{30.08}}}{1.37 \times 3.008 \times 3.65 \times 2.4 \times 3.6} = 1.24 \times 10^{9}
$$
\nQ17. In the following circuit the input voltage  $V_{in}$  is such that  $|V_{in}| < |V_{out}|$ , where  $V_{sat}$  is the saturation voltage of the op-amp. (Assume that the diode is an ideal one and  $R_L C$  is much large than the duration of the measurement.)

voltage of the op-amp. (Assume that the diode is an ideal one and  $R_{\!L\!C}$  is much large than the duration of the measurement.)  $V_{in}$ 



for the input voltage as shown in the figure above, the output voltage  $V_{_{out}}$  is best represented



for the input voltage as shown in the figure above, the output voltage  $V_{out}$  is best represented by

# Topic: Electronics

# Sub Topic: OPAMP

# Ans. : (a)

Solution: Given that  $R_{LC}$  is much larger than the duration of measurement. Once capacitor is charged to **Education**<br>
CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
Sub Topic: OPAMP<br>
a)<br>
a: Given that  $R_{LC}$  is much larger than the duration of measurement. Once capacitor is charged to<br>
input value  $(v_i = 6V)$ , It make output to remain fixed at  $11V$ .

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Q18. To measure the height  $h$  of a column of liquid helium in a container, a constant current I is sent through an  $NbTi$  wire of length I, as shown in the figure. The normal state resistance of the  $NbTi$  wire is R . If the superconducting transition **Practice CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br>
(SUR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
(sub Topic: OPAMP)<br>
(i)<br>
(ii) in that  $R_{LC}$  is much larger than the duration of measure described by the expression CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
Sub Topic: OPAN<br>
that  $R_{LC}$  is much larger than the duration of measurement. Once capacitor<br>
lue  $(v_i = 6V)$ , It will be remain at 6V until further increase in v CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics<br>
sub Topic: OPAMP<br>
en that  $R_{LC}$  is much larger than the duration of measurement. Once capacitor is calue  $(v_i = 6V)$ , It will be remain at  $6V$  until further inc **Example 19 The Uniter State (V<sub>i</sub>** = 6*V*), It will be remain at 6*V* until further increase in voltage to 11.<br>
Sub Topic: OPAMP<br>
en that  $R_{LC}$  is much larger than the duration of measurement. Once capacitor is challed ST, TIFR and GRE for Physics<br>
Sub Topic: OPAMP<br>
of measurement. Once capacitor is charged to<br>
ntil further increase in voltage to 11V which<br>
um in a container, a constant<br>
, as shown in the figure. The<br>
e superconducting Sub Topic: OPAMP<br>
of measurement. Once capacitor is charged to<br>
ntil further increase in voltage to 11*V* which<br>
um in a container, a constant<br>
', as shown in the figure. The<br>
e superconducting transition<br>
sured voltage en that  $R_{LC}$  is much larger than the duration of measurement. Once capacitor is<br>value  $(v_i = 6V)$ , It will be remain at 6 V until further increase in voltage to<br>output to remain fixed at 11V.<br>asure the height h of a colum en that  $R_{LC}$  is much larger than the duration of measurement. Once capacitor is<br>value  $(v_i = 6V)$ , It will be remain at 6 V until further increase in voltage to<br>output to remain fixed at  $11V$ .<br>asure the height h of a col current *I* is sent through an *NbTi* wire of length *I*, as shown in the figure. The<br>normal state resistance of the *NbTi* wire is *R*. If the superconducting transition<br>temperature of *NbTi* is  $\approx 10 K$  then the measure

(a) 
$$
IR\left(\frac{1}{2} - \frac{2h}{l}\right)
$$
 (b)  $IR\left(1 - \frac{h}{l}\right)$  (c)  $IR\left(\frac{1}{2} - \frac{h}{l}\right)$  (d)  $IR\left(1 - \frac{2h}{l}\right)$   
\nTopic: Solid State Physics  
\n(b)  $IR\left(1 - \frac{2h}{l}\right)$  (e)  $IR\left(1 - \frac{2h}{l}\right)$   
\nTopic: Solid State Physics  
\nSub Topic: Superved  
\n(b)  $IR\left(1 - \frac{2h}{l}\right)$  (f)  $IR\left(1 - \frac{2h}{l}\right)$   
\nTopic: Solid State Physics  
\nSub Topic: Superved  
\nSub Topic: Superved  
\nSub Topic:  $Sp(1 - \frac{2h}{l})$   
\nTopic:  $Sp(1 - \frac{2h}{l})$   
\nTopic: <



