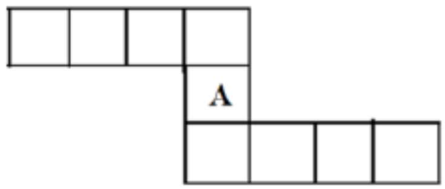
# NET-JRF (June 2022) [Solution]

## PART A

**Question ID:- 515**

The squares in the following sketch are filled with digits 1 to 9, without any repetition, such that the numbers in the

two horizontal rows add up to 20 each. What number appears in the square labelled A in the vertical column?




It cannot be ascertained in the absence of the sum of the numbers in the column , Option ID :- 2057,

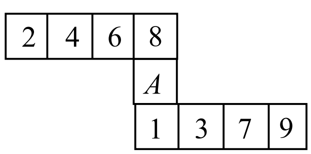
3, Option ID :- 2058,

5, Option ID :- 2059,

7, Option ID :- 2060,

**Ans.: Option ID-2059**

**Solution:**




From the diagram we see that number 5 should come in place of $A$.

**Question ID:- 516**

Sections A, B, C and D of a class have 24, 27, 30 and 36 students, respectively. One section has boys and girls who

are seated alternately in three rows, such that the first and the last positions in each row are occupied by boys. Which

section could this be?

A, Option ID :- 2061, B, Option ID :- 2062,

C, Option ID :- 2063, D, Option ID :- 2064,

**Ans.: Option ID-2062**

**Solution:** Since in each row boys and girls are seated alternately and the first and last position is occupied by boys, then must be bad number of students in each row. Now

$$\text{Odd} + \text{Odd} + \text{Odd} = \text{Odd}$$

This means the required section could be option B.

**Question ID:- 517**

In a round-robin tournament, after each team has played exactly four matches, the number of wins/ losses of 6 participating teams are as follows

| Team | Win | Loss |
|---|---|---|
| A | 4 | 0 |
| B | 0 | 4 |
| C | 3 | 1 |
| D | 2 | 2 |
| E | 0 | 4 |
| F | 3 | 1 |

Which of the two teams have certainly NOT played with each other?

A and B, Option ID :- 2065, C and F, Option ID :- 2066,

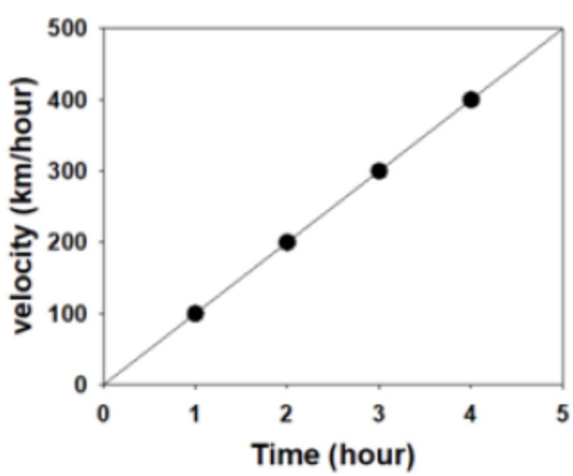
E and D, Option ID :- 2067, B and E, Option ID :- 2068

**Ans. : Option ID: 2068**

**Solution:** $B$ hes lost all 4 matches and $E$ has also lost all 4 matches. This means the has been no match between $B$ and $E$. If the was a match between $B$ and $E$, either $B$ or $E$ must have at least one win.

**Question ID:- 518**

Given plot describes the motion of an object with time.



Which one of the following statements is CORRECT?

The object is moving with a constant velocity, Option ID :- 2069,

The object covers equal distance every hour, Option ID :- 2070,

The object is accelerating, Option ID :- 2071,

Velocity of the object doubles every hour, Option ID :- 2072

**Ans.: Option ID: 2071**

**Solution:** Velocity time graph is a straight line having non-zero slope. This means the object is accelerating uniformly.

**Question ID:- 519**

If one letter each is drawn at random from the words CAUSE and EFFECT, the chance that they are the same is

1/30 , Option ID :- 2073,

1/11 , Option ID :- 2074,

1/10 , Option ID :- 2075,

2/11, Option ID :- 2076,

**Ans.: Option ID: 2075**

**Solution:** The total number of outcomes $= 5 \times 6 = 30$

For the letters to be same.

| Number from the letter CAUSE | Number form the letter EFFECT |
|---|---|
| C | C |
| E | E |
| E | E |

Hence favorite outcomes $= 3$

$$\text{Hence required probability } = \frac{3}{30} = \frac{1}{10}$$

**Question ID:- 520**

A vehicle has tyres of diameter 1 m connected by a shaft directly to gearwheel A which meshes with gearwheel B as shown in the diagram. A has 12 teeth and B has 8. If points *x* on A and *y* on B are initially in contact, they will again be in contact after the vehicle has travelled a distance (in meters)

2π, Option ID :- 2077,

3π, Option ID :- 2078,

4π, Option ID :- 2079,

12π, Option ID :- 2080

**Ans.: Option ID-2077**

**Question ID:- 521**

A liar always lies and a non-liar, never. If in a group of *n* persons seated around a round-table everyone calls his/her left neighbor a liar, then

all are liars, Option ID :- 2081,

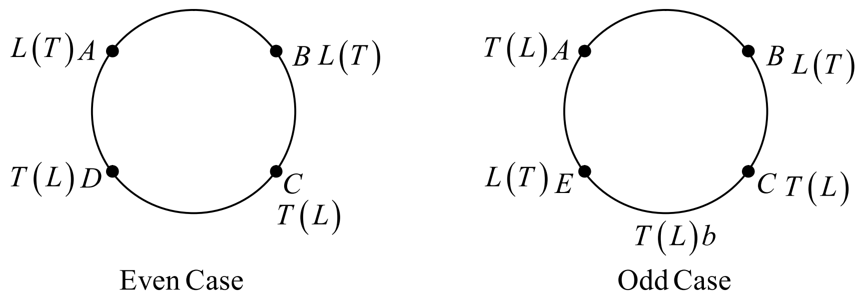
*n* must be even and every alternate person is a liar, Option ID :- 2082,

*n* must be odd and every alternate person is a liar, Option ID :- 2083,

*n* must be a prime, Option ID :- 2084

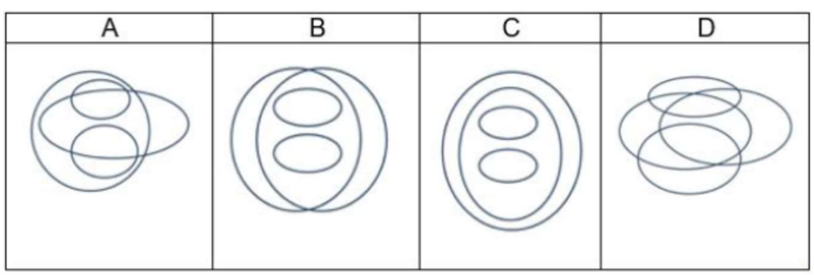
**Ans.: Option ID- 2082**

**Solution:** $n$ must be even and every alternate person is a liar.




**Question ID:- 522**

The correct pictorial representation of the relations among the categories PLAYERS, FEMALE CRICKETERS, MALE FOOTBALLERS and GRADUATES is




A, Option ID :- 2085,

B, Option ID :- 2086,

C, Option ID :- 2087,

D, Option ID :- 2088

**Ans.: Option ID: 2085**

**Question ID:- 523**

What is the product of the number of capital letters and the number of small letters of the English alphabet in the

following text?

17, Option ID :- 2089, 37, Option ID :- 2090,

53, Option ID :- 2091, 63, Option ID :- 2092,

**Ans.: Option ID-2092**

**Solution:** No. of capital letters $= 9$, No of small letters $= 7$

Required product $= 9 \times 7 = 63$

**Question ID:- 524**

On a track of 200 m length, S runs from the starting point and R starts 20 m ahead of S at the same time. Both reach the end of the track at the same time. S runs at a uniform speed of 10 m/s. If R also runs at a uniform speed, what is R's speed (in m/s)?

9, Option ID :- 2093, 10, Option ID :- 2094,

12, Option ID :- 2095, 8, Option ID :- 2096,

**Ans.: Option ID-2093**

**Solution:** Time taken by both $S$ and $R$ to reach the end point is the same.

$$\text{Time taken by} S = \frac{200}{10} = 20 \text{m/s}$$

$$\text{Time taken by } R = \frac{180}{v} \Rightarrow 20 = \frac{180}{v} \Rightarrow v = 9 \text{ m/s.}$$

**Question ID:- 525**

A plant grows by 10% of its height every three months. If the plant's height today is 1 m, its height after one year is

the closest to

1.10 m, Option ID :- 2097, 1.21 m, Option ID :- 2098,

1.33 m, Option ID :- 2099, 1.46 m, Option ID :- 2100,

**Ans.: Option ID - 2100**

**Solution:** The height of the tree after 1 year

$$= 1\left(1 + \frac{10}{100}\right)^4 = \left(\frac{11}{10}\right)^4 = 1.4641$$

**Question ID:- 526**

Starting from the top of a page and pointing downward, an ant moves according to the following commands

Start

↓

*Repeat actions in curly brackets 3 times*

{

*Repeat actions in the box 4 times*

Move forward 100 units

↓

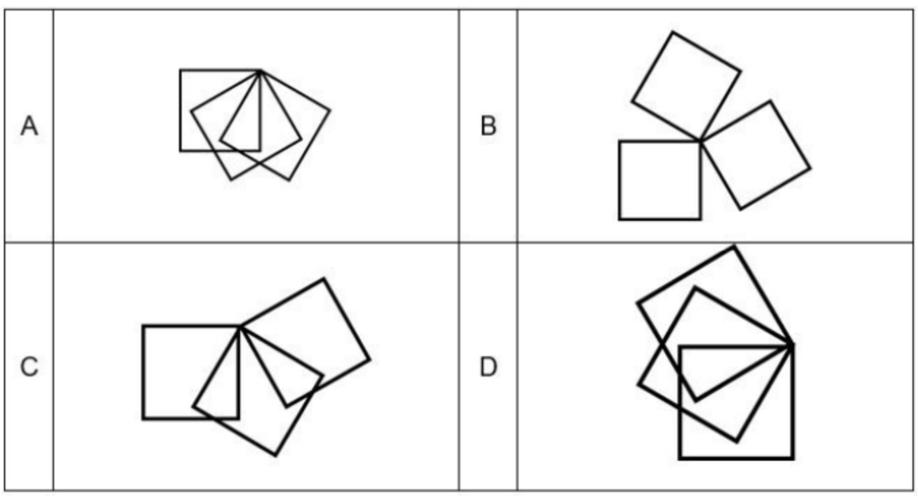
Turn right by 90 degrees

↓

Turn left by 30 degrees

}

Of the following paths




Which is the current path of the ant?

Options:-

A, Option ID :- 2101, B, Option ID :- 2102,

C, Option ID :- 2103, D, Option ID :- 2104

**Ans.: Option ID: 2101**

**Question ID:- 527**

In a four-digit PIN, the third digit is the product of the first two digits and the fourth digit is zero. The number of such

PINs is

42, Option ID :- 2105, 41, Option ID :- 2106,

40, Option ID :- 2107, 39, Option ID :- 2108,

**Ans.: Option ID: 2105**

**Solution:** Let the four digit number be $abc0$. From the question $a \times b = c$.

| Third Place | A and B can be filled in ways |
|---|---|
| 0 | 19 |
| 1 | 1 |
| 2 | 2 |
| 3 | 2 |
| 4 | 3 |
| 5 | 2 |
| 6 | 4 |
| 7 | 2 |
| 8 | 4 |
| 9 | 3/42 |

**Question ID:- 528**

After 10:00:00 the hour hand and minute hand of a clock will be perpendicular to each other for the first time at

12:16:21, Option ID :- 2109,

12:15:00, Option ID :- 2110,

13:22:21, Option ID :- 2111,

12:48:08, Option ID :- 2112

**Ans.: Option ID:2109**

**Solution:** Suppose $t$ minutes After $12:00$, the Hand and the minute holes hand are perpendicular to each other.

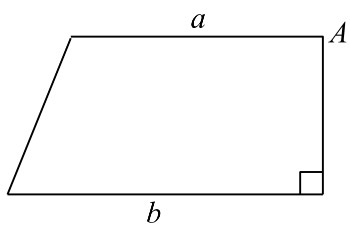
The hour hand moves $0.5'$ per minute and the minute hand moves $6''$ per minute. From the question

$$6t + 0.5t = 90^\circ \Rightarrow 5.5\text{t} = 90$$
$$\Rightarrow t + \frac{90}{5.5} = \frac{900}{55} = \frac{180}{11} = 16\frac{4}{11} \text{ minutes}$$
$$= 16 \text{ minutes and } \frac{4}{11} \times 60$$
$$= 16 \text{ minutes } 21 \text{ seconds nearly}$$

**Question ID:- 529**

At what horizontal distance from A should a vertical line be drawn so as to divide the area of the trapezium shown in

the figure into two equal parts ? (a and b are lengths of the parallel sides.)




