

JNU PhD PAPER 2020

A sphere of radius R carries a polarization $\vec{P}(r) = k\vec{r}$ where k is a constant and \vec{r} is the vector from the center of the sphere. Answer the following three questions for this problem.

Q1. The surface bound charge σ_b is:

- | | |
|---------------------------|--|
| (a) $\frac{kr}{4\pi R^2}$ | (b) $\frac{1}{4\pi\epsilon_0} \frac{kr}{4\pi R^2}$ |
| (c) $kR\hat{r}$ | (d) $kR\hat{r}$ |

Ans. (d)

Q2. The volume bound charge (p_b) is:

- | | |
|--|------------------------|
| (a) $\frac{1}{4\pi\epsilon_0} \frac{3k}{4\pi R^3}$ | (b) $-3kr$ |
| (c) $-3k$ | (d) $9k^3 r^3 \hat{r}$ |

Ans. (c)

Q3. The electric field outside the sphere is:

- | | |
|-----------------|--|
| (a) $4\pi kR^2$ | (b) $\frac{4}{3} \pi kR^3 + 4\pi kR^2$ |
| (c) 0 | (d) $\frac{1}{3\epsilon_0} \vec{r}$ |

Ans. (c)

Q4. Consider the differential equation $\frac{d^2y}{dx^2} + \omega^2 y = 0$. The solution of this equation can be expressed in the series form as: $y(x) = \sum_n c_n x^n$. Which of the following is the correct recursion relation for the coefficients of this series?

- | | |
|--|---|
| (a) $c_{n+2} = -\frac{\omega^2}{(n+2)(n+1)} c_n$ | (b) $c_n = -\frac{\omega^2}{n(n+1)} c_{n+1}$ |
| (c) $c_n = \frac{\omega^2}{n(n-1)} c_{n-1}$ | (d) $c_{n+2} = \frac{\omega^2}{(n+2)(n+1)} c_n$ |

Ans. (a)

Q5. For an atom with an electronic configuration np^2 (where n is the principal quantum number of a shell), the possible values of total angular momentum L and total spin S in the ground state are:

- | | |
|-------------------------|-------------------------|
| (a) $L = 2$ and $S = 0$ | (b) $L = 2$ and $S = 1$ |
| (c) $L = 1$ and $S = 1$ | (d) $L = 1$ and $S = 0$ |

Ans. (c)

Q6. Which one of the following two-particle state $\psi(\vec{r}_1, \vec{r}_2)$ correctly describes two identical bosons in the plane wave states given by the wave-vectors \vec{k}_1 and \vec{k}_2 ?

(a) $\psi(\vec{r}_1, \vec{r}_2) = e^{i(\vec{k}_1, \vec{r}_1 + \vec{k}_2, \vec{r}_2)}$

(b) $\psi(\vec{r}_1, \vec{r}_2) = e^{i\vec{k}_1, \vec{r}_1 + \vec{k}_2, \vec{r}_2}$

(c) $\psi(\vec{r}_1, \vec{r}_2) = e^{i(\vec{k}_1, \vec{r}_1 + \vec{k}_2, \vec{r}_2)} + e^{i(\vec{k}_1, \vec{r}_2 + \vec{k}_2, \vec{r}_1)}$

(d) $\psi(\vec{r}_1, \vec{r}_2) = e^{i(\vec{k}_1, \vec{r}_1 + \vec{k}_2, \vec{r}_2)} - e^{i(\vec{k}_1, \vec{r}_2 + \vec{k}_2, \vec{r}_1)}$

Ans. (c)

Q7. Electrons are ejected from calcium surface when monochromatic light of wavelength 488 nm falls on it. The work function of calcium is 2.28 eV . What is the maximum kinetic energy of the emitted electron?

(Planck's constant, $h = 4.14 \times 10^{-15} \text{ eV sec}$; speed of light, $c = 3 \times 10^8 \text{ m/sec}$)

- (a) 0.026 eV (b) 26 eV (c) 2.6 eV (d) 0.26 eV

Ans. (d)

Q8. Which one of the following is not true about the superconductors?