 Topic: Solid State Physics Sub Topic: Superconductivity

#### Ans. : (d)

Solution: From given diagram it is clearly noticeable that if liquid Helium is fully occupied then the superconducting state. temperature of *NbTi* is  $\approx 10 K$  then the measured voltage  $V(h)$  is best<br>
described by the expression<br>
(a)  $IR\left(\frac{1}{2} - \frac{2h}{l}\right)$ <br>
(b)  $IR\left(1 - \frac{h}{l}\right)$ <br>
(c)  $IR\left(\frac{1}{2} - \frac{h}{l}\right)$ <br>
(d)  $IR\left(1 - \frac{2h}{l}\right)$ <br>
Topic: Solid temperature of *NbTi* is  $\approx 10 K$  then the measured voltage  $V(h)$  is best<br>described by the expression<br>(a)  $IR\left(\frac{1}{2} - \frac{2h}{l}\right)$ <br>(c)  $IR\left(\frac{1-h}{2}\right)$ <br>(c)  $IR\left(\frac{1-h}{2}\right)$ <br>(d)  $IR\left(1-\frac{2h}{l}\right)$ <br>Topic: Solid State Physics<br>Su

From given option, only (a) will satisfy that  $V(h_{\text{max}}) = IR\left(1 - \frac{2l/2}{l}\right) = 0$ .

$$
V(h) = IR(1 - \frac{2h}{l}) \Rightarrow V(h) = IR(1 - \frac{2 \times 0}{l}) = IR
$$

(c)  $IR\left(1-\frac{R}{l}\right)$  (d)  $IR\left(1-\frac{2R}{l}\right)$  (d)  $IR\left(1-\frac{2R}{l}\right)$ <br>
Topic: Solid State Physics<br>
Ans.: (d) Solution: From given diagram it is clearly noticeable that if liquid Helium is fully occupied then the<br>
maximum heigh  $n = 1, 2, 3,...$  . At a temperature T, the free energy F can be expressed in terms of the average energy  $E$  by From given diagram it is clearly noticeable that if liquid Helium is fully occurrium height will be  $h_{\text{max}} = l/2$ . At this time, voltage  $V(h)$  will be zerroonducting state.<br>
m given option, only (a) will satisfy that  $V(h_{$ f liquid Helium is fully occupied then the<br>
, voltage  $V(h)$  will be zero due to the<br>  $IR\left(1-\frac{2l/2}{l}\right)=0$ .<br>
all conductor. Which will satisfy the Ohm's<br>  $\int_{-2}^{2} f(t) dt = IR$ <br>
m are  $\epsilon_n = nE_0$ , where  $E_0$  is a constant and<br> hat if liquid Helium is fully occupied then the<br>
time, voltage  $V(h)$  will be zero due to the<br>  $\lim_{n \to \infty}$  =  $IR\left(1 - \frac{2l/2}{l}\right) = 0$ .<br>
normal conductor. Which will satisfy the Ohm's<br>  $\frac{2 \times 0}{l}$  =  $IR$ <br>
ystem are  $\epsilon_n = nE$ 

(a) 
$$
E_0 + k_B T \ln \frac{E}{E_0}
$$
 (b)  $E_0 + 2k_B T \ln \frac{E}{E_0}$ 

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(c) 
$$
E_0 - k_B T \ln \frac{E}{E_0}
$$
 \t\t (d)  $E_0 - 2k_B T \ln \frac{E}{E_0}$ 

Topic: Thermodynamics & Statistical Mechanics

Sub Topic: Concept of Free Energy in Statistical Physics

Ans. : (c)

Solution: Partition function can be written as follows

**PROBLEM SET 15.2 Equation 16.2 Example 2.3 Example 3.4 Example 4.4 Example 4.5 Example 5.6 Example 6.6 Example 7.7 Example 8. Statistical Mechanism Sub Topic: Thermodynamics & Statistical Mechanism Sub Topic: Concept of Free Energy in Statistical Statistical Method 17.8 Sub Topic: Concept of Free Energy in Statistical Method 27.8 Example 3.8 Stationary Example 4.9 Example 5.1 Example 6.1 Example 7.1 Example 8.1 Sub Topic: Concept of Free Energy in Statistical Method 17.8 Sub Topic: Concept of Free Energy in Statistical Method 27.8 Sub Topic: Concept of Free Energy in Statistical Method 37.8 Sub Topic: Concept of Free Energy in Statistical Method 47.8 Sub Topic: Concept of Free Energy in Statistical Method 57.8 Sub Topic: 
$$
C = \frac{E}{E_0}
$$
 Sub Taylor Example 8.1 Sub Topic:  $C = \frac{E}{E_0}$  Sub Topic:  $C = \frac{E}{E_0}$  Sub Topic:  $C = \frac{E}{E_0}$  Sub logic:  $C = \frac{E$** 