$(a+b)/4$, Option ID :- 2113,

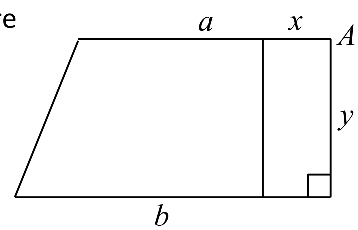
$(a+b)/3$, Option ID :- 2114,

$(a+b)/2$, Option ID :- 2115,

$(2a+b)/2$, Option ID :- 2116,

**Ans.: Option ID -2113**

**Solution:** Let the required distance be $x$. From the figure




$$xy = \frac{1}{2}(a - x + b - x)y \Rightarrow x = \frac{1}{2}(a + b - 2x)$$

$$\Rightarrow 2x = a + b - 2x \Rightarrow 4x = a + b \Rightarrow x = \frac{a+b}{4}$$

**Question ID:- 530**

I have a brother who is 4 years elder to me, and a sister who was 5 years old when my brother was born. When my sister was born, my father was 24 years old. My mother was 27 years old when I was born. How old (in years) were my father and mother, respectively, when my brother was born?

29 and 23, Option ID :- 2117,

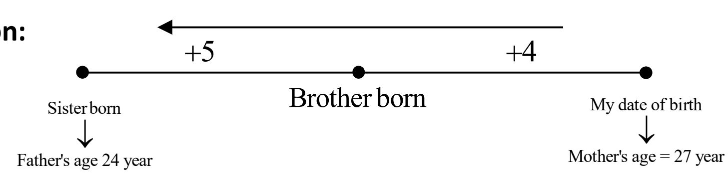
27 and 25, Option ID :- 2118,

27 and 23, Option ID :- 2119,

29 and 25, Option ID :- 2120

**Ans.: Option ID: 2117**

**Solution:**




From the figure we see that when my brother was born, my father's age was $24 + 5 = 29$ years. Also, we see that when my brother was born my mother's age was $27 - 4 = 23$ years.

**Question ID:- 531**

A boy has kites of which all but 9 are red, all but 9 are yellow, all but 9 are green, and all but 9 are blue. How many kites does he have?

2, Option ID :- 2121,
15, Option ID :- 2122,
9, Option ID :- 2123,
18, Option ID :- 2124

**Ans.: Option ID-2121**

**Solution:** Let then be $N$ balls of which $R$ balls are red $\gamma$ balls are yellow, G balls are green and B balls are blue from the question

$$\begin{aligned} N-R &= 9 \quad \text{(I)} \\ N-Y &= 9 \quad (II) \\ N-W &= 9 \quad \text{(III)} \\ N-B &= 9 \quad (IV) \end{aligned}$$

Adding all equations gives

$$4N-(N+\gamma+(G+B)=36$$

$$\Rightarrow 4N-N=36 \Rightarrow N=12$$

**Question ID:- 532**

Tokens numbered from 1 to 25 are mixed and one token is drawn randomly. What is the probability that the number on the token drawn is divisible either by 4 or by 6?

8/25, Option ID: 2125
10/25, Option ID: 2126
9/25, Option ID: 2127
12/25, Option ID: 2128

**Ans.: Option ID: 2125**

**Solution:** let $n(A), n(B)$ and $n(A \cap B)$ denote the numbers divisible by $4$, numbers divisible by 6 and the numbers by both 4 and 6 respectively, then

$$\begin{aligned} n(A \cup B) &= n(A)+n(B)-n(A \cap B) \\ &= 6+4-2=8 \end{aligned}$$

Hence required probability $=\frac{8}{25}$

**Question ID:- 533**

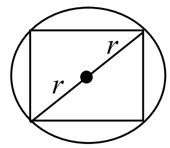
A beam of square cross-section is to be cut out of a wooden log. Assuming that the log is cylindrical, what approximately is the largest fraction of the wood by volume that can be fruitfully utilized as the beam?

49%, Option ID :- 2129,
64%, Option ID :- 2130,
71%, Option ID :- 2131,
81%, Option ID :- 2132

**Ans.: Option ID – 2130**

**Solution:** Let $l$ be the length of the wood and $r$ its radius of cross section. The cross section of the beam will be

$$(\sqrt{2}r)^2 = 2r^2$$



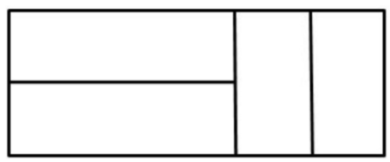

Hence the volume of bear will be

The fore required percentage

$$= 2r^2 l = \frac{2r^2 l}{\pi r^2 l} \times 100 = \frac{2}{\pi} \times 100 \approx 64\%$$

**Question ID:- 534**

How many rectangles are there in the given figure?



6, Option ID :- 2133,

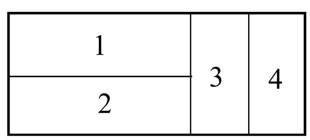
7, Option ID :- 2134,

8, Option ID :- 2135,

9, Option ID :- 2136

**Ans.: Option ID: 2135**

**Solution:**




Let us count the rectangles 1,2,3,4, 1+2, 1+2+3, 1+2+3+4, 3+4

So there are a total of eight rectangles.

**PART B**

**Question ID:- 1**

The value of the integral $\int_0^\infty dx e^{-x^{2m}}$, where $m$ is a positive integer, is

**Options:-**

$\Gamma\left(\frac{m+1}{2m}\right)$, Option ID :- 1,

$\Gamma\left(\frac{m-1}{2m}\right)$, Option ID :- 2,

$\Gamma\left(\frac{2m+1}{2m}\right)$, Option ID :- 3,

$\Gamma\left(\frac{2m-1}{2m}\right)$, Option ID :- 4,

**Topic – Mathematical Physics**

**Subtopic – Integration**

**Ans.:** $\Big)\frac{2m+1}{2m}$

**Solution:** $\int_0^\infty e^{-x^{2m}} dx$

Take $m=1$

$$\int_0^\infty e^{-x^2} dx = \frac{1}{2}\sqrt{\pi} \quad \left[ Since, \int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \right]$$

By substituting $m=1$, option ID 3 is giving the correct option.

Let us check option,

$$\Big)\frac{2m+1}{2m} = \Big)\frac{3}{2} = \frac{1}{2}\sqrt{\pi}$$

**Question ID:- 2**

At $z = 0$, the function $\frac{1}{z - \sin z}$ of a complex variable $z$ has

**Options:**

No singularity , Option ID: 5

A simple pole , Option ID: 6

A pole of order 2 , Option ID: 7

A pole of order 3 , Option ID: 8

**Topic – Mathematical Physics**

**Subtopic – Complex Analysis**

**Ans.:** A pole of order 3

**Solution:** $\frac{1}{z - \sin z} = \frac{1}{z - \left(z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots\right)} = \frac{1}{\frac{z^3}{3!} - \frac{z^5}{5!} + \frac{z^7}{7!} \cdots}$

At $z = 0$, there is a pole of order 3.

**Question ID:- 3**

Two $n \times n$ invertible real matrices $A$ and $B$ satisfy the relation

$$(AB)^T = -(A^{-1}B)^{-1}$$

If $B$ is orthogonal then $A$ must be

**Options:**

Lower triangle; Option ID : 9

Orthogonal; Option ID : 10

Symmetric; Option ID : 11

Anti-Symmetric; Option ID : 12

**Topic – Mathematical Physics**

**Subtopic – Matrices**

**Ans.:** Anti-Symmetric

**Solution:** $(AB)^{\top} = -(A^{-1}B)^{-1}$

$B^{\top}A^{\top} = -B^{-1}A$

$B^{-1}A^{\top} = -B^{-1}A$ (Since, B is orthogonal )

$A^{\top} = -A$ , A is asymmetric matrix

**Question ID:- 4**

The infinite series $\sum_{n=0}^{\infty}(n^2+3n+2)x^n$ evaluated at $x = \frac{1}{2}$, is

**Options:**

16; Option ID: 13

32; Option ID: 14

8; Option ID: 15

24; Option ID: 16

**Topic – Mathematical Physics**

**Subtopic – Series**

**Ans.: 16**

**Solution**: $\sum_{n=0}^{\infty}(n^2+3n+2)x^n = S_n$

We know $\sum x^n = \frac{1}{1-x}$

$\sum nx^{n-1} = \frac{1}{(1-x)^2}; \sum nx^n = \frac{x}{(1-x)^2}$ ……..(1)

diff. (1) w.r.t. $x$,

$$\sum n \cdot nx^{n-1} = \frac{1}{(1-x)^2} + x \cdot -2(1-x)^{-3} \cdot (-1) = \frac{1}{(1-x)^2} + \frac{2x}{(1-x)^3}$$

$$\sum n^2x^{n-1} = \frac{1+x}{(1-x)^3}$$

$$\sum n^2x^n = \frac{x(1+x)}{(1-x)^3}$$

$$\sum_{n=1}^{\infty}(n^2x^n + 3nx^n + 2x^n) = \frac{x(1+x)}{(1-x)^3} + \frac{3x}{(1-x)^2} + \frac{2}{1-x}$$

At $x = \frac{1}{2}$; $\frac{x(1+x)}{(1-x)^3} + \frac{3x}{(1-x)^2} + \frac{2}{1-x} = 16$

**Question ID:- 5**

If $z = i^{i^{i\cdots\cdots}}$ (note that the exponent continues indefinitely), then a possible value of $\frac{1}{z}\ln\ z$ is

**Options:-**

$2i\ln\ i$, Option ID :- 17,

$\ln\ i$, Option ID :- 18,

$i\ \ln i$,Option ID :- 19,

$2\ln\ i$ Option ID :- 20,

Topic – Mathematical Physics

Subtopic – Complex Analysis

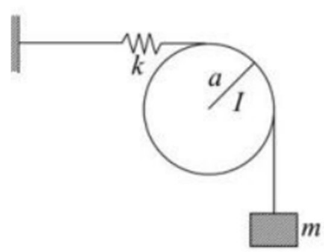
**Ans.** $\ln\ i$

**Solution:** $z = i^{i^{i}}$ $z = i^{Z}$; $\ln z = z\log\ i$

$$\frac{\ln\ z}{z} = \log\ i$$

**Question ID:- 6**

A wire, connected to a massless spring of spring constant $k$ and a block of mass $m$, goes around a disc of radius $a$ and moment of inertia $I$, as shown in the figure.




