JNU MSc 2018

Q1. Given the points $P(0,0,0), Q(1,1,1), R(1,1,0)$ and $S(0,1,1)$ in three-dimensional space, what is the shortest distance between lines PQ and RS?

(a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{6}}$

Q2. The exponential function $e^{i\theta A} A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is given by which oe of the following matrices? Here θ is an arbitrary real number.

(a) $\begin{pmatrix} 0 & e^{i\theta} \\ e^{i\theta} & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & e^{i\theta} \\ e^{-i\theta} & 1 \end{pmatrix}$

(c) $\begin{pmatrix} \cos\theta & i\sin\theta \\ -i\sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos\theta & -i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$

Q3. What is the solution of the differential equation $y'' + y' = bx$, with the boundary conditions $y(0) = 1$ and $y'(0) = 0$? Here b is a constant, $y' = \frac{dy}{dx}$ and $y'' = \frac{d^2y}{dx^2}$.

(a) $y(x) = (1-b) + \frac{b}{2}x(x+2) + be^{-x}$ (b) $y(x) = (1-b) - \frac{b}{2}x(x-2) + be^{-x}$

(c) $y(x) = (1+b) - \frac{b}{2}x(x+2) - be^{-x}$ (d) $y(x) = (1-b) + \frac{b}{2}x(x-2) + be^{-x}$

Q4. What is the value of the following complex integral?

$$\int_C x^2 e^{1/z} dz$$

Here C denotes the unit circle $|z| = 1$ traversed counter-clockwise

(a) $2\pi i$ (b) πi (c) $\frac{2\pi i}{5}$ (d) $\frac{\pi i}{3}$

Q5. A point particle of mass m is placed on the inner surface of a frictinless bowl which has the shape of a aparaboloid of revolution given by the equation $z = a(x^2 + y^2)$. The gravity g is acting vertically downwards (along the negative z direction). At what angular speed should the the bowl be rotated about the vertical axis so that the particle remains stationary?

(a) $\sqrt{2ag}$ (b) $\sqrt{ag}$ (c) $\sqrt{\frac{2g}{a}}$ (d) $\frac{1}{\sqrt{2ag}}$

Q6. The stiffness k_c of a spring when compressed from its equilibrium length is given to be different from its stiffness k_s when stretched. The potential energy $V(x)$, stored in theis spring for an arbitrary compression or stretching x, is given by $V(x)=k_c x^2/2$ for $x<0$ and $V(x)=k_c x^2/2$ for $x \geq 0$. A particle of mass m is attached to the one end of this spring, while the other end is fixed. The mass of the spring is negligible. Starting at time $t=0$, with the compressed spring an the particle at rest at $x(0)=-1$, this system undergoes an oscillatory motion. What is the angular frequency of its oscillation?

(a) $\sqrt{\dfrac{k_c+k_s}{2m}}$ (b) $\dfrac{2}{\sqrt{m}}\dfrac{\sqrt{k_c k_s}}{\sqrt{k_c+\sqrt{k_s}}}$

(c) $\dfrac{1}{2}\left[\sqrt{\dfrac{k_c}{m}}+\sqrt{\dfrac{k_s}{m}}\right]$ (d) $\sqrt{\dfrac{2k_c k_s}{m(k_c+k_s)}}$

Q7. A uniform string of length $2m$ and mass $10g$ is under a tension of $8N$, with both ends fixed. If the string is plucked transversely and then touched at a point $80cm$ away from one end, the vibrational modes of what frequencies will persist?

(a) All integer multiples of $10Hz$ (b) All integer multiples of $20Hz$

(c) All integer multiples of $25Hz$ (d) All integer multiples of $50Hz$

Q8. A concave-convex lens of negligible thickness has refractive index of 1.5, and its radii of curvature are $10cm$ (for the convex surface) and $20cm$ (for the concave surface). The lens is placed horizontally with concave surface facing upwards which is filled with an oil of refractive index 1.6. What is the focal length of this lens filled with oil?

(a) $18.18cm$ (b) $40.12cm$ (c) $20.5cm$ (d) $6.3cm$

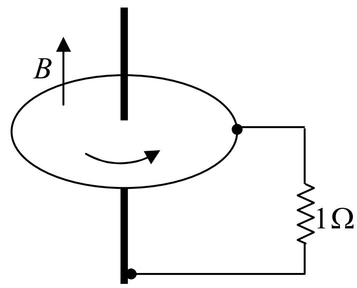
Q9. Consider the fringes produced with monochromatic light of wavelength $5880\,\mathring{A}$ in a Young's double-slit experiment. When a thin glass plate of refractive index 1.5 is placed normally in front of one of the slits, the central bright fringe is shifted to a position where the third bright fringe from the centre was before the plate was placed. What is the thickness of this glass plate?

