Chapter 12

Random Walk Problem

3. Ficks Law

Fick's law discuss Brownian motion from point of view of Diffusion. We denote no density of Brownian Particles in the fluid by the symbol $n(r,t)$ and their current density by $j(r,t)\{=n(r,t)v(r,t)\}$ then according to Fick's Law

$$j(r,t)=-D\nabla n(r,t)$$

Where D denotes the Diffusion coefficient of the medium. we also have equation of continuity

Namely $\vec{\nabla}.\vec{j}(r,t)+\frac{\partial n(r,t)}{\partial t}=0$ putting the value $j(r,t)=-D\nabla n(r,t)$

$$\nabla^2 n(r,t)-\frac{1}{D}\frac{\partial n(r,t)}{\partial t}=0$$

For Three-Dimensional System

For three dimensional system where $r^2 = x^2 + y^2 + z^2$

$$n(r,t) = \frac{N}{(4\pi Dt)^{3/2}} \exp\left(-\frac{r^2}{4Dt}\right) \text{ and } \int_0^\infty n(r,t) 4\pi r^2 dr = N$$

$$\langle r^2 \rangle = \frac{\int_0^\infty r^2 n(r,t) 4\pi r^2 dr}{\int_0^\infty n(r,t) 4\pi r^2 dr} = 6Dt$$ so rms value of r is given by $\sqrt{6Dt}$

For Two Dimensional System

For three dimensional system where $r^2 = x^2 + y^2$

$$n(r,t) = \frac{N}{(4\pi Dt)^{2/2}} \exp\left(-\frac{r^2}{4Dt}\right) \text{ and } \int_0^\infty n(r,t) 2\pi r dr = N$$

$$\langle r^2 \rangle = \frac{\int_0^\infty r^2 n(r,t) 2\pi r dr}{\int_0^\infty n(r,t) 2\pi r dr} = 4Dt$$ so rms value of r is given by $\sqrt{4Dt}$

For One Dimensional System

$$n(r,t) = \frac{N}{(4\pi Dt)^{1/2}} \exp\left(-\frac{x^2}{4Dt}\right)$$

$$\langle x^2 \rangle = \frac{\int_{-\infty}^\infty x^2 n(x,t) dx}{\int_{-\infty}^\infty n(x,t) dx} = 2Dt$$ so rms value of r is given by $\sqrt{2Dt}$