- (a) Type II superconductors realize a mixed state between the critical magnetic field H_{c1} and H_{c2} .
- (b) Type II superconductors, the penetration depth (λ) is smaller than the coherence length (ζ)
- (c) According to BCS theory, the copper pairs are formed due to electron-phonon interaction
- (d) Superconductivity is characterized by strongly paramagnetic behavior

Ans. (d)

Q9. Consider a vector $\vec{v} = x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3$ in a real three dimensional vector space spanned by three basis vectors \vec{a}_1, \vec{a}_2 and \vec{a}_3 . Consider a new basis of three vectors: $\vec{b}_1 = \vec{a}_1, \vec{b}_2 = \vec{a}_1 + \vec{a}_2$, and $\vec{b}_3 = \vec{a}_1 + \vec{a}_2 + \vec{a}_3$. Let the vector \vec{v} given above be denoted in this new basis as: $\vec{v} = y_1\vec{b}_1 + y_2\vec{b}_2 + y_3\vec{b}_3$. If the transformation matrix V between the components of the vector \vec{v} in the two bases is defined as: $x_i = \sum_{j=1}^3 V_{ij}y_j$ for $i = 1, 2, 3$, then

(a) $V = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (b) $V = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $V = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ (d) $V = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Ans. (b)

Q10. Which of the following expressions is correct for the Helmholtz free energy $F(T, V, N)$ of a thermodynamic system in canonical ensemble? Here, P is pressure, V is volume, N is the number of particles, μ is chemical potential, and T is temperature.

- (a) $F = PV + \mu N$ (b) $F = PV + \mu N$
(c) $F = -PV - \mu N$ (d) $F = \mu N$

Ans. (a)

Q11. Let the angular momentum eigenstates with quantum number j be denoted as $|j, m\rangle$, where $m = -j, -j+1, \dots, j-1, j$. For a system of two angular momenta j_1 and j_2 , any state can be described as linear superposition of their product states $|j_1, m_1\rangle |j_2, m_2\rangle$. For $j_1 = 1$ and $j_2 = \frac{1}{2}$, which of the following is the correct expression for the total angular momentum eigenstate with quantum number $j_{total} = \frac{3}{2}$ and $m_{total} = \frac{1}{2}$?

- (a) $|j_{total} = \frac{3}{2}, m_{total} = \frac{1}{2}\rangle = \frac{1}{\sqrt{3}}(|1, 1\rangle |1/2, -1/2\rangle + \sqrt{2}|1, 0\rangle |1/2, 1/2\rangle)$
(b) $|j_{total} = \frac{3}{2}, m_{total} = \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}(|1, 1\rangle |1/2, -1/2\rangle + |1, 0\rangle |1/2, 1/2\rangle)$
(c) $|j_{total} = \frac{3}{2}, m_{total} = \frac{1}{2}\rangle = |1, 0\rangle |1/2, 1/2\rangle$
(d) $|j_{total} = \frac{3}{2}, m_{total} = \frac{1}{2}\rangle = |1, 1\rangle |1/2, -1/2\rangle$

Ans. (a)

Q12. Consider a gas of N free electrons confined in a volume V . (m is the electron mass, \hbar is Planck's constant and k_B is Boltzmann's constant)

Answer the following three questions on the free electron gas problem.

What is the density of states for the free electrons?

- (a) $\frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{1/2} E^{3/2}$ (b) $\frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right) E^{3/2}$
(c) $\frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$ (d) $\frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right) E^{1/2}$

Ans. (c)

Q13. What is the Fermi energy in terms of N and V ?

- (a) $\left(\frac{3\pi^2 N}{V}\right)^{1/2}$ (b) $\frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{1/3}$ (c) $\frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$ (d) $\left(\frac{3\pi^2 N}{V}\right)^{3/2}$

Ans. (c)

Q14. How does the specific heat (C_V) of free electron gas vary with temperature (T) at low temperature?