(a) $1.176\,\mu m$ (b) $1.764\,\mu m$ (c) $2.841\,\mu m$ (d) $3.528\,\mu m$

Q10. Consider a solid cube with a uniform charge density. Let thee electrostatic potential be V_1 at its centre, and V_2 at any corner of the cube. The potential is zero at infinity. What is $\dfrac{V_1}{V_2}$?

(a) 2 (b) $\dfrac{2}{\sqrt{3}}$ (c) $\dfrac{1}{2}$ (d) 8

Q11. A metallic disc of radius $0.1m$ rotates about its vertical axis with a rotational speed of connected between the axle and the outer edge of the disc, then the current flowing through the resistor is



(a) $5\,mA$ (b) $31.4\,mA$ (c) $16.7\,mA$ (d) $10\,mA$

Q12. The photoelectric effect can be demonstrated by shining light on a metallic surface in an evacuated chamber which results in a current of photoeletrons. To stop the photoelectric current (i_p), a negative voltage is applied between cathode and anode using a variable voltage source (V) and the corresponding stopping potential (V_s) is with fixed intensity (I) and frequency (v) of the incident light. Based on the resultant findings sketched in graphs $(A),(B),(C)$ below, which of the following conclusions are correct?

(i) The work function of tungsten (W) is more than of potassium (K).

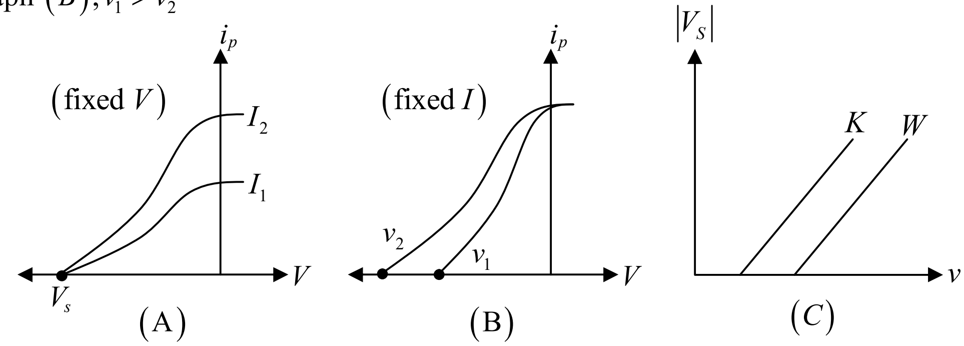
(ii) The saturation current is independent of the frequency of light for affixed I

(iii) The V_s depends on the intensity of light

(iv) The Planck constant can be estimated from graph (C)

(v) The graph (C) is explainable by classical electrodynamics.

(vi) In graph $(B), v_1 > v_2$



(a) (i), (ii) and (iv) are the correct conclusions

(b) (i), (iv) and (v) are the correct conclusions

(c) (ii), (iii) , (v) and (vi) are the correct conclusions

(d) (i), (ii) , (iv) and (vi) are the correct conclusions

Q13. A particle in one dimension is given to be in the quantum state $\psi(x)=e^{(a+ib)x-\frac{1}{2}x^2}$, where a and b are real-valued constants. What are the position and momentum expectation values $\langle x\rangle$ and $\langle p\rangle$ respectively, of the particle in this state?

(a) $\langle x\rangle=a$ and $\langle p\rangle=\frac{\hbar}{2a}$ (b) $\langle x\rangle=a$ and $\langle p\rangle=\hbar b$

(c) $\langle x\rangle=ae^{a^2}\sqrt{\pi}$ and $\langle p\rangle=\hbar be^{-b^2}\sqrt{\pi}$ (d) $\langle x\rangle=ae^{a^2}\sqrt{\pi}$ and $\langle p\rangle=\hbar be^{a^2}\sqrt{\pi}$

Q14. The unstable oxygen nuclei ${}^{14}_{8}O$ and ${}^{19}_{8}O$ undergo beta decay. Which of the following do you expect to happen?

(a) ${}^{14}_{8}O$ undergoes positive beta decay and ${}^{19}_{8}O$ undergoes negative beta decay

(b) ${}^{14}_{8}O$ undergoes negative beta decay and ${}^{19}_{8}O$ undergoes positive beta decay

(c) Both ${}^{14}_{8}O$ and ${}^{19}_{8}O$ undergo positive beta decay

(d) Both ${}^{14}_{8}O$ and ${}^{19}_{8}O$ undergo negative beta decay

Q15. An X-ray tube operates at an applied voltage of $15000V$. What is the shortest wavelength of the X-ray emitted by this tube?