Average energy can be written as follows

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\n(c) 
$$
E_0 - k_B T \ln \frac{E}{E_0}
$$
  
\nTopic: Thermodynamics & Statistical Mechanics  
\nSub Topic: Concept of Free Energy in Statistical Physics  
\n: Partition function can be written as follows  
\n
$$
Z = e^{-\beta E_0}
$$
\n
$$
= e^{-\beta E_0}
$$
\n
$$
= \frac{e^{-\beta E_0}}{(1 - e^{-\beta E_0})}
$$
\n
$$
= \frac{e^{-\beta E_0}}{(1 - e^{-\beta E_0})}
$$
\n
$$
E = -\frac{d}{d\beta} \ln(Z) = -\frac{d}{d\beta} \ln\left(\frac{e^{-\beta E_0}}{(1 - e^{-\beta E_0})}\right)
$$
\n
$$
E = -\frac{d}{d\beta} \ln\left(\frac{e^{-\beta E_0}}{(1 - e^{-\beta E_0})}\right)
$$
\n
$$
E = -\frac{d}{d\beta} \ln\left(\frac{e^{-\beta E_0}}{(1 - e^{-\beta E_0})}\right) = -\frac{d}{d\beta}[-\beta E_0 - \ln(1 - e^{-\beta E_0})] = E_0 + \frac{E_0 e^{-\beta E_0}}{(1 - e^{-\beta E_0})} = \frac{E_0}{(1 - e^{-\beta E_0})}
$$
\n
$$
\frac{E}{E_0} = \frac{1}{(1 - e^{-\beta E_0})} \Rightarrow \ln\left(\frac{E}{E_0}\right) = -\ln(1 - e^{-\beta E_0})
$$
\nFree energy can be written as follows  
\n
$$
F = -kT \ln(Z) = -kT \ln \frac{e^{-\beta E_0}}{(1 - e^{-\beta E_0})} = -kT \ln e^{-\beta E_0} + kT \ln(1 - e^{-\beta E_0})
$$
\n
$$
F = E_0 + kT \ln(1 - e^{-\beta E_0}) = E_0 - kT \ln\left(\frac{E}{E_0}\right)
$$
 [Since  $\ln(1 - e^{-\beta E_0}) = -\ln\left(\frac{E}{E_0}\right)$ ]  
\nA polymer made up of *N* monomers, is in thermal equilibrium at temperature *T*. Each monomer could be of length *a* or 2*a*. The first contributes zero energy, while the second one

Free energy can be written as follows

$$
F = -kT \ln(Z) = -kT \ln \frac{e^{-\beta E_0}}{(1 - e^{-\beta E_0})} = -kT \ln e^{-\beta E_0} + kT \ln(1 - e^{-\beta E_0})
$$
  

$$
F = E_0 + kT \ln(1 - e^{-\beta E_0}) = E_0 - kT \ln\left(\frac{E}{E_0}\right) \text{ [Since } \ln(1 - e^{-\beta E_0}) = -\ln\left(\frac{E}{E_0}\right) \text{]}
$$

Q20. A polymer made up of N monomers, is in thermal equilibrium at temperature T. Each monomer could be of length  $a$  or  $2a$ . The first contributes zero energy, while the second one contributes  $\epsilon$ . The average length (in units of  $Na$ ) of the polymer at temperature B T k  $=\frac{\in}{\infty}$  is

(a) 
$$
\frac{5+e}{4+e}
$$
 (b)  $\frac{4+e}{3+e}$  (c)  $\frac{3+e}{2+e}$  (d)  $\frac{2+e}{1+e}$ 

 Topic: Thermodynamics & Statistical Mechanics Sub Topic: Boltzmann's Distribution Law

Ans. : (d)

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Solution: Probability of length a is  $P_a$ 

Probability of length 2a is  $P_{2a}$ 

We know that the probability  $P \alpha e^{-\beta \varepsilon}$ 

Thus, 
$$
\frac{P_{2a}}{P_a} = e^{-\beta \varepsilon} = e^{-\frac{1}{kT}\varepsilon}
$$