- (a) $C_V \propto T^3$
 (b) $C_V \propto e^{\frac{-\Delta}{k_B T}}$, where Δ is the energy gap
 (c) $C_V \propto T^2$
 (d) $C_V \propto T$

Ans. (d)

Consider the function $f(z) = e^{1/z}$ of a complex variable $z = x + iy$ in a complex plane. Answer the following three questions on this function

Q15. The function $f(z) = e^{1/z}$ has:

- (a) no singularity at $z = 0$ (b) an essential singularity at $z = 0$
 (c) a simple pole at $z = 0$ (d) a branch point at $z = 0$

Ans. (b)

Q16. Evaluate the integral $\oint dz e^{1/z}$ over the closed contour given by the unit circle $|z|=1$ centered around the origin of the complex plane.

- (a) π (b) $i\pi$ (c) $i2\pi$ (d) 2π

Ans. (c)

Q17. The equation of the contour corresponding to a fixed value, A is:

- (a) $\left(x - \frac{1}{2 \ln A}\right)^2 + y^2 = \frac{1}{4(\ln A)^2}$ (b) $\left(x + \frac{1}{2 \ln A}\right)^2 + y^2 = \frac{1}{4(\ln A)^2}$
 (c) $\left(x - \frac{1}{\ln A}\right)^2 + y^2 = \frac{1}{(\ln A)^2}$ (d) $\left(x + \frac{1}{\ln A}\right)^2 + y^2 = \frac{1}{(\ln A)^2}$

Ans. (a)

Q18. For a classical system described by a pair of canonical q and momentum p , consider the transformation $Q = -\sqrt{2p} \cos q$ and $P = \sqrt{2p} \sin q$. The poisson bracket of the new variables Q and P is equal to:

- (a) $-\cos 2q$ (b) $\cos 2q$ (c) 1 (d) 0

Ans. (c)

Answer the following three questions on the relativistic corrections to the hydrogen problem.

Q19. The leading relativistic correction to the kinetic energy term in the hydrogen atom Hamiltonian is:

- (a) $\frac{p^4}{8m^3c^2}$ (b) $-\frac{p^3}{8m^3c^2}$ (c) $-\frac{p^4}{8m^3c^2}$ (d) $\frac{p^5}{8m^3c^2}$

Ans. (c)

Q20. The relativistic correction to the hydrogen atom problem leading to spin-orbit interaction is given by:

- (a) $\xi(r) \vec{L} \cdot \vec{S}$, where $\xi(r) \propto r$
 (b) $\xi(r) \vec{L} \cdot \vec{S}$, where $\xi(r) \propto r^{-3}$
 (c) $\xi(r) \vec{L} \cdot \vec{S}$, where $\xi(r) \propto r^{-2}$
 (d) $\xi(r) \vec{L} \cdot \vec{S}$, where $\xi(r) \propto r^{-1}$

Ans. (b)

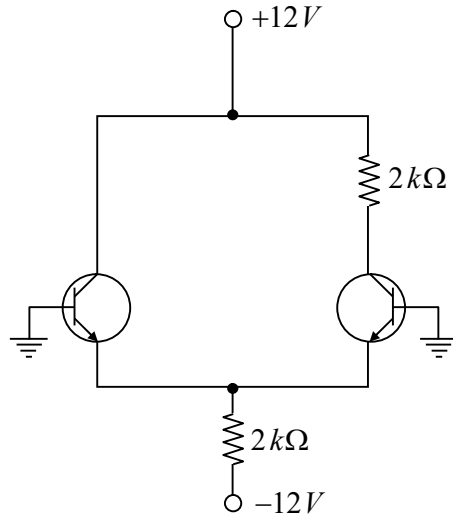
Q21. The relativistic correction due to Darwin to the hydrogen atom problem is given by $\frac{1}{8\epsilon_0} \left(\frac{\hbar e}{mc} \right)^2 \delta(\vec{r})$

where $\delta(\vec{r})$ is Dirac delta function. Which of the following atomic states will be affected by the Darwin correction term?

- (a) only $l = 0$ states
 (b) only $l = 1$ states
 (c) only $l = 2$ states
 (d) All l states

Ans. (a)

For a single ended differential amplifier as given in the figure, answer the following three questions.