Assume that the spring remains horizontal, the pully rotates freely and there is no slippage between the wire and the pully. The angular frequency of oscillation of the disc is

**Options:-**

$\sqrt{\frac{2ka^2}{ma^2+I}}$, Option ID: – 21 $\sqrt{\frac{ka^2}{ma^2+I}}$, Option ID: – 22

$\sqrt{\frac{ka^2}{ma^2+2I}}$, Option ID : – 23, $\sqrt{\frac{ka^2}{2ma^2+I}}$, Option ID: – 24

Topic –Classical Mechanics

Subtopic – Rotational Motion

**Ans.**: $\omega = \sqrt{\dfrac{ka^2}{ma^2 + I}}$

**Solution:** $v = \omega R$

$$\omega = \frac{v}{R} = \frac{\dot{x}}{a}$$

$$L = T - V == \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\omega^2 - \frac{1}{2}kx^{\wedge}2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\frac{\dot{x}^2}{a^2} - \frac{1}{2}kx^{\wedge}2$$

Equation of motion

$$\left.\begin{aligned} &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \\ \Rightarrow\quad & m\ddot{x} + \frac{I}{a^2}\ddot{x} + kx = 0 \\ \Rightarrow\quad & \ddot{x}\left(m + \frac{I}{a^2}\right) + kx = 0 \\ \Rightarrow\quad & \ddot{x} + \left(\frac{ka^2}{ma^2 + I}\right)x = 0 \\ & \ddot{x} + \omega^2 x = 0 \end{aligned}\right\}$$

$$\omega = \sqrt{\frac{ka^2}{ma^2 + I}}$$

**Question ID:- 7**

The Lagrangian of a system described by three generalized coordinates $q_1, q_2$ and $q_3$ is $L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2) + M\dot{q}_1\dot{q}_2 + k\dot{q}_1 q_3$, where $m, M$ and $k$ are positive constants. Then, as a function of time

**Options:**

Two coordinates remain constant and one evolves linearly; Option ID: 25,

One coordinates remain constant, one evolves linearly and Third evolves as a quadratic function; Option ID: 26,

One evolves linearly and Two evolves as a quadratic function; Option ID: 27,

Al three evolves linearly; Option ID: 28

**Topic – Classical Mechanics**

**Subtopic – Lagrangian Formulation**

**Ans.:** Two coordinates remain constant and one evolves linearly

**Solution:** $L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2) + M\dot{q}_1\dot{q}_2 + k\dot{q}_1 q_3$

$$\frac{d}{dt}[m\dot{q}_1 + M\dot{q}_2 + q_3] = 0$$

$$\Rightarrow m\ddot{q}_1 + m\ddot{q}_2 + k\dot{q}_3 = 0.....(1)$$

$$\frac{d}{dt}(m\dot{q}_2 + M\dot{q}_1) = 0$$

$$m\ddot{q}_2 + M\ddot{q}_1 = 0......(2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_3}\right) - \frac{\partial^2}{\partial q_3} = 0$$

$$\dot{q}_1 = 0$$

$$q_1 = A......(3)$$

$$\ddot{q}_2 = 0$$

$$q_2 = At + C$$

$$\dot{q}_2 = 0$$

$$q_3 = D$$

**Which implies two coordinates remain constant and one evolves linearly**

**Question ID:- 8**

The periods of oscillation of a simple pendulum at the sea level and at the top of a mountain of height $6\text{ km}$ are $T_1$ and $T_2$, respectively. If the radius of earth is approximately $6000\text{ km}$, then $\frac{(T_2 - T_1)}{T_1}$ is closest to

**Options:-**

$-10^{-4}$, Option ID :- 29,
$-10^{-3}$, Option ID :- 30,
10 , Option ID :- 31,
$10^{-3}$, Option ID :- 32

**Topic – Classical Mechanics**

**Subtopic – Gravitation**

**Ans.:** $10^{-3}$

**Solution:** $T_1 = 2\pi\sqrt{l/g}$ , $T_2 = 2\pi\sqrt{\frac{l}{g'}}$

$$g' = \frac{GM}{(R+h)^2} \Rightarrow g' = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2} \Rightarrow g' = \frac{g}{\left(1+\frac{R}{R}\right)^2}$$

$$\left(\frac{T_2 - T_1}{T_1}\right) \Rightarrow \left(\frac{T_2}{T_1} - 1\right)$$

$$\sqrt{\frac{g}{g'}} - 1$$

$\left(1+\frac{h}{R}\right)-1$

$\frac{h}{R}=\frac{6}{6000}=10\hat{}-3$

**Question ID:- 9**

A particle of rest mass $m$ is moving with a velocity $v\hat{k}$, with respect to an inertial frame $S$. The energy of the particle as measured by an observer $S'$, who is moving with a uniform velocity $u\hat{\imath}$ with respect to $S$ (in terms of $\gamma_u = 1/\sqrt{1-u^2/c^2}$ and $\gamma_v = 1/\sqrt{1-v^2/c^2}$ ) is

**Options:**

$\gamma_u\gamma_v m(c^2-uv)$, Option ID :- 33,

$\gamma_u\gamma_v mc^2$, Option ID :- 34,

$\frac{1}{2}(\gamma_u+\gamma_v)mc^2$, Option ID :- 35,

$\frac{1}{2}(\gamma_u+\gamma_v)m(c^2-uv)$, Option ID :- 36,

**Topic – Classical Mechanics**

**Subtopic – STR**

**Ans.:** $\gamma_u\gamma_v mc^2$

**Solution:** $v_x=0;\ v_y=0; v_z=v;\ v=u$

$$v'_x=\frac{v_x-u}{1-\frac{uv_x}{c^2}};=-u,\qquad v'_y=\frac{v_y\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{uv_x}{c^2}}=0;\ v'_z=\frac{v_z\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{uv_x}{c^2}}=v\sqrt{1-\frac{u^2}{c^2}}$$

$$v=\sqrt{u^2+v^2\left(1-\frac{u^2}{c^2}\right)}$$

$$E=\frac{mc^2}{\sqrt{1-\frac{u^2+v^2(1-u^2/c^2)}{c^2}}}\Rightarrow E=\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}-\frac{v^2}{c^2}+\frac{v^2u^2}{c^4}}}$$

$$E=\frac{mc^2}{\sqrt{\left(1-\frac{u^2}{c}\right)\left(1-\frac{v^2}{c^2}\right)}}\Rightarrow E=\gamma_u\gamma_v mc^2$$

**Question ID:- 10**

An electromagnetic wave is incident from vacuum normally on a planar surface of a non-magnetic medium. If the amplitude of the electric field of the incident wave is $E_0$ and that of the transmitted wave is $2E_0/3$, then neglecting any loss, the refractive index of the medium is

**Options:-**

1.5, Option ID:- 37

2.0, Option ID:- 38

2.4, Option ID:- 39

2.7, Option ID:- 40

**Topic – Electromagnetic Theory**

**Subtopic – Electromagnetic Wave**

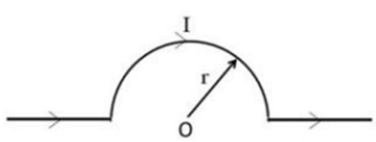
**Ans.: 2**

**Solution:** Reflected amplitude $= E_0 - \frac{2}{3}E_0 = \frac{1}{3}E_0$

Reflectivity for normal incidence $R = \frac{(n-1)^2}{(n+1)^2} = \frac{I_R}{I_I} = \frac{1}{9} \Rightarrow \frac{(n-1)}{(n+1)} = \frac{1}{3} \Rightarrow n = 2$

**Question ID:- 11**

A part of an infinitely long wire, carrying a current $I$, is bent in a semicircular arc of radius $r$ (as shown in the figure).




The magnetic field at the centre O of the arc is

**Options:-**

$\frac{\mu_0 I}{4r}$, Option ID:- 41,

$\frac{\mu_0 I}{4\pi r}$, Option ID:- 42,

$\frac{\mu_0 I}{2r}$, Option ID:- 43,

$\frac{\mu_0 I}{2\pi r}$, Option ID:- 44

**Topic – Electromagnetic Theory**

**Subtopic – Magnetostatics / Ampere's Circuital Law**

**Ans.:** $\frac{\mu_0 I}{4r}$

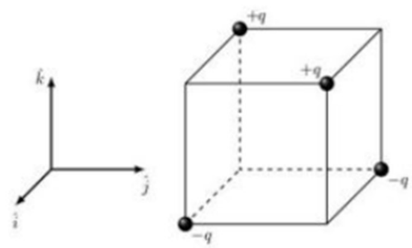
**Solution:** Magnetic field at o due to both line segments is zero.

Only magnetic field due to semi-circle

$$B = \frac{\mu_0 I}{2r}\frac{(2\pi - \pi)}{2\pi} = \frac{\mu_0 I}{4r}$$

**Question ID:- 12**

Two positive and two negative charges of magnitude $q$ are placed on the alternate vertices of a cube of side $a$ (as shown in the figure).




The electric dipole moment of this charge configuration is

**Options:-**

$-2qa\hat{\mathbf{k}}$, Option ID :- 45,

$2qa\hat{\mathbf{k}}$, Option ID :- 46,

$2qa(\hat{\mathbf{i}}+\hat{\mathbf{j}})$, Option ID :- 47,

$2qa(\hat{\mathbf{i}}-\hat{\mathbf{j}})$, Option ID :- 48

Topic – Electromagnetic Theory

Subtopic –Multipole Expansion

**Ans. :** $\bar{p} = 2qa\hat{\mathbf{k}}$

**Solution:**

$\vec{p} = \sum_i q_i r_i$

$\bar{p} = -qa\mathrm{i} - qa\mathrm{j} + qa\hat{k} + qa(i + j + k)$

$\bar{p} = 2qa\hat{k}$

**Question ID:- 13**

The electric and magnetic fields in an inertial frame are $\mathbf{E} = 3a\hat{\mathbf{i}} - 4\hat{j}$ and $\mathbf{B} = \frac{5a}{c}\hat{\mathbf{k}}$, where $a$ is a constant. A massive charged particle is released from rest. The necessary and sufficient condition that there is an inertial frame, where the trajectory of the particle is a uniform-pitched helix, is

**Options:-**

$1 < a < \sqrt{2}$ , Option ID : − 49,

$-1 < a < 1$, Option ID :- 50,

$a^2 > 1$, Option ID :- 51,

$a^2 > 2$ , Option ID : − 52,

Topic – Electromagnetic Theory

Subtopic – Relativistic Electrodynamics

**Ans.:** - $a^2 > 1$,

**Question ID:- 14**

If the expectation value of the momentum of a particle in one dimension is zero, then its (box-normalizable) wavefunction may be of the form

**Options:-**

$\sin\ kx$, Option ID :- 53,

$e^{ikx}\sin\ kx$, 0ption ID :- 54,

$e^{ikx}\cos\ kx$, Option ID :- 55,

$\sin\ kx+e^{ikx}\cos\ kx$., Option ID :- 56

Topic – Quantum Mechanics

Subtopic – Particle in Box

**Ans.:** $\sin\ kx$

**Solution:** $\psi(x)=\sin\ kx$

$\langle p_x\rangle=\int_{-\infty}^{\infty}\psi^*(x)p_x\psi(x)dx=\int_{L}^{L}\sin kx\left(-i\hbar\frac{d}{dx}\sin kx\right)kx\,dx$

$=-i\hbar k\int_{-L}^{L}\sin\ kx\cos\ kxdx=-i\hbar k/2\int_{-L}^{L}\sin\ 2kxdx=0$

**Question ID:- 15**

In terms of a complete set of orthonormal basis kets $|n\rangle$, $n=0,\pm1,\pm2,\cdots$, the Hamiltonian is

$$H=\sum_n\left(E|n\rangle\langle n|+\epsilon|n+1\rangle\langle n|+\epsilon|n\rangle\langle n+1|\right)$$

where $E$ and $\epsilon$ are constants. The state $|\varphi\rangle=\sum_n e^{in\varphi}|n\rangle$ is an eigenstate with energy

**Options:-**

$E+\epsilon\cos\varphi$, Option ID : – 57,

$E-\epsilon\cos\varphi$, Option ID : – 58,

$E+2\epsilon\cos\ \varphi$, Option ID :- 59,

$E-2\epsilon\cos\varphi$, Option ID : – 60,

Topic – Quantum Mechanics

Subtopic – Energy Eigen Value

**Ans.** $\boldsymbol{E+2\in\cos\ \varphi}$

**Solution:**

$$H=\sum_n\left(E|n\rangle\langle n|+\epsilon|n+1\rangle\langle n|+\epsilon|n\rangle\langle n+1|\right)$$

$|\varphi\rangle=\sum_n e^{in\varphi}|n\rangle$

$H|\varphi\rangle=E_\varphi|\varphi\rangle$

$H|\varphi\rangle=\sum_n\sum_m E|n\rangle\langle n\mid m\rangle e^{im\varphi}+\epsilon|n+1\rangle\langle n\mid m\rangle e^{im\varphi}+\epsilon\ \mathrm{l}n\rangle\langle n+1|\mathrm{m}\rangle e^{(m\phi)}$

$=\sum_n\sum_m E|n\rangle\delta_{n,m}e^{im\phi}+\epsilon\mid\mathrm{n}+1\rangle\delta_{n,m}e^{im\phi}+\epsilon\ \mathrm{l}n\rangle\delta_{n+1,m}e^{im\varphi}$

$=\sum_n E|n\rangle e^{(n\varphi}+\epsilon\ \mathrm{l}n+1\rangle e^{in\varphi}+\epsilon\ \ \mathrm{l}n\rangle e^{i(n+1)\varphi}$

$\sum_n E\ \ln\rangle e^{in\phi} + \epsilon\ \ln+1\rangle e^{in\varphi} + \epsilon\ \ \ln\rangle e^{i(n+1)\phi} = E_\phi \sum_n \ \ln\rangle e^{in\varphi}$

$\sum_n \left(E e^{in\varphi} + \epsilon e^{i(n+1)\varphi}\right) \mid n) + \sum_n \in (n+1) e^{in\varphi} = E_\varphi \sum_n e^{in\varphi} \ln)$

$\sum_n \left(E e^{in\varphi} + \epsilon e^{i(n+1)\varphi}) n\right) + \sum_n \epsilon \ln) e^{i(n-1)\varphi} = E_\phi \sum e^{in\varphi} \mid n)$