Given that,  $T = \frac{\varepsilon}{k}$  $=\frac{\varepsilon}{\cdot}$ 

$$
P_{2a} = e^{-1} P_a \dots (1)
$$

Also, total probability

$$
P_{2a}+P_a=1......(2)
$$

From (1) and (2), we will get

 ${}^{1}P_{a} + P_{a} = 1 \Rightarrow P_{a} = \frac{e}{e+1}$  $e^{-1}P_a + P_a = 1 \Rightarrow P_a = \frac{e}{e+1}$ . Thus,  $P_{2a} = \frac{1}{e+1}$  $P_{2a} = \frac{1}{e+1}$  $=\frac{1}{e+1}$ [Since  $p_{2a} = ep_a$ ] Average length, et<br>
et<br>  $P_{2a}$ <br>  $P_{2a} = \frac{1}{e+1}$  [Since  $p_{2a} = ep_a$ ]<br>  $P_{2a} = Na(P_a + 2P_{2a}) = Na\left(\frac{e}{e+1} + 2\frac{1}{e+1}\right) = Na\left(\frac{e+2}{e+1}\right)$  $2NaP_{2a} = Na(P_a + 2P_{2a}) = Na\left(\frac{e}{1} + 2\frac{1}{1}\right) = Na\left(\frac{e+2}{1}\right)$  $\sum_{i} I_{i} P_{i} = \text{N} a P_{a} + \text{2} \text{N} a P_{2a} = \text{N} a (P_{a} + \text{2} P_{2a}) = \text{N} a \left( \frac{1}{e+1} + \frac{1}{e+1} \right) = \text{N} a \left( \frac{1}{e+1} \right)$  $|l\rangle = \sum l_i P_i = NaP_a + 2NaP_{2a} = Na(P_a + 2P_{2a}) = Na\left(\frac{e}{e} + 2\frac{1}{4}\right) = Na\left(\frac{e+2}{4}\right)$  $\overline{e+1}$ +2 $\overline{e+1}$  $\left| \overline{-1}u\right|$  $\overline{e+1}$  $=\sum_{i}l_{i}P_{i} = NaP_{a} + 2NaP_{2a} = Na(P_{a} + 2P_{2a}) = Na\left(\frac{e}{e+1} + 2\frac{1}{e+1}\right) = Na\left(\frac{e+2}{e+1}\right)$ 

In the unit of Na it will be  $2^{^{\degree}}$ 1 e e  $(e+2)$  $\left(\frac{e+2}{e+1}\right)$ 

Q21. Balls of ten different colours labeled by  $a = 1, 2, ..., 10$  are to be distributed among different coloured boxes. A ball can only go in a box of the same colour, and each box can contain at most one ball. Let  $n_a$  and  $N_a$  denote, respectively the numbers of balls and boxes of colour  $a$ . Assuming that  $N_a \gg n_a \gg 1$ , the total entropy (in units of the Boltzmann constant) can be best approximated by

(a) 
$$
\sum_{a} (N_a \ln N_a + n_a \ln n_a - (N_a - n_a) \ln (N_a - n_a))
$$
  
\n(b)  $\sum_{a} (N_a \ln N_a - n_a \ln n_a - (N_a - n_a) \ln (N_a - n_a))$   
\n(c)  $\sum_{a} (N_a \ln N_a - n_a \ln n_a + (N_a - n_a) \ln (N_a - n_a))$   
\n(d)  $\sum_{a} (N_a \ln N_a + n_a \ln n_a + (N_a - n_a) \ln (N_a - n_a))$ 

 Topic: Thermodynamics & Statistical Mechanics Sub Topic: Microcanonical Ensemble

Ans. : (b)

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1: Number of ways such that  $n_x$  particle will be adjusted in same colour  $N_a$  box<br>  $W = \prod_a \frac{|N_a|}{|n_a|N_a - n_a|}$ <br>
So, entropy  $S = k_a \ln W = k_a \left[ \$ 

Solution: Number of ways such that  $n_a$  particle will be adjusted in same colour  $N_a$  box

$$
W = \prod_{a} \frac{N_a}{\left| n_a \right| N_a - n_a}
$$

So, entropy  $S = k_B \ln W = k_B \left[ \ln \left| N_a - \ln \left| n_a - \ln \left| N_a - n_a \right| \right. \right] \right]$ 

Using Sterling formula  $\ln |n = n \ln - n|$ 

$$
S = k_B \left[ N_a \ln N_a - N_a - n_a \ln n_a + n_a - (N_a - n_a) \ln (N_a - n_a) + (N_a - n_a) \right]
$$
  

$$
S = k_B \left( N_a \ln N_a - n_a \ln n_a - (N_a - n_a) \ln (N_a - n_a) \right)
$$