(a) $0.83\,\mathring{A}$ (b) $0.66\,\mathring{A}$ (c) $1.2\,\mathring{A}$ (d) $2.1\,\mathring{A}$

Q16. The work (in mega electron volts) that must be done to increase the speed of an electron from $0.4c$ to $0.8c$ (where c is the speed of light) is approximately

(a) $0.1\,MeV$ (b) $0.3\,MeV$ (c) $0.5\,MeV$ (d) $0.7\,MeV$

Q17. Consider a three-level atom with energy levels $0,\varepsilon_1$ and ε_2. What is the probability of finding the atom in the energy level ε_2 at temperature T (with $\beta=\frac{1}{k_BT}$)?

(a) 0 (b) $e^{-\beta\varepsilon_2}$

(c) $\frac{e^{-\beta\varepsilon_1}}{\left(1+e^{-\beta e_1}+e^{-\beta\varepsilon_2}\right)}$ (d) $\frac{e^{-\beta\varepsilon_2}}{\left(1+e^{-\beta e_1}+e^{-\beta\varepsilon_2}\right)}$

Q18. Foe a gas at fixed temperature pressure and particle number (T,P,N) the natural thermodynamic potential is $G=U-TS+PV$, where U is the internal energy, V is volume and S is entropy. Which of the following is the corresponding potential for a gas at fixed (T,V,μ)? Here μ is the chemical potential.

(a) $G+TS$ (b) $G-PV$ (c) $G-PV+\mu N$ (d) $G-PV-\mu N$

Q19. Two moles of an ideal gas is kept at a temperature of $300\,K$ in a container of $1\,m^3$ volume. At this temperature, particle number and volume, what is the rate of change of its entropy with pressure in SI units?

(a) 300 (b) -600 (c) $\frac{-1}{300}$ (d) $\frac{1}{600}$

Q20. What is the number of lattice points contained in a primitive unit cell of the f.c.c. (face-centred cubic) lattice?

(a) 1 (b) 2 (c) 3 (d) 4

Q21. Which one of the following is not true for the specific heat (C_v) of solids?

(a) The C_v decreases as temperature (T) decreases

(b) According to Debye model, $C_v \propto T^3$ at low temperatures

(c) The Einstein model accounts for the contribution to C_v from the acoustic modes of lattice vibrations

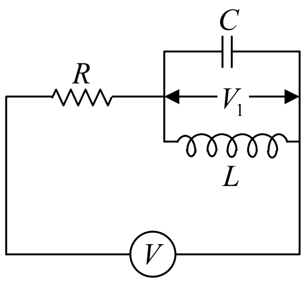
(d) The Debye model accounts for the contribution to C_v from the acoustic modes of lattice vibrations

Q22. What is the concentration of electrons and holes at a temperature of $300\,K$ in an intrinsic semiconductor with band gap $1.1\,eV$? In this semiconductor, the effective mass of electrons is given to be $0.7\,m_e$ and the effective mass of holes is $0.7\,m_e$. Here m_e denotes the rest mass of a free electron.

(a) $2.1\times10^{17}\,m^{-3}$ (b) $8.5\times10^{16}\,m^{-3}$

(c) $1.1\times10^{16}\,m^{-3}$ (d) $1.1\times10^{15}\,m^{-3}$

Q23. Consider the circuit drawn below of a resistor (R), capacitor (C) and inductor (L) with an applied time-dependent voltage $V(t)$. Let V_1 be the resultant voltage at any time t across the inductor. Which one of the following differential equations correctly applies to this circuit? Here $\dot{V}_1 = \frac{dV_1}{dt}$ and $\ddot{V}_1 = \frac{d^2V_1}{dt^2}$



(a) $RC\ddot{V}_1 + \dot{V}_1 + \left(\frac{R}{L}\right)V_1 = \dot{V}(t)$ (b) $RC\ddot{V}_1 + \dot{V}_1 - \left(\frac{R}{L}\right)V_1 = \dot{V}(t)$

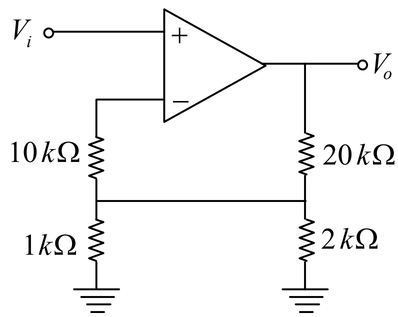
(c) $RC\ddot{V}_1 + \left(\frac{R}{L}\right)\dot{V}_1 + V_1 = V(t)$ (d) $RC\ddot{V}_1 - \left(\frac{R}{L}\right)\dot{V}_1 + V_1 = V(t)$

Q24. The Boolean expression $B.(A+B)+A.(\vec{B}+A)$ can be realized using a minimum number of

(a) 1 NAND gate (b) 2 AND gates

(c) 1 OR gate (d) 1 NOR gate

Q25. For an input voltage $V_i = 1\ mV$, what is the output voltage V_0 in the following circuit of an ideal OP-AMP (operational amplifier)?



(a) $-10\,mV$ (b) $62\,mV$ (c) $31\,mV$ (d) $11\,mV$