$E e^{in\varphi} + \epsilon\left(e^{i(n+1)\varphi} + e^{i(n-1)\varphi}\right) = E_\varphi e^{i(n\varphi)}$

$E + \epsilon 2 \left(\frac{e^{i\phi} + e^{-i\phi}}{2}\right) = E_\phi$

$E_\phi = E + 2 \in \cos\ \varphi$

**Question ID:- 16**

The momentum space representation of the Schrödinger equation of a particle in a potential $V(\vec{r})$ is $\left(|\mathbf{p}|^2 + \beta\left(\nabla_p^2\right)^2\right)\psi(\mathbf{p},t) = i\hbar\frac{\partial}{\partial t}\psi(\mathbf{p},t)$, where $\left(\boldsymbol{\nabla}_p\right)_i = \frac{\partial}{\partial p_i}$, and $\beta$ is a constant. The potential is (in the following $V_0$ and $a$ are constants)

**Options:-**

$V_0 e^{-r^2/a^2}$ Option ID :- 61, $V_0 e^{-r^4/a^4}$ Option ID :- 62,

$V_0 \left(\frac{r}{a}\right)^2$, Option ID : − 63, $V_0 \left(\frac{r}{a}\right)^4$, Option ID :- 64

Topic – Quantum Mechanics

Subtopic – Relativistic Quantum Mechanics

**Ans.:** $V_0 \left(\frac{r}{a}\right)^4$,

**Question ID:- 17**

Consider the Hamiltonian $H = AI + B\sigma_x + C\sigma_y$, where $A, B$ and $C$ are positive constants, $I$ is the $2\times 2$ identity matrix and $\sigma_x, \sigma_y$ are Pauli matrices. If the normalized eigenvector corresponding to its largest energy eigenvalue is $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ y\end{pmatrix}$, then $y$ is

**Options:-**

$\frac{B+iC}{\sqrt{B^2+C^2}}$, Option ID : − 65, $\frac{A-iB}{\sqrt{A^2+B^2}}$, Option ID : − 66,

$\frac{A-iC}{\sqrt{A^2+C^2}}$, Option ID : − 67, $\frac{B-iC}{\sqrt{B^2+C^2}}$, Option ID : − 68

Topic – Quantum Mechanics

Subtopic – Spin Angular Momentum

**Ans.:** $\frac{B+iC}{\sqrt{B^2+C^2}}$

**Solution:**

$$H = AI + B\sigma_x + C\sigma_y$$
$$= A\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} + B\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} + C\begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$$
$$= \begin{pmatrix}A & B-iC\\B+iC & A\end{pmatrix}$$

$$|H-\lambda I| = 0$$

$$\therefore \begin{vmatrix}A-\lambda & B-iC\\B+iC & A-\lambda\end{vmatrix} = 0$$

$$\therefore (A-\lambda)^2 - (B^2+C^2) = 0$$
$$\Rightarrow A-\lambda = \pm\sqrt{B^2+C^2}$$
$$\therefore \lambda = A-\sqrt{B^2+C^2}, A+\sqrt{B^2+C^2}$$
$$\lambda\ (\text{largest}) = A+\sqrt{B^2+c^2}$$
$$HX = \lambda X$$
$$\Rightarrow \begin{pmatrix}A & B-iC\\B+iC & A\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\y\end{pmatrix} = \left(A+\sqrt{B^2+C^2}\right)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\y\end{pmatrix}$$
$$\Rightarrow \begin{pmatrix}A+y(B-ic)\\B+iC+Ay\end{pmatrix} = \begin{pmatrix}A+\sqrt{B^2+C^2}\\y\left(A+\sqrt{B^2+C^2}\right)\end{pmatrix}$$

$$A+y(B-iC) = A+\sqrt{B^2+C^2}$$
$$\Rightarrow y = \frac{\sqrt{B^2+C^2}}{(B-iC)}\times\frac{(B+iC)}{B+iC)}$$
$$= \frac{\sqrt{B^2+C^2}}{B^2+C^2}\times B+iC = \frac{B+iC}{\sqrt{B^2+C^2}}$$

**Question ID:- 18**

If the average energy $\langle E\rangle_T$ of a quantum harmonic oscillator at a temperature $T$ is such that $\langle E\rangle_T = 2\langle E\rangle_{T\to 0}$, then $T$ satisfies

**Options:-**

$\coth\left(\frac{\hbar\omega}{k_BT}\right) = 2$, Option ID : − 69,

$\coth\left(\frac{\hbar\omega}{2k_BT}\right) = 2$, Option ID : − 70,

$\coth\left(\frac{\hbar\omega}{k_BT}\right) = 4$, Option ID : − 71,

$\coth\left(\frac{\hbar\omega}{2k_BT}\right) = 4$, Option ID : − 72

Topic – Statistical Mechanics

Subtopic – Partition Function

**Ans.**: $\cot\frac{\hbar\omega}{2}=2$

**Solution:** For quantum Harmonic oscillator

It is given that

$$\langle E\rangle_T = 2\langle E\rangle_{T\to 0}$$

The energy of Harmonic oscillator can be written as

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega,\quad n=0,1,2,3......$$

The partition function can be written as follows

$$Z_1=\sum_n e^{-\beta E_n}=e^{-\frac{\beta\hbar\omega}{2}}+e^{-\frac{3\beta\hbar\omega}{2}}+e^{-\frac{5\beta\hbar\omega}{2}}.......=e^{-\frac{\beta\hbar\omega}{2}}\left(1+e^{-\beta\hbar\omega}+.......\right)$$

$$Z_1=\sum_n e^{-\beta E_n}=e^{-\frac{\beta\hbar\omega}{2}}+e^{-\frac{3\beta\hbar\omega}{2}}+e^{-\frac{5\beta\hbar\omega}{2}}.......=e^{-\frac{\beta\hbar\omega}{2}}\left(1+e^{-\beta\hbar\omega}+.......\right)$$

$$Z_1=e^{-\frac{\beta\hbar\omega}{2}}\frac{1}{\left(1-e^{-\beta\hbar\omega}\right)}=\frac{1}{2\frac{\left(e^{\frac{\beta\hbar\omega}{2}}-e^{-\frac{\beta\hbar\omega}{2}}\right)}{2}}=\frac{1}{2\sinh\left(\frac{\beta\hbar\omega}{2}\right)}$$

$$\langle\varepsilon\rangle=-\frac{\partial}{\partial\beta}\ln z$$

$$\ln z=\ln 1-\ln\left(2\sinh\left(\frac{\beta\hbar\omega}{2}\right)\right)$$

$$\ln z=-\ln\left(2\sinh\left(\frac{\beta\hbar\omega}{2}\right)\right)$$

$$\langle\varepsilon\rangle=\frac{\hbar\omega 2\cosh\left(\frac{\beta\hbar\omega}{2}\right)}{2\left\{2\sinh\left(\frac{\beta\hbar\omega}{2}\right)\right.}$$

$$\langle E\rangle_T=\frac{\hbar\omega}{2}\coth\frac{\beta\hbar\omega}{2}$$

$$\langle E\rangle_{T\to 0}=\frac{\hbar\omega}{2}$$

$$\langle E\rangle_T=2\langle E\rangle_{T\to 0}$$

$$\frac{\hbar\omega}{2}\coth\frac{\hbar\omega}{2}=2\frac{\hbar\omega}{2}$$

$$\cot\frac{\hbar\omega}{2}=2$$

**Question ID:- 19**

A thermally isolated container, filled with an ideal gas at temperature $T$, is divided by a partition, which is clamped initially, as shown in the figure below.


The partition does not allow the gas in the two parts to mix. It is subsequently released and allowed to move freely with negligible friction . The final pressure at equilibrium is

Options:

5P/3, Option ID:73, 5P/4, Option ID:74

3P/5, Option ID:75, 4P/5, Option ID:75

**Topic – Thermodynamics & Statistical Mechanics**

**Subtopic – Thermodynamic Process**

**Ans.:** 5p/3

**Solution:** $PV = nRT$

$$n = \frac{PV}{RT}$$

$$n = n_1 + n_2$$

$$\frac{P'V'}{T} = \frac{PV}{T} + \frac{2P2V}{T}$$

$$p' = \frac{pv + 4pv}{3\text{v}} = 5\text{p}/3$$

**Question ID:- 20**

A walker takes steps, each of length $L$, randomly in the directions along east, west, north and south. After four steps its distance from the starting point is $d$. The probability that $d \leq 3L$ is

**Options**

63/64, Option ID: 77, 59/64, Option ID: 78

57/64, Option ID: 79, 55/64, Option ID: 80

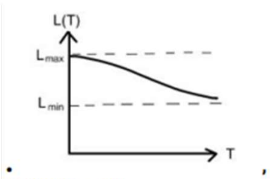
**Topic – Statistical Mechanics**

**Subtopic – Random Walk Problem**

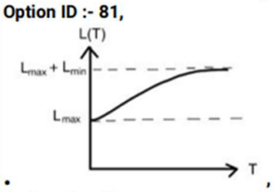
**Ans. 55/64**

**Question ID:- 21**

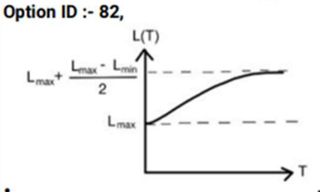
An elastic rod has a low energy state of length $L_{\max}$ and high energy state of length $L_{\min.}$. The best schematic representation of the temperature $(T)$ dependence of the mean equilibrium length $L(T)$ of the rod, is



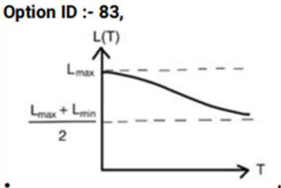

**Option ID :- 81,**




**Option ID :- 82,**




**Option ID :- 83,**

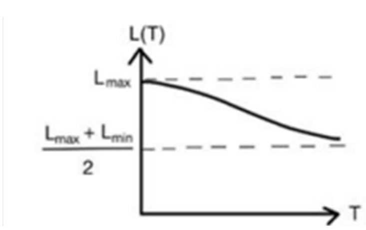



**Option ID :- 84,**

**Topic – Thermodynamics & Statistical Mechanics**

**Subtopic – Thermodynamics Equilibrium**

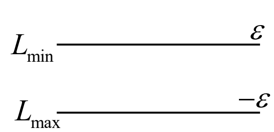
**Ans.**




**Solution:**

$T \to 0$

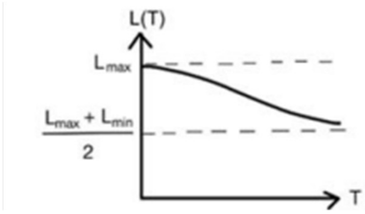
$P(-\varepsilon) \to 1$




$T \to \infty$

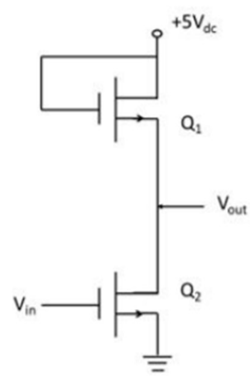
$$P(-\varepsilon) = P(\varepsilon) = \frac{1}{2}$$

The following graph is satisfying the above condition




**Question ID:- 22**

The circuit containing two $n$-channel MOSFETs shown below, works as




**Options:-**

a buffer, Option ID :- 85,

a non-inverting amplifier, Option ID :- 86,

an inverter, Option ID :- 87 ,

a rectifier, Option ID :- 88

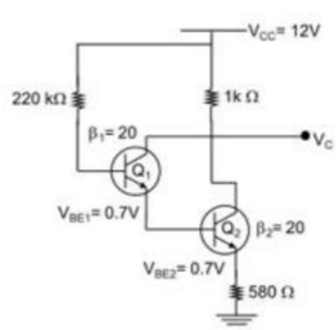
Topic – Electronics

Subtopic – MOSFET

**Ans.** An inverter

**Question ID:- 23**

The figure below shows a circuit with two transistors, $Q_1$ and $Q_2$, having current gains $\beta_1$ and $\beta_2$ respectively.