Total entropy in the unit of  $k_{B}$ 

$$
S = \sum (N_a \ln N_a - n_a \ln n_a - (N_a - n_a) \ln (N_a - n_a))
$$

Q22. The dispersion relation of a gas of non-interacting bosons in  $\,d\,$  dimensions is  $\,E(k)\!=\!ak^{_s}$  , where

 $a$  and  $s$  are positive constants. Bose-Einstein condensation will occur for all values of

(a)  $d > s$  (b)  $d + 2 > s > d - 2$ 



 Topic: Thermodynamics & Statistical Mechanics Sub Topic: Bose-Einstein Condensation

Ans. : (a)

Solution: Given that  $E(k) = Ak^s$ 

Now, the density of states  $\rho(E) \alpha E^{d/s-1}$ ,  $d =$  dimension

For Bose Einstein condensation

 $d / s - 1 > 0 \Rightarrow d / s > 1 \Rightarrow d > s$ 

Q23. Lead is superconducting below  $7 K$  and has a critical magnetic field  $800 \times 10^{-4}$  tesla close to 0 K . At 2 K the critical current that flows through a long lead wire of radius 5  $mm$  is closest to (a) 1760 A (b) 1670 A (c) 1950 A (d) 1840 A

 Topic: Solid State Physics Sub Topic: Superconductivity

Ans. : (d)

Solution:  $T_c = 7K$ 

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**PROBLEM SET 18.1 CAUTION**  
\nCSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics  
\n
$$
H_C(0) = 800 \times 10^{-4} = 8 \times 10^{-2} \times 79700 \text{ A/m}
$$
\n
$$
H_C(T = 2K) = 8 \times 10^{-2} \left\{ 1 - \left(\frac{2}{7}\right)^2 \right\} \times 79700
$$
\n= 585551.02×10<sup>-2</sup> A/m  
\n $I_C = 2\pi r H_C = 2 \times 3.14 \times 5 \times 10^{-3} \times 585551.02 \times 10^{-2} A = 1840 \text{ A}$ 

- Q24. Potassium chloride forms an FCC lattice, in which K and Cl occupy alternating sites. The density of KCl is 1.98  $g / cm^3$  and the atomic weights of K and Cl are 39.1 and 35.5, respectively. The angles of incidence (in degrees) for which Bragg peaks will appear when  $X$ -ray of wavelength 0.4  $nm$  is shone on a  $KCl$  crystal are
	- (a) 18.5, 39.4 and 72.2 (b) 19.5 and 41.9 (c)  $12.5, 25.7, 40.5$  and  $60.0$  (d) $13.5, 27.8, 44.5$  and  $69.0$ Topic: Solid State Physics

Sub Topic: Crystal Structure, X-Ray Diffraction

Ans.  $:(a)$ 

Solution: We know for cubic lattice,  $a^3 = \frac{n_{\text{eff}} M_{A1} + n_{\text{eff}2} M_{A2}}{2M} = \frac{4(39.1) + 4(35.5)}{1.08 \times 6.23 \times 10^{23}} = 242 \times 10^{-24}$  $1.98 \times 6.23 \times 10^{2}$  $_{eff}$ <sub>1</sub> $M$ <sub>A1</sub> +  $n_{eff}$ <sub>2</sub> $M$ <sub>A2</sub> A  $n_{\text{eff}1}M_{A1} + n_{\text{eff}2}M_{A}$  $a^3 = \frac{n_{eff1}M_{Al} + n_{eff2}M_{Al}}{M} = \frac{4(39.1) + 4(33.3)}{1.28 \times 10^{23}} = 242 \times 10^{-24} \text{ cm}$  $\rho N$  $=\frac{n_{\text{eff}}M_{A1}+n_{\text{eff2}}M_{A2}}{N_{\text{eff2}}M_{\text{H2}}}=\frac{4(39.1)+4(35.5)}{1.00\times10^{23}}=242\times10^{-3}$  $\times 6.23\times 10$ 

 $a = 6.22 A^0$ 

The interplanar spacing can be written as follows

$$
d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}
$$

For KCl crystal, the (h, k, l) values for which Bragg's diffraction observed are (2, 0, 0),

 $(2, 2, 0), (2, 2, 2),$ 

From Bragg's law,  $2d \sin(\theta) = \lambda$ 

Given that, 
$$
\lambda = 0.4nm = 4A^0
$$

$$
2 \frac{a}{\sqrt{h^2 + k^2 + l^2}} \sin(\theta) = \lambda \implies 2 \frac{6.22}{\sqrt{h^2 + k^2 + l^2}} \sin(\theta) = 4
$$
  