Q22. The tail current is:

- (a) 5 mA (b) 10 mA (c) 6 mA (d) 8 mA

Ans. (c)

Q23. The value of emitter current is:

- (a) 1 mA (b) 2 mA (c) 3 mA (d) 4 mA

Ans. (c)

Q24. The value of the collector voltage:

- (a) 4V (b) 6V (c) 8V (d) 10V

Ans. (b)

Q25. Which one of the following elements cannot be used as dopants in silicon to make it *n*-type semiconductor?

- (a) Arsenic (b) Phosphorus (c) Boron (d) Antimony

Ans. (c)

Q26. Consider a particle in a state given by the wavefunction $\psi(x, y, z) = (y + iz)^2$. The wavefunction is an eigenfunction of the angular momentum operator L_x with eigenvalues.

- (a) $-2\hbar$ (b) $-\hbar$ (c) $+\hbar$ (d) $+2\hbar$

Ans. (d)

Q27. By doing an elastic scattering experiment with a beam of electrons of momentum $p \geq 120 \text{ MeV}/c$, we can determine:

(Planck's constant, $h = 6.63 \times 10^{-34} \text{ J.s}$; speed of light, $c = 3 \times 10^8 \text{ m/s}$; electro charge, $e = 1.6 \times 10^{-19} \text{ C}$)

- (a) The size of a biomolecule (b) The lattice constants of a crystal of gold
 (c) The size of an atomic nucleus (d) None of the above

Ans. (c)

A “two-level” atom is considered to have only two energy levels with energies 0 and ϵ . For a system of N non-interacting two-level atoms with total energy E , answer the following three questions.

Q28. What is the number of microstates $\Omega(N, E)$?

- (a) $\frac{N!}{\left(N + \frac{E}{\epsilon}\right)! \left(\frac{E}{\epsilon}\right)!}$ (b) $\frac{N!}{\left(N - \frac{E}{\epsilon}\right)! \left(\frac{E}{\epsilon}\right)!}$
- (c) $\frac{N!}{\left(N - \frac{E}{\epsilon}\right)! \left(N + \frac{E}{\epsilon}\right)!}$ (d) $\frac{N!}{\left(N - \frac{\epsilon}{E}\right)! \left(\frac{\epsilon}{E}\right)!}$

Ans. (b)

Q29. What is the entropy per particle in the limit of large N ?

- (a) $-k_B \left[\left(1 - \frac{E}{N\epsilon}\right) \ln \left(1 - \frac{E}{N\epsilon}\right) - \left(\frac{E}{N\epsilon}\right) \ln \left(\frac{E}{N\epsilon}\right) \right]$
- (b) $+k_B \left[\left(1 - \frac{E}{N\epsilon}\right) \ln \left(1 - \frac{E}{N\epsilon}\right) + \left(\frac{E}{N\epsilon}\right) \ln \left(\frac{E}{N\epsilon}\right) \right]$
- (c) $-k_B \left[\left(1 - \frac{E}{N\epsilon}\right) \ln \left(1 - \frac{E}{N\epsilon}\right) + \left(\frac{E}{N\epsilon}\right) \ln \left(\frac{E}{N\epsilon}\right) \right]$
- (d) $+k_B \left[\left(1 + \frac{E}{N\epsilon}\right) \ln \left(1 + \frac{E}{N\epsilon}\right) - \left(\frac{E}{N\epsilon}\right) \ln \left(\frac{E}{N\epsilon}\right) \right]$

Ans. (c)

Q30. What is the corresponding temperature T ?

- (a) $\frac{1}{T} = \frac{k_B}{\epsilon} \ln \left(\frac{N\epsilon}{E} - 1 \right)$ (b) $\frac{1}{T} = \frac{k_B}{\epsilon} \ln \left(\frac{N\epsilon}{E} + 1 \right)$
- (c) $\frac{1}{T} = \frac{k_B}{\epsilon} \ln \left(\frac{E}{N\epsilon} + 1 \right)$ (d) $\frac{1}{T} = \frac{k_B}{\epsilon} \ln \left(\frac{E}{N\epsilon} - 1 \right)$

Ans. (a)

Q31. The