The collector voltage $V_C$ will be closest to

**Options:-**

0.9 V, Option ID :- 89, 2.2 V , Option ID :- 90,

2.9 V, Option ID :- 91, 4.2 V, Option ID :- 92,

$-12 + I_{B_1}(220\text{k}\Omega) + 0.7 + 0.7 - 21 \times 21 \times 580 I_{B_1} = 0$

$I_{B_1} = \dfrac{10.6}{475.78\text{k}\Omega} = 22.27\mu\text{A}$

$I_{B_2} = (B_1 + 1)I_{B_1} = 21 \times 22.27\mu\text{A} = 457.67\text{MA}$

$I_{c_1} = 20 \times 22.27 \times 10^{-6}\ \text{A}$

$= 4451\ \ \mu\text{A}$

$I_{c_2} = 20 \times 467.67\ \mu\text{A}$

$= 9.353\ mA$

$I = I_{c_1} + I_{c_2}$

$= 9.798\ mA$

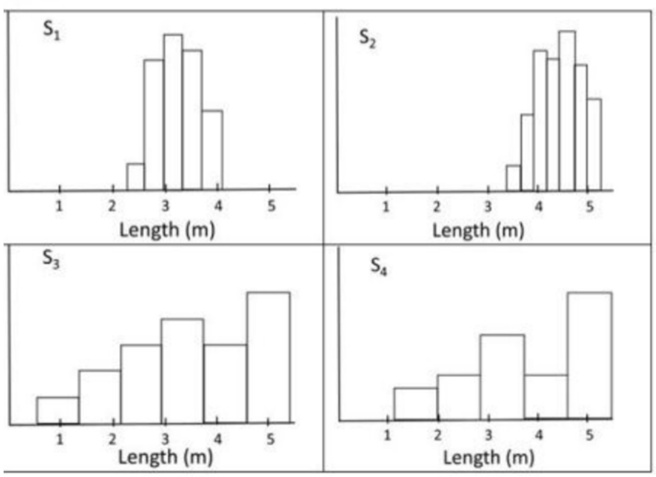
$V_c = 12 - 9.798 = 2.202\ \text{V}$

**Topic – Electronics**

**Subtopic – Transistor**

**Question ID:- 24**

Four students ($\text{S}_1$, $\text{S}_2$, $\text{S}_3$ and $\text{S}_4$) make multiple measurements on the length of a table. The binned data are plotted as histograms in the following figures




**Options:**

$S_2$, Option ID :- 93, $S_1$, Option ID :- 94

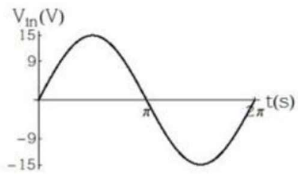
$\text{S}_4$, Option ID :- 95, S, Option ID :- 96

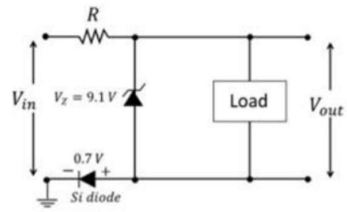
**Topic – Mathematical Physics**

**Subtopic – Probability**

**Question ID:- 25**

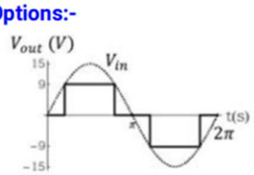
A high impedence load network  is connected in the circuit as shown below



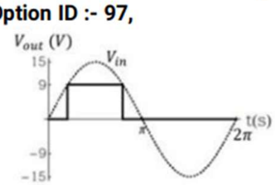

The forward voltage drop for silicon diode is 0.7 V and the Zener voltage is 9.10 V. If the input voltage ($V_{in}$) is sine wave with an amplitude of 15 V (as shown in the figure above), which of the following waveform qualitatively describes the output voltage ($V_{out}$) across the load?

**Options:-**

- 

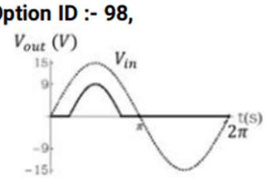
,

**Option ID :- 97,**

- 

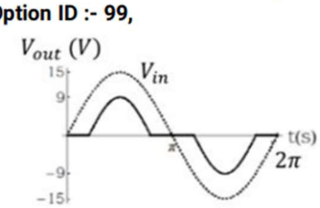
,

**Option ID :- 98,**

- 

,

**Option ID :- 99,**

- 

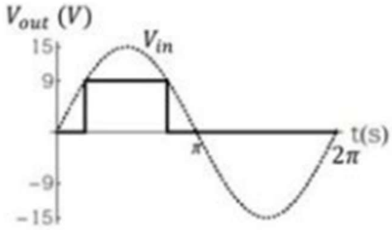
,

**Option ID :- 100,**

**Topic – Electronics**

**Subtopic – Zener Diode**

**Ans.**




**Solution:** In the positive half cycle, the Zener diode is not conducting up to the input voltage of 9.1 V. However, above 9.1 V we can think the Zener diode as a constant pow supply of 9.1 V. Therefore, we will get constant out put voltage across load whenever input greater than equal to 9.1 V.

But, in negative cycle , the Si-diode is non conducting. As a result, we will get zero voltage across load, for negative half cycle.

**Question ID:- 26**

A bucket contains 6 red and 4 blue balls. *A* ball is taken out of the bucket at random and two balls of the same colour are put back. This step is repeated once more. The probability that the numbers of red and blue balls are equal at the end, is

**Options:**

4/11, Option ID: 101 | 2/11, Option ID: 102

1/4, Option ID: 103 | 3/4, Option ID: 104

**Topic – Mathematical Physics**

**Subtopic – Probability**

**Ans.** 2/11

**Question ID:- 27**

The value of the integral $\int_{-\infty}^{\infty} \frac{\cos \alpha x}{x^2+1} dx$, for $\alpha > 0$, is

Options:-

$\pi e^{\alpha}$, Option ID :- 105, | $\pi e^{-\alpha}$, Option ID :- 106,

$\pi e^{-\alpha/2}$, Option ID :- 107 | $\pi e^{\alpha/2}$, Option ID :- 108,

**Topic – Mathematical Physics**

**Subtopic – Complex Analysis**

**Ans.** $\pi e^{-\alpha}$

**Solution**: $\int_{-\infty}^{\infty} \frac{\cos \alpha x}{x^2+1}$

$\int_{-\infty}^{\infty} \frac{\cos az}{z^2+1}$

$z^2+1=0$

$z=\pm i$

Only $z=+i$ will be located inside the semicircle.

Residue at $z=i$, $\lim_{z\to i}(z-i)\frac{e^{iaz}}{(z-i)(z+i)}$

$\lim_{z\to i}\frac{e^{iaz}}{z+i}$

$\frac{e^{iai}}{i+i} \Rightarrow \frac{e^{-a}}{2i}$

$\int_{-\infty}^{\infty}\frac{\cos ax}{x^2+1} = 2\pi i \times \frac{e^{-a}}{2i} = \pi e^{-a}$

**Question ID:- 28**

The Laplace transform $L[f](y)$ of the function $f(x)=\begin{cases}1 & \text{for } 2n\le x\le 2n+1 \\ 0 & \text{for } 2n+1\le x\le 2n+2\end{cases}$,

$n=0,1,2,\cdots$ is

**Options:-**

$\frac{e^{-y}(e^{-y}+1)}{y(e^{-2y}+1)}$, Option ID : – 109,

$\frac{e^{y}-e^{-y}}{y}$, Option ID : – 110,

$\frac{e^{y}+e^{-y}}{y}$, Option ID : – 111,

$\frac{e^{y}(e^{y}-1)}{y(e^{2y}-1)}$, Option ID : – 112

Topic – Mathematical Physics

Subtopic – Laplace Transform

**Ans.** $\frac{e^{y}(e^{y}-1)}{y(e^{2y}-1)}$,

**Solution**: $\frac{1}{1-e^{-2T}}\int_0^2 F(t)e^{-yt}dt$

$=\frac{1}{1-e^{-2y}}\int_0^1 F(\text{t})e^{-yt}dt+\int_1^2 F(t))e^{-yt}dt$

$=\frac{1}{1-e^{-2y}}\int_0^1 F(\text{t})e^{-y}\ dt$

$=\frac{1}{(1-e^{-2y})}\frac{e^{-yz}}{-y}\Big|_0^1 = \frac{1}{(1-e^{-2y})}\cdot\frac{1}{(-y)}(e^{-y}-1) = \frac{1}{y(1-e^{-2y})}(1-e^{-y})$

$=\frac{e^{y}(e^{y}-1)}{y(e^{2y}-1)}$

**Question ID:- 29**

The matrix corresponding to the differential operator $\left(1+\frac{d}{dx}\right)$ in the space of polynomials of degree at most two, in the basis spanned by $f_1 = 1, f_2 = x$ and $f_3 = x^2$, is

Options:-

$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$, Option ID :- 113,

$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$, Option ID : − 114,

$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$, Option ID : − 115,

$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$, Option ID : − 116

Topic – Mathematical Physics

Subtopic – Matrices

**Ans.** $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$,

**Question ID:- 30**

The Lagrangian of a system of two particles is $L = \frac{1}{2}\dot{x}_1^2 + 2\dot{x}_2^2 - \frac{1}{2}(x_1^2 + x_2^2 + x_1 x_2)$. The normal frequencies are best approximated by

**Options:-**

1.2 and 0.7, Option ID :- 117,

1.5 and 0.5, Option ID :- 118,

1.7 and 0.5, Option ID :- 119,

1.0 and 0.4, Option ID :- 120,

Topic – Classical Mechanics

Subtopic – Small Oscillations

**Ans.:** $\omega_1 \simeq 1, \quad \omega_2 = 0.4$

**Solution:**

$$L = \frac{1}{2}\dot{x}_1^2 + 2x_2^2 - \frac{1}{2}(x_1^2 + x_2^2 + x_1 x_2)$$

$$T = \frac{1}{2}\dot{x}_1^2 + 2x_2^2$$

$$= \frac{1}{2}\dot{x}_1^2 + \frac{1}{2}4\dot{x}_2^2$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$$

$$V = \frac{1}{2}(x_1^2 + x_2^2 + x_1 x_2)$$

$$= \frac{1}{2}\left(x_1^2 + x_2^2 + \frac{x_1 x_2}{2} + \frac{x_2 x_1}{2}\right)$$

$$V = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$$

$$|V - \omega^2 T| = 0$$

$$\begin{pmatrix} 1-\omega^2 & 1/2 \\ 1/2 & 1-4\omega^2 \end{pmatrix} = 0$$

$$(1-\omega^2)(1-4\omega^2) - 1/4 = 0$$

$$1 - 5\omega^2 + 4\omega^4 - 1/4 = 0$$

$$4\omega^4 - 5\omega^2 + 3/4 = 0$$

$$\omega^2 = \frac{5 \pm \sqrt{25 - 4\cdot 4 \times \frac{3}{4}}}{2\cdot 4}$$

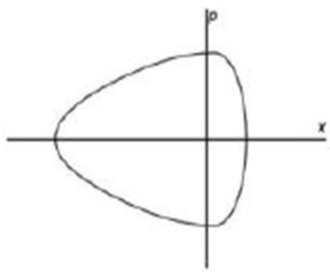
$$\omega^2 = \frac{5 \pm \sqrt{13}}{8}$$

$$\omega_1 \simeq 1$$

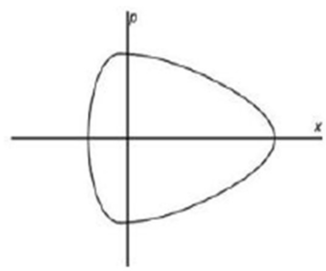
$$\omega_2 = 0.4$$

**Question ID:- 31**

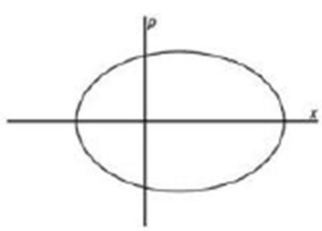
The Lagrangian of a particle in one dimension is $L = \frac{m}{2}\dot{x}^2 - ax^2 - V_0 e^{-10x}$ where $a$ and $V_0$ are positive constants. The best qualitative representation of a trajectory in the phase space is



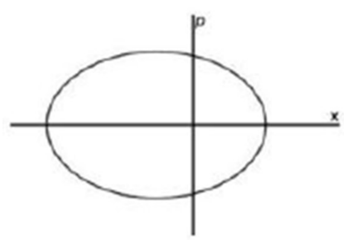

**Option ID:- 121,**




**Option ID:- 122,**




**Option:- 123,**



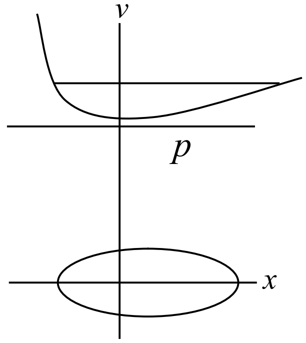

**Option:- 124**

Topic – Classical Mechanics

Subtopic – Phase Space

**Ans.: Option:- 122**

**Solution:**




**Question ID:- 32**

Earth may be assumed to be an axially symmetric freely rotating rigid body. The ratio of the principal moments of inertia about the axis of symmetry and an axis perpendicular to it is $33:32$. If $T_0$ is the time taken by earth to make one rotation around its axis of symmetry, then the time period of precession is closest to

**Options:-**

33 $T_0$, Option ID :- 125, $33T_0/2$, Option ID :- 126,

$32T_0$, Option ID :- 127, $16T_0$, Option ID :- 128,

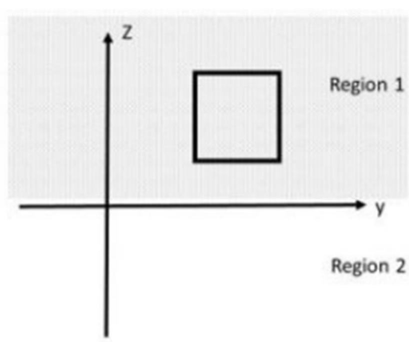
Topic – Classical Mechanics

Subtopic – Moment of Inertia

**Ans.: $32T_0$, Option ID :- 127**

**Question ID:- 33**

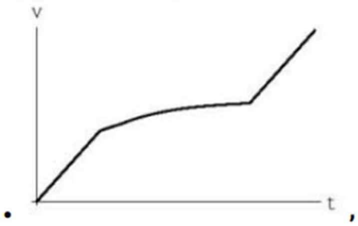
A square conducting loop in the $yz$-plane, falls downward under gravity along the negative $z$-axis. Region 1 , defined by $z > 0$ has a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{i}}$, while region 2 (defined by $z < 0$) has no magnetic field.