\n
$$
\implies \frac{1}{\sqrt{h^2 + k^2 + l^2}} \sin(\theta) = \frac{4}{6.22 \times 2} = 0.31
$$
  
\n
$$
\implies \sin(\theta) = 0.31\sqrt{h^2 + k^2 + l^2} \implies \theta = \sin^{-1}\left(0.31\sqrt{h^2 + k^2 + l^2}\right)
$$
  
\nFor (h, k, 1) = (2, 0, 0),  
\n
$$
\implies \theta = \sin^{-1}\left(0.31 \times 2\right) = \sin^{-1}\left(0.62\right) = 39.4^{\circ}
$$

Q25. In the reaction  $p + n \rightarrow p + K^+ + X$ , mediated by strong interaction, the baryon number B,

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strangeness  $S$  and third component of isospin  $|I_3|$  of the particle  $X$  are, respectively

(a)  $-1$ , 1 and  $-1$  (b)  $+1$ , 1 and  $-1$ (c)  $+1, -2$  and  $-\frac{1}{2}$ (d)  $-1, -1$  and 0

> Topic: Nuclear & Particle Physics Sub Topic: Fundamental Interaction in Particle Physics

Ans. : (b)

Solution:  $p+n \to p+K^+ + X$ 

B: 1+1=1+0+B 
$$
\Rightarrow
$$
 B = 1  
\nS: 0+0=0+1+S  $\Rightarrow$  S = -1  
\n $I_3$ :  $(\frac{1}{2})+(-\frac{1}{2})=(\frac{1}{2})+(\frac{1}{2})+I_3 \Rightarrow I_3 = -1$ 

- Q26. A <sup>60</sup> C<sub>o</sub> nucleus  $\beta$  -decays from its ground state with  $J^P = 5^+$  to a state of <sup>60</sup> Ni with  $J^P = 4^+$ . From the angular momentum selection rules, the allowed values of the orbital angular momentum  $L$  and the total spin  $S$  of the election-antineutrino pair are
	- (a)  $L = 0$  and  $S = 1$  (b)  $L = 1$  and  $S = 0$
	- (c)  $L = 0$  and  $S = 0$  (d)  $L = 1$  and  $S = 1$

 Topic: Nuclear & Particle Physics Sub Topic: Beta Decay

Ans. : (a)

Solution:  ${}^{60}Co \rightarrow {}^{60}Ni + e^- + \overline{v_e}$ 

 $5^+ \rightarrow 4^+ + (e^- + \overline{v_e})$ 

We know that the parity is defined as  $(-1)^L$ 

From parity conservation, we can say that the orbital angular momentum of  $(e^- + \overline{v_e})$  is zero.

On the, the other hand  $e^-$  and  $v_e$  both are spin  $\frac{1}{2}$  (Fermions)

$$
{}^{60}Co \rightarrow {}^{60}Ni + e^- + \overline{v_e}
$$
  

$$
5^+ \rightarrow 4^+ + (e^- + \overline{v_e})
$$

Total S of  $(e^- + \overline{v_e})$  is either 0 or 1

But, from angular momentum conservation S should  $(+1)$ 

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**CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics**<br>So total  $S = 1/2 + 1/2 = 1$   $L = 0$ ,  $S = 1$ <br>The Q -value of the  $\alpha$ -decay of  $\frac{225}{H}$  to the ground state of  $\frac{228}{H}Ra$  is 4082  $keV$ . The maximum<br>possible k Q27. The  $Q$  -value of the  $\alpha$  -decay of <sup>232</sup>Th to the ground state of <sup>228</sup>Ra is 4082 keV. The maximum possible kinetic energy of the  $\alpha$ -particle is closest to **EXECUTE: CONSERVERTLANT CONSERVERTLANT SOLUTION**<br>
So total  $S = 1/2 + 1/2 = 1$ <br>
227. The *Q* -value of the *α*-decay of <sup>332</sup>Th to the ground state of <sup>238</sup>Ra is 4082 keV. The maximum<br>
possible kinetic energy of the *A*-p

(a)  $4082 \, keV$  (b)  $4050 \, keV$  (c)  $4035 \, keV$  (d)  $4012 \, keV$ 

 Topic: Nuclear & Particle Physics Sub Topic: Alpha Decay

Ans. : (d)

$$
E_{th} = \frac{(A-4)}{A}Q = \frac{228}{232}4082 = 4012 \,\text{keV}
$$

COCIR EULCATION<br>
So total  $S = 1/2 + 1/2 = 1$ <br>  $L = 0$ ,  $S = 1$ <br>
Q27. The Q -value of the  $\alpha$ -decay of <sup>332</sup>  $Th$  to the ground state of <sup>331</sup>  $Ra$  is 4082  $keV$ . The maximum<br>
possible kinetic energy of the  $\alpha$ -particle is clos decays to the state  $|1,0,0\rangle$  via two dipole transitions. The transition route and the corresponding probability are The Q -value of the  $\alpha$ -decay of  $^{332}Th$  to the ground state of  $^{238}Ra$  is  $4082$  keV. The maximum<br>possible kinetic energy of the  $\alpha$ -particle is closest to<br>(a)  $4082 \text{ keV}$  (b)  $4050 \text{ keV}$  (c)  $4035 \text{ keV}$  (d)  $4012$ to the ground state of  $^{238}Ra$  is 4082  $keV$ . The maximum<br>
le is closest to<br>
(c) 4035  $keV$  (d) 4012  $keV$ <br>
Topic: Nuclear & Particle Physics<br>
Sub Topic: Alpha Decay<br>
(d)  $n, l, m \rangle$ ) of the *H*-atom in the non-relativistic t (a)  $4082 \, keV$  (b)  $4050 \, keV$  (c)  $4035 \, keV$  (d)  $4012 \, keV$ <br>
Topic: Nuclear & Particle Physics<br>
(d)  $4012 \, keV$ <br>
Topic: Nuclear & Particle Physics<br>
(d)  $4012 \, keV$ <br>
Topic: Alpha Decay<br>
(d)  $4012 \, keV$ <br>
Topic: Alpha Decay<br>
(d (c) 4035 keV (d) 4012 keV<br>
Topic: Nuclear & Particle Physics<br>
Sub Topic: Alpha Decay<br>
(d)<br>
(d) 4012 keV<br>
Sub Topic: Alpha Decay<br>
(d)  $|n,l,m\rangle$ ) of the H-atom in the non-relativistic theory<br>
ole transitions. The transition Solution:  $\frac{325(4)T_1}{A} \rightarrow \frac{228(4-4)}{232}4082 = 4012 \text{ keV}$ <br>
228. The  $\left| 3, 0, 0 \right\rangle$  state (in the standard notation  $\left| n, m \right\rangle$ ) of the *H*-atom in the non-relativistic theory<br>
decays to the state  $\left| 1, 0, 0 \right\rangle$ The  $|3,0,0\rangle$  state (in the standard notation  $|n,l,m\rangle$ ) of the *H*-atom in the non-relativistic theory<br>
decays to the state  $|1,0,0\rangle$  via two dipole transitions. The transition route and the corresponding<br>
robability ar

decays to the state 
$$
|1,0,0\rangle
$$
 via two dipole transitions. The transition route and the corresponding  
probability are  
\n(a)  $|3,0,0\rangle \rightarrow |2,1,-1\rangle \rightarrow |1,0,0\rangle$  and  $\frac{1}{4}$   
\n(b)  $|3,0,0\rangle \rightarrow |2,1,1\rangle \rightarrow |1,0,0\rangle$  and  $\frac{1}{4}$   
\n(c)  $|3,0,0\rangle \rightarrow |2,1,0\rangle \rightarrow |1,0,0\rangle$  and  $\frac{1}{3}$   
\n(d)  $|3,0,0\rangle \rightarrow |2,1,0\rangle \rightarrow |1,0,0\rangle$  and  $\frac{2}{3}$   
\nTopic: Atomic & Molecular Physics  
\nSub Topic: Dipole Transition  
\n1: Selection rule Δ*I* = ±1, Δ*m*<sub>*l*</sub> = 0, ±1  
\nThe possible decay mode for  $|3,0,0\rangle$   
\n $|3,0,0\rangle \rightarrow |2,1,1\rangle$ ,  $|3,0,0\rangle \rightarrow |2,1,0\rangle$ ,  $|3,0,0\rangle \rightarrow |2,1,-1\rangle$   
\nEach are equally probable. So, probability for  $|3,0,0\rangle \rightarrow |2,1,0\rangle$  will be 1/3  
\nA linear diatomic molecule consists of two identical small electric dipoles with an equilibrium  
\nseparation *R*, which is assumed to be a constant. Each dipole has charges ±*q* of mass *m*

 Topic: Atomic & Molecular Physics Sub Topic: Dipole Transition

Ans.  $: (c)$ 

The possible decay mode for  $|3,0,0\rangle$ 

$$
\big|3,0,0\big\rangle\!\rightarrow\!\big|2,1,1\big\rangle,\ \big|3,0,0\big\rangle\!\rightarrow\!\big|2,1,0\big\rangle,\ \big|3,0,0\big\rangle\!\rightarrow\!\big|2,1,-1\big\rangle
$$

Q29. A linear diatomic molecule consists of two identical small electric dipoles with an equilibrium separation R, which is assumed to be a constant. Each dipole has charges  $\pm q$  of mass M separated by  $r$  when the molecule is at equilibrium. Each dipole can execute simple harmonic motion of angular frequency  $\omega$ .