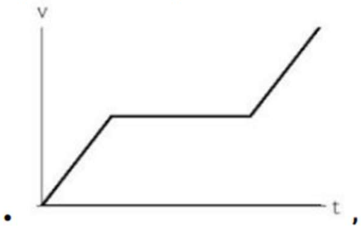

The time dependence of the speed $v(t)$ of the loop, as it starts to fall from well within the region 1 and passes into the region 2 , is best represented by

**Options:-**

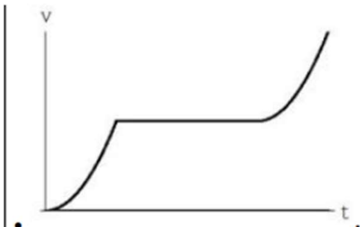

Option ID :- 129,




Option ID :- 130,




Option ID :- 131,




Option ID :- 132,

**Topic – Electromagnetic Theory**

**Subtopic – Electrodynamics**

**Ans.:** Option ID: 130

**Question ID:- 34**

Two small metallic objects are embedded in a weakly conducting medium of conductivity $\sigma$ and dielectric constant $\epsilon$. A battery connected between them leads to a potential difference $V_0$. It is subsequently disconnected at time $t = 0$. The potential difference at a later time $t$ is

**Options:-**

$V_0 e^{-\frac{t\sigma}{4\epsilon}}$, Option ID :- 133,

$V_0 e^{-\frac{t\sigma}{2\epsilon}}$, Option ID :- 134,

$V_0 e^{-\frac{3t\sigma}{4\epsilon}}$, Option ID :- 135,

$V_0 e^{-\frac{t\sigma}{\epsilon}}$, Option ID :- 136,

Topic – Electromagnetic Theory

Subtopic – Continuity Equation

**Ans.** $V_0 e^{-\frac{t\sigma}{\epsilon}}$

**Solution:** $V(t) = IR = I(t)R = \frac{q(t)}{t}R = \frac{q(t)}{t}R = \frac{\rho(t)V}{t}R = \frac{\rho_0 e^{-\frac{\sigma t}{\varepsilon}}V}{t}R = \frac{\rho_0 e^{-\frac{\sigma t}{\varepsilon}}V}{t}R = V_0 e^{-\frac{\sigma t}{\varepsilon}}$

**Question ID:- 35**

A stationary magnetic dipole $\mathbf{m} = m\hat{\mathbf{k}}$ is placed above an infinite surface $(z = 0)$ carrying a uniform surface current density $\boldsymbol{\kappa} = \kappa\hat{\mathbf{i}}$. The torque on the dipole is

Options:-

$\frac{\mu_0}{2} m\kappa\hat{\mathbf{i}}$, Option ID :- 137,

$-\frac{\mu_0}{2} m\kappa\hat{\mathbf{i}}$, Option ID :- 138,

$\frac{\mu_0}{2} m\kappa\hat{j}$, Option ID :- 139,

$-\frac{\mu_0}{2} m\kappa\hat{j}$, Option ID :- 140

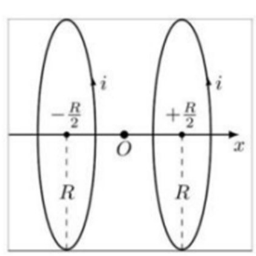
Topic – Electromagnetic Theory

Subtopic – Magnetostatics

**Ans.:** $\frac{\mu_0}{2} m\kappa\hat{\mathbf{i}}$

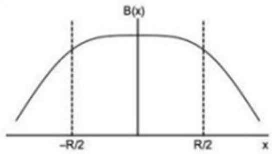
**Question ID:- 36**

Two parallel conducting rings, both of radius $R$, are separated by a distance $R$. The planes of the rings are perpendicular to the line joining their centres, which is taken to be the $x$-axis.



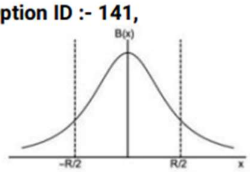
If both the rings carry the same current $i$ along the same direction, the magnitude of the magnetic field along the $x$ axis is best represented by

**Options:-**

- 

,

**Option ID :- 141,**

- 

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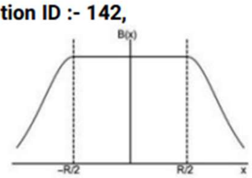
**Option ID :- 142,**

- 

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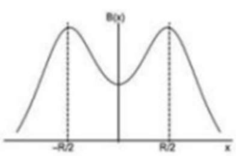
**Option ID :- 143,**

- 

,

**Option ID :- 144,**

**Topic – Electromagnetic Theory**

**Subtopic – Magnetostatics**

**Ans.:** Option ID 141

**Solution:** We know that the magnetic field on the axis of circular loop at a distance $x$ from center

$$B = \frac{\mu_0 I}{2} \frac{R^2}{\left(R^2 + x^2\right)^{3/2}}$$

The field will be maximum at the center of loop .

Now there are two loops. So, the magnetic field in the region in between two loops will be

$B_{net} = \frac{\mu_0 I}{2} \frac{R^2}{\left(R^2 + \left(x - \frac{R}{2}\right)^2\right)^{3/2}} + \frac{\mu_0 I}{2} \frac{R^2}{\left(R^2 + \left(x + \frac{R}{2}\right)^2\right)^{3/2}}$, The field will be maximum

(Helmotz Coil ) in the region in between two loops. However, there will be decay of field on either side of loops. So, the correct option is ID 141

**Question ID:- 37**

At time $t=0$, a particle is in the ground state of the Hamiltonian $H(t)=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2+\lambda x\sin\frac{\omega t}{2}$ where $\lambda,\omega$ and $m$ are positive constants. To $O(\lambda^2)$, the probability that at $t=\frac{2\pi}{\omega}$, the particle would be in the first excited state of $H(t=0)$ is

**Options:-**

$\frac{9\lambda^2}{16m\hbar\omega^3}$, Option ID :- 145,

$\frac{9\lambda^2}{8m\hbar\omega^3}$, Option ID : − 146,

$\frac{16\lambda^2}{9m\hbar\omega^3}$, Option ID : − 147,

$\frac{8\lambda^2}{9m\hbar\omega^3}$, Option ID : − 148

**Topic – Quantum Mechanics**

**Subtopic – Time Dependent Perturbation Theory**

**Ans.**: $\frac{8\lambda^2}{9m\omega^3\hbar}$

**Solution:**

$$H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2+\lambda x\sin\frac{\omega t}{2}$$

$$P_{0-1}=\frac{|V_{10}|^2}{\hbar^2}\left|\int_0^{t=\frac{2\pi}{\omega}} f(t)e^{\frac{2\pi}{\omega}}dt\right|^2$$

$$\omega_0=\frac{E_f-E_i}{\hbar}=\frac{E_1-E_0}{\hbar}$$

$$=\frac{3}{2}\hbar\omega-\frac{1}{2}\hbar\omega$$

$$\omega_0=\omega$$

$$f(t)=\sin\ \omega t$$

$$V_{10}=\langle 1\mid\lambda\mathrm{x}\mid 0\rangle$$

$$x=\sqrt{\frac{\hbar}{2m\omega}}(a+a^+)$$

$$V_{10}=\sqrt{\frac{\hbar}{2m\omega}}\lambda\langle 1|a|0\rangle+\langle 1|a^+|0\rangle$$

$$V_{10}=\lambda\left|\frac{\hbar}{2m\omega}\right|^{1/2}$$

$$|V_{10}|^2=\lambda^2\frac{\hbar}{2m\omega}$$

$I = \int_0^{\frac{2\pi}{\omega}} f(t)e^{i\omega\cdot t}dt = \int_0^{\frac{2\pi}{\omega}} \sin\frac{\omega t}{2}e^{i\omega t}dt$

$= \frac{1}{2i} \quad \int_0^{\frac{2\pi}{\omega}} e^{3i\omega t/2}dt - \int_0^{\frac{2\pi}{\omega}} e^{i\omega t/2}dt = 4/3\omega$

$I^2 = \left(\frac{4}{3\omega}\right)^2 = \frac{16}{9\omega\text{^}2}$

$P_{0-1} = \frac{8\lambda^2}{9m\omega^3\hbar}$

**Question ID:- 38**

To first order in perturbation theory, the energy of the ground state of the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 + \frac{\hbar\omega}{\sqrt{512}}\exp\left[-\frac{m\omega}{\hbar}x^2\right]$$

(treating the third term of the Hamiltonian as a perturbation) is

**Options:-**

$\frac{15}{32}\hbar\omega$, Option ID:- 149,

$\frac{17}{32}\hbar\omega$, Option ID: $-$ 150,

$\frac{19}{32}\hbar\omega$, Option ID: $-$ 151,

$\frac{21}{32}\hbar\omega$, Option ID: 152

**Topic – Quantum Mechanics**

**Subtopic – Time Independent Perturbation Theory**

**Ans.**: $\frac{17}{32}\hbar\omega$

**Solution:**

$E(Q_g) = \frac{\hbar\omega}{2}$

$|Q_g\rangle = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$

$H' = \frac{\hbar\omega}{\sqrt{512}}emh\left(-\frac{m\omega x^2}{\hbar}\right)$

$E_g' = \langle Q_g|w|Q_g\rangle$

$= \int_{-\infty}^{\infty} \hat{\varphi}_g H' \varphi_g dx$

$= \frac{\hbar\omega}{\sqrt{512}}\left|\frac{m\omega}{\pi\hbar}\right|^{1/2} \int_{-\infty}^{-\infty} e^{\frac{-2m\omega x^2}{\hbar}}dx$

$= \frac{\hbar\omega}{\sqrt{512}} \times \left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\sqrt{\frac{\pi\hbar}{2m\omega}} = \frac{\hbar\omega}{\sqrt{1024}} = \frac{\hbar\omega}{32}$

$E_g = E_g^0 + E_g' = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{32} = \frac{17}{32}\hbar\omega$

**Question ID:- 39**

The energy/energies $E$ of the bound state(s) of a particle of mass $m$ in one dimension in the potential $V(x) = \begin{cases} \infty, & x \leq 0 \\ -V_0, & 0 < x < a \\ 0, & x \geq a \end{cases}$ (where $V_0 > 0$) is/are determined by

**Options:-**

$\cot^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right) = \frac{E-V_0}{E}$, Option ID : – 153,

$\tan^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right) = -\frac{E}{E+V_0}$, Option ID : – 154,

$\cot^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right) = -\frac{E}{E+V_0}$, Option ID : – 155,

$\tan^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right) = \frac{E-V_0}{E}$, Option ID : – 156

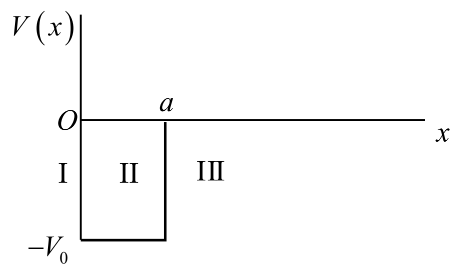
**Topic – Quantum Mechanics**

**Subtopic – Finite Potential Well**

**Ans.** $\cot^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right) = -\frac{E}{E+V_0}$

Solution:

$V = +\text{ve}$
$E = -ve$
$E < -V$




Region I $\rightarrow \psi_I(x) = 0$

Region II $\rightarrow$

$$\frac{-\hbar^2}{2m}\nabla^2\psi - V\psi = E\psi \rightarrow \frac{-\hbar^2}{2m}\nabla^2\psi - (V+E)\psi = 0$$

$$\rightarrow \nabla^2\psi + \frac{2m}{\hbar^2}(V+E)\psi = 0 \rightarrow D^2 + r^2 = 0$$

$$\psi_\pi(x) = Ae^{i\gamma x} + Be^{-i\gamma x}$$

here , $\gamma = \sqrt{\frac{2m(E+V)}{\hbar^2}} > 0$

$\psi_{II}(x=0) = 0 \Rightarrow \; A = -B$

$\Rightarrow \psi_{\text{II}} \; = D\sin\,\gamma x$

Region III →

$$\frac{-\hbar^2}{2m}\nabla^2\psi = E\psi \rightarrow \; \nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0 \rightarrow \nabla^2\psi - \left(\frac{-2mE}{\hbar^2}\right)\psi = 0$$

$\rightarrow \nabla^2\psi - k^2\psi = 0 \rightarrow \psi_{II}(x) = Ae^{kx} + Be^{-k} \;\; \rightarrow \psi_{\text{III}}\,(x \rightarrow \infty) = 0$

$\rightarrow \psi_{III}(x) = Be^{-kx} \rightarrow \; \psi_{II}(x=a) = \psi_{III}(x=a)$

$\rightarrow D\sin\,\gamma a = Be^{-k}$

$\Psi'_{\text{II}}\,(x=a) = \Psi'_{\text{III}}\,(x=a)$

$\gamma\cot\,\gamma a = -k$

$\cot^2\,\gamma a = \frac{k^2}{\gamma^2}$

$$\cot^2\left(a\sqrt{\frac{2m(E+V)}{\hbar^2}}\right) = \frac{-E}{E+V}$$

**Question ID:- 40**

The energy levels of a system, which is in equilibrium at temperature $T = 1/(k_B\beta)$, are $0, \epsilon$ and $2\epsilon$. If two identical bosons occupy these energy levels, the probability of the total energy being $3\epsilon$, is

**Options:-**

$\frac{e^{-3\beta\epsilon}}{1+e^{-\beta\epsilon}+e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta}}$ , Option ID : – 157,

$\frac{e^{-3\beta\epsilon}}{1+2e^{-\beta}\;+2e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta\epsilon}}$ , Option ID : – 158,

$\frac{e^{-3\beta\epsilon}}{e^{-\beta}\;+2e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta\epsilon}}$ , Option ID : – 159,

$\frac{e^{-3\beta}}{1+e^{-\beta\epsilon}+2e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta\epsilon}}$ , Option ID : – 160

Topic – Statistical Mechanics

Subtopic – Partition Function

**Ans.**: $P = \frac{e^{-3\beta\epsilon}}{1+e^{-\beta}\;+2e^{-2\beta\epsilon}+e^{-3\beta\epsilon}+e^{-4\beta\epsilon}}$

**Solution:** Partition function

$$z = 1 + \underbrace{e^{-\beta\epsilon}}_{\epsilon} + \underbrace{2e^{-2\beta\epsilon}}_{2\epsilon} + e^{-3\beta\epsilon} + e^{-4\beta\epsilon}$$

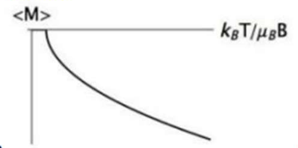
The probability of the total energy being $3\epsilon$

$$P = \frac{e^{-3\beta\epsilon}}{Z} = \frac{e^{-3\beta\epsilon}}{1 + e^{-\beta} + 2e^{-2\beta\epsilon} + e^{-3\beta\epsilon} + e^{-4\beta\epsilon}}$$

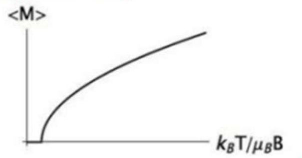
**Question ID:- 41**

A paramagnetic salt with magnetic moment per ion $\mu_{\pm} = \pm\mu_B$ (where $\mu_B$ is the Bohr magneton) is in thermal equilibrium at temperature $T$ in a constant magnetic field $B$. The average magnetic moment $\langle M \rangle$, as a function of $\frac{k_B T}{\mu_B B}$, is best represented by

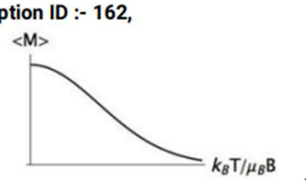
**Options:-**



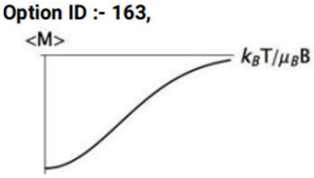

Option ID :- 161,




Option ID :- 162,




Option ID :- 163,




Option ID :- 164,

Topic – Thermodynamics & Statistical Mechanics

Subtopic – Magnetism

**Ans.: Option ID 163**

**Solution:** The partition function can be written as follows

$$E_1 = -\mu_B B;\; E_2 = \mu_B B; \qquad Z = e^{-\beta E_1} + e^{-\beta E_2} = e^{-\beta\mu_B B} + e^{\beta\mu_B B} = 2\cosh(B\mu\beta)$$

$$F = -kT\ln Z = -kT\ln\left(2\cosh\mu_0\mu_B\beta H\right) = -kT\ln\left(2\cosh\left(\mu_0\mu_B\beta H\right)\right)$$

Free energy $M = -\frac{dF}{dH} = \mu_0 \mu_B \tanh\left(\mu_0 \mu_B \beta H\right)$

So, the correct graph is represented in option ID 163.

**Question ID:- 42**

A system of $N$ non-interacting particles in one-dimension, each of which is in a potential $V(x) = gx^6$ where $g > 0$ is a constant and $x$ denotes the displacement of the particle from its equilibrium position. In thermal equilibrium, the heat capacity at constant volume is

**Options:-**

$\frac{7}{6}Nk_B$, Option ID : − 165,

$\frac{4}{3}Nk_B$ , Option ID : − 166,

$\frac{3}{2}Nk_B$, Option ID : − 167,

$\frac{2}{3}Nk_B$, Option ID : − 168

Topic – Statistical Mechanics

Subtopic – Heat Capacity

**Ans.:** $\frac{2}{3}Nk_B$, Option ID:- 168

**Solution:** Using virial theorem

If $V(x)\ \alpha\ x^{n+1}$

then $\langle T\rangle = \frac{n+1}{2}\langle v\rangle, \langle T\rangle = \frac{6}{2}\langle V\rangle \Rightarrow \langle T\rangle = 3\langle V\rangle$

$\langle E\rangle = \langle T + \langle v\rangle = \langle T\rangle + \frac{\langle T\rangle}{3} = \frac{4}{3}(T)$

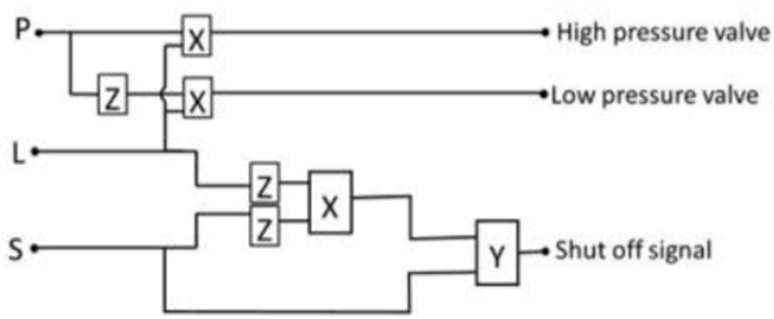
$\langle E\rangle = \frac{4}{3} \times \frac{1}{2} K_B T = \frac{2}{3} K_B T$

$C = \frac{d\langle E\rangle}{dT} = \frac{2}{3}k_B$

For N Particle, $C = \frac{2}{3}Nk_B$

**Question ID:- 43**

A liquid oxygen cylinder system is fitted with a level-sensor ($L$) and a pressure-sensor ($P$), as shown in the figure below. The outputs of $L$ and $P$ are set to logic high ($S = 1$) when the measured values exceed the respective preset threshold values. The system can be shut off either by an operator by setting the input $S$ to high, or when the level of oxygen in the tank falls below the threshold value.




The logic gates $X, Y$ and $Z$, respectively, are

**Options:-**

OR, AND and NOT OR, Option ID:- 169,

AND, OR and NOT Option ID :- 170,

NAND, OR and NOT, Option ID :- 171,

NOR, AND and NOT, Option ID :- 172,

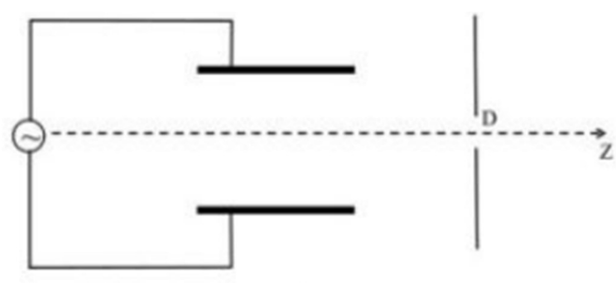
**Topic – Electronics**

**Subtopic – Digital Electronics**

**Ans.: Option ID:-170**

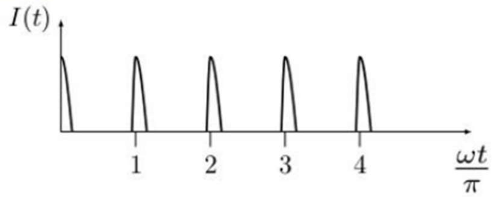
**Question ID :- 44**

A high frequency voltage signal $V_i = V_m \sin\ \omega t$ is applied to a parallel plate deflector as shown in the figure

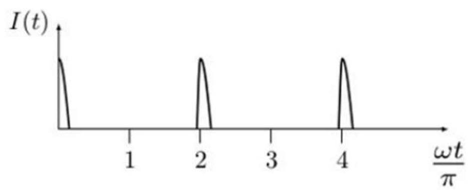


An electron beam is passing through the deflector along the central line. The best qualitative representation of the intensity $I(t)$ of the beam after it goes through the narrow circular aperture D, is

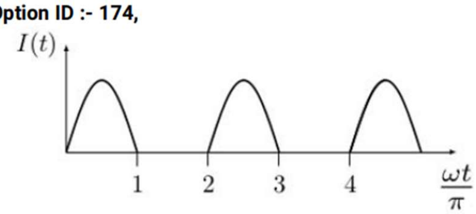
**Options:-**



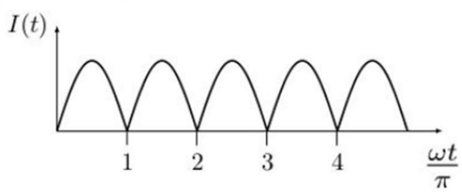

- Option ID :- 173,




- Option ID :- 174,




- Option ID :- 175,



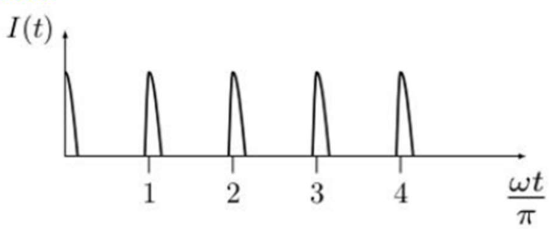

- Option ID :- 176,

Topic – Electromagnetic Theory

Subtopic – Faraday's Law

**Ans.:**

**Options:-**




- Option ID :- 173,

**Solution:** $V_i = V_m \sin\ \omega t$

The applied voltage is having zero value when $\omega t = \pi, 2\pi, 3\pi....$

Or $\frac{\omega t}{\pi} = 1, 2, 3....$

At zero applied voltage, there will be no deflection of charge particle. As a result, we will get $I$. But, in other value of $\frac{\omega t}{\pi}$, the applied voltage is non zero. Thus, there will be deflection of charge particle. So, it can not reach D. Thus, I(t) will be zero.