H.N. 28 A/1, Jia Sarai, Near IIT-Delhi, Hauz Khas, New Delhi-110016 #: +91-89207-59559 Website: www.pravegaa.com | Email: pravegaaeducation@gmail.com

**Vegaal Education** 

Recall that the interaction potential between two dipoles of moments  $p_1$  and  $p_2$ , separated by

$$
R_2 = R_1 \hat{n} \text{ is } (p_1 \cdot p_2 - 3(p_1 \cdot \hat{n})(p_2 \cdot \hat{n})) / (4\pi \in R_1^3).
$$
  
Assume that  $R >> r$  and let  $\Omega^2 = \frac{q^2}{4\pi \in R_0^3}$ . The angular frequencies of small oscillations of the  
diatomic molecule are  
(a)  $\sqrt{\omega^2 + \Omega^2}$  and  $\sqrt{\omega^2 - \Omega^2}$   
(b)  $\sqrt{\omega^2 + 3\Omega^2}$  and  $\sqrt{\omega^2 - 3\Omega^2}$   
(c)  $\sqrt{\omega^2 + 4\Omega^2}$  and  $\sqrt{\omega^2 - 4\Omega^2}$   
(d)  $\sqrt{\omega^2 + 2\Omega^2}$  and  $\sqrt{\omega^2 - 2\Omega^2}$ 

Topic: Electromagnetic Theory

Sub Topic: Dipole Interaction

# Ans.  $: (c)$

Solution: The interaction energy between electric dipole is given as

$$
U = \frac{\left(p_1 \cdot p_2 - 3\left(p_1 \cdot \hat{n}\right)\left(p_2 \cdot \hat{n}\right)\right)}{4\pi\varepsilon_0 R_{12}^3}
$$

Since,  $p_1 = qr, p_2 = qr, p_1 || p_2$ 

$$
U = \frac{\left(p_1 \cdot p_2 - 3\left(p_1 \cdot \hat{n}\right)\left(p_2 \cdot \hat{n}\right)\right)}{4\pi\varepsilon_0 R_{12}^3} = \frac{1}{4\pi\varepsilon_0 R_{12}^3} (1 - 3)q^2 r^2 = -2m\Omega^2 r^2
$$

The potential energy of one dipole will be

$$
U = \frac{1}{2} m \omega^2 r^2 + 2m\Omega^2 r^2 = \frac{1}{2} m \omega^{12} r^2 \Rightarrow \omega^1 = \sqrt{\omega^2 + 4\Omega^2}
$$

The potential energy of another dipole should be

$$
U = \frac{1}{2} m \omega^2 r^2 - 2m\Omega^2 r^2 = \frac{1}{2} m \omega^{12} r^2 \Rightarrow \omega = \sqrt{\omega^2 - 4\Omega^2}
$$
 [From energy conservation]

Q30. Diffuse hydrogen gas within a galaxy may be assumed to follow a Maxwell distribution at temperature 10<sup>6</sup> K, while the temperature appropriate for the  $H$  gas in the inter-galactic space, following the same distribution, may be taken to be  $10^4 K$ . The ratio of thermal broadening  $\Delta v_{_G}$  /  $\Delta v_{_{1G}}$  of the Lyman- $\alpha$  line from the  $H$ -atoms within the galaxy to that from the intergalactic space is closest to

(a) 100 (b) 
$$
\frac{1}{100}
$$
 (c) 10 (d)  $\frac{1}{10}$ 

10

 Topic: Atomic & Molecular Physics Sub Topic: Broadening of Spectral Lines

Ans. (c)

**PraVegam Education** CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics

Solution: The thermal means Doppler broadening

We know that

$$
\Delta v_D = \frac{v_0}{c} \sqrt{\frac{2kT}{m}} \Rightarrow \Delta v_D \alpha \sqrt{T}
$$

$$
\frac{\Delta v_G}{\Delta v_{IG}} = \frac{\sqrt{10^6}}{\sqrt{10^4}} = 10
$$