**Question ID:- 45**

An amplifier with a voltage gain of 40 dB without feedback is used in an electronic circuit. A negative feedback with a fraction 1/40 is connected to the input of this amplifier. The net gain of the amplifier in the circuit is closest to

**Options**

40 dB, Option ID :- 177, 37 dB, Option ID :- 178,

29 dB, Option ID :- 179, 20 dB, Option ID :- 180,

Topic – Electronics

Subtopic – OPAMP

**Ans. 29 dB**

**Solution:** $40 = 20\log A \Rightarrow A = 10^2 = 100$

In negative feedback

$$A_{cl} = \frac{A}{1+\beta A} = \frac{100}{1+100\frac{1}{40}} = 28.9$$

Gain in dB $20\log 28.9\ dB = 29\ \ dB$

**Question ID:- 46**

A receiver operating at 27°c has an input resistance of 100Ω. The input thermal noise voltage for this receiver with a bandwidth of 100kHz is closest to

**Options:-**

0.4 nV , Option ID :- 181, 0.6 pV , Option ID :- 182,

40mV, Option ID :- 183, $0.4\mu V$, Option ID :- 184,

Topic – Electronics

Subtopic – Instrumentation

**Ans.** $0.4\ \mu V$

**Solution:** $V = \sqrt{4kTBR}$

$V = \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 1 \times 10^{5} \times 100}$

$V = 0.4\ \mu V$

**Question ID:- 47**

The Raman rotational-vibrational spectrum of nitrogen molecules is observed using an incident radiation of wavenumber 12500 $\text{cm}^{-1}$. In the first shifted band, the wavenumbers of the observed lines (in $\text{cm}^{-1}$ ) are 10150 , 10158, 10170, 10182 and 10190 . The values of vibrational frequency and rotational constant (in $\text{cm}^{-1}$ ), respectively, are

**Options**

2330 and 2, Option ID: 185,

2350 and 2, Option ID: 186

2350 and 3, Option ID: 187,

2330 and 3, Option ID: 187

**Topic – Atomic & Molecular Physics**

**Subtopic – Raman Spectra**

**Ans.** 2330 and 2

Solution: Central one will be the corresponding value of vibrational frequency

10150 , 10158, 10170, 10182 and 10190 , Here 10170 is the Stokes line corresponding to vibrational one.

$\varepsilon = 12500\,cm^{-1}$

$\varepsilon_{Stoke} = 10170\ cm^{-1}$

$\varepsilon_{stoke} = \varepsilon - \omega_0 \Rightarrow \omega_0 = 12500 - 10170 = 2330\,cm^{-1}$

10150 and 10158 are rotational Raman Stokes lines. Also, 10182 and 10190 are rotational Raman Stokes lines.

We know, $1058 - 1050 = 8cm^{-1} = 4B \Rightarrow B = 2cm^{-1}$

**Question ID:- 48**

The electronic configuration of $^{12}\text{C}$ is $1s^2 2s^2 2p^2$. Including LS coupling, the correct ordering of its energies is

**Options:-**

$E(^3P_2) < E(^3P_1) < E(^3P_0) < E(^1D_2)$, Option ID :- 189,

$E(^3P_0) < E(^3P_1) < E(^3P_2) < E(^1D_2)$, Option ID :- 190,

$E(^1D_2) < E(^3P_2) < E(^3P_1) < E(^3P_0)$, Option ID :- 191,

$E(^3P_1) < E(^3P_0) < E(^3P_2) < E(^1D_2)$, Option ID :- 192,

**Topic – Atomic & Molecular Physics**

**Subtopic – L-S Coupling**

**Ans.:** $E(\,^3P_0) < E(\,^3P_1) < E(\,^3P_2) < E(\,^1D_2)$

**Solution:** $1s^2 2s^2 2p^2$

$2p^2,\ l_1 = 1,\ l_2 = 1,\ s_1 = \frac{1}{2};\ s_2 = \frac{1}{2},\ L = |l_1 + l_2|.......|l_1 - l_2| = 2,1,0, S = |s_1 + s_2|.......|s_1 - s_2| = 1,0$

The lower energy belong from higher S (=1), less than half field ($2p^2$)

$S = 1,\ L = 1,\ J = |L+S|.......|L-S| = 2,1,0\ \ Term = {}^3P_{0,1,2}$

$S = 0,\ L = 0,\ J = |L+S|.......|L-S| = 0\ \ Term = {}^1S_0$

$S = 0,\ L = 2,\ J = |L+S|.......|L-S| = 2\ \ Term = {}^1D_2$

According to Hund's rule, Higher S , lower energy. Higher L, lower will be energy and Higher J, higher will be the energy.

By applying Hund's rule $E(\,^3P_0) < E(\,^3P_1) < E(\,^3P_2) < E(\,^1D_2)$

**Question ID:- 49**

In the absorption spectrum of H-atom, the frequency of transition from the ground state to the first excited state is $v_H$. The corresponding frequency for a bound state of a positively charged muon ($\mu^+$)and an electron is $v_\mu$. Using $m_\mu = 10^{-2}$ kg, $m_e = 10^{-30}$ kg and $m_p \gg m_e, m_\mu$, the value of $(v_\mu - v_H)/v_H$ is

**Options: -**

0.001, Option ID :- 193,

0.001, Option ID :- 194

-0.01, Option ID :- 195,

0.01, Option ID :- 196,

**Topic – Atomic & Molecular Physics**

**Subtopic – Bohr Model**

**Ans.:** - 0.01

**Solution:** $E_\mu = \frac{m_\mu m_e}{m_\mu + m_e} = 0.99 E_H$

For Hydrogen atom, $v_H = \frac{E_2 - E_1}{h}$

For muonium

$$v_\mu = \frac{0.99E_2 - 0.99E_1}{h} = 0.99 v_H$$

$$\frac{\nu_\mu - \nu_H}{\nu_H} = \frac{0.99\nu_H - \nu_H}{\nu_H} = -0.01$$

**Question ID:- 50**

The energies of a two-level system are $\pm E$. Consider an ensemble of such non-interacting systems at a temperature T. At low temperatures, the leading term in the specific heat depends on $T$ as

**Options:-**

$\frac{1}{T^2}e^{-E/k_BT}$, Option ID : − 197,

$\frac{1}{T^2}e^{-\frac{2E}{k_BT}}$, Option ID : − 198,

$T^2 e^{-E/k_BT}$, Option ID : − 199,

$T^2 e^{-\frac{2E}{k_BT}}$, Option ID : − 200

Topic – Statistical Mechanics

Subtopic – Partition Function

**Ans.: Option ID:- 198**

**Solution:** The partition function can be written as follows

$$Z = e^{-\beta E} + e^{\beta E} = 2\cosh \beta E$$

$$Z = e^{-\beta E} + e^{\beta E} = 2\cosh \beta E$$

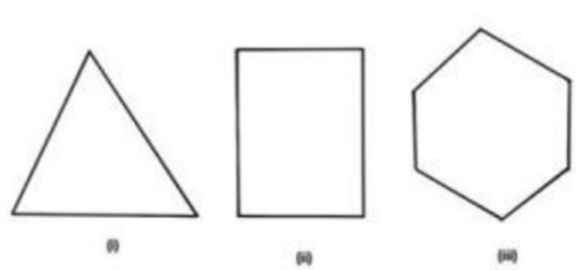
$$u = -\frac{d}{d\beta}\ln\left(2\cosh\beta E\right) = -\tanh\left(\beta E\right)$$

$$c = \frac{d}{dT}\tanh\left(\frac{E}{k_BT}\right) = \frac{1}{k_BT^2}\text{sech}^2\left(\frac{E}{k_BT}\right) = \frac{1}{k_BT^2}\frac{4}{\left(e^{\frac{E}{k_BT}} + e^{\frac{-E}{k_BT}}\right)^2}; \quad At\ low T;\ e^{\frac{-E}{k_BT}} \Rightarrow 0$$

$$C \propto \frac{1}{T^2}\frac{1}{\left(e^{\frac{E}{k_BT}}\right)^2} \propto \frac{1}{T^2}e^{\frac{-2E}{k_BT}}$$

**Question ID:- 51**

The Figures (i), (ii) and (iii) below represent an equilateral triangle, a rectangle and a regular hexagon, respectively.



Which of these can be primitive unit cells of a Bravais lattice in two dimensions?

**Options:-**

Only (i) and (iii) but not (ii), Option ID :- 201,

Only (i) and (ii) but not (iii), Option ID :- 202,

Only (ii) and (iii) but not (i), Option ID :- 203,

All of them, Option ID :- 204

**Topic – Condensed Matter Physics**

**Subtopic – Crystallography**

**Ans.** only (ii) and (iii) but not (i)

**Question ID:- 52**

The Hamiltonian for a spin-1/2 particle in a magnetic field $\mathbf{B} = B_0\hat{k}$ is given by $H = \lambda \mathbf{S} \cdot \mathbf{B}$, where $\mathbf{S}$ is its spin (in units of $\hbar$ ) and $\lambda$ is a constant. If the average spin density is $\langle \mathbf{S} \rangle$ for an ensemble of such non- interacting particles, then $\frac{d}{dt}\langle S_x \rangle$

**Options:-**

$\frac{\lambda}{\hbar} B_0 \langle S_x \rangle$, Option ID : − 205,

$\frac{\lambda}{\hbar} B_0 \langle S_y \rangle$, Option ID : − 206,

$-\frac{\lambda}{\hbar} B_0 \langle S_x \rangle$, Option ID : − 207,

$-\frac{\lambda}{\hbar} B_0 \langle S_y \rangle$, Option ID : − 208,

**Topic – Quantum Mechanics**

**Subtopic – Spin**

**Ans.:** $-\lambda B_0 < S_y >$

**Solution**: $\frac{d\langle A \rangle}{dt} = \frac{1}{i\hbar}\langle [A, H] \rangle + \left\langle \frac{\partial A}{\partial t} \right\rangle$

$$\frac{d\langle Sx \rangle}{dt} = \frac{1}{i\hbar}\langle [Sx, Hi] \rangle + 0$$

$$H = \lambda \vec{S} \cdot \vec{B}$$
$$= \lambda \hat{S}_z \cdot B_0$$

$$[S_x, H] = \lambda [S_x, (S_z B_0]$$
$$= \lambda B_0 [S_x, S_z]$$
$$= \lambda B_0 \mathrm{x} - \left(i\hbar S_y\right)$$

$$\frac{d\langle Sx \rangle}{dt} = \frac{1}{i\hbar}\langle [Sx, H] \rangle + 0 = \frac{1}{i\hbar} \lambda B_0 x \text{-} i\hbar < S_y > = - \lambda B_0 < S_y >$$

**Question ID:- 53**

The tensor component of the nuclear force may be inferred from the fact that deuteron nucleus $^2_1H$

**Options:**

has only one bound state with total spin $S = 1$, Option ID : – 208

has a non-zero electric quadrupole moment in its ground state, Option ID :- 209,

Is stable while triton $^3_1$H is unstable, Option ID :- 210,

Is the only two nucleon bound state , Option ID :- 212,

**Topic – Nuclear & Particle Physics**

**Subtopic – Deuteron Problem**

**Ans.** has a non-zero electric quadrupole moment in its ground state

**Solution:** The tensor component of the nuclear force may be inferred from the fact $^2_1H$ has a non-zero electric quadrupole moment in its ground state

**Question ID:- 54**

The elastic scattering process $\pi^- p \to \pi^- p$ may be treated as a hard-sphere scattering. The mass of $\pi^-, m_\pi \simeq \frac{1}{6} m_p$, where $m_p \simeq 938\text{MeV}/c^2$ is the mass of the proton. The total scattering cross-section is closest to

**Options: -**

0.01 milli-barn, Option ID:- 213,

1 milli-barn, Option ID:- 214,

0.1 barn, Option ID:- 215,

10 barn, Option ID:- 216,

**Topic – Nuclear & Particle Physics**

**Subtopic – Particle Physics**

**Ans.: 0.1 barn, Option ID:- 215**

**Solution:** $\sigma = 2\pi R^2;\ R = R_p + R_\pi;\ R_p = R_0 A^{1/3} = R_0 1^{1/3} = 1\,fm;\ R_\pi^{\ 3} = R_p^{\ 3}\dfrac{m_\pi}{m_n}$

$\Rightarrow R_\pi = 0.4\,fm \Rightarrow R = 1.40\,fm$

$\sigma = 2\pi R^2 = 0.15 \approx 0.1 barn$

**Question ID:- 55**

Thermal neutrons may be detected most efficiently by a

**Options:-**

$Li^6$ loaded plastic scintillator, Option ID :- 217,

Geiger-Müller counter, Option ID :- 218,

inorganic scintillatorCaF2, Option ID :- 219,

silicon detector, Option ID :- 220,

**Topic – Nuclear & Particle Physics**

**Subtopic – Nuclear Detector**

**Ans.** $Li^6$ loaded plastic scintillator

**Solution** : Thermal neutrons may be detected most efficiently by 6 Li loaded plastic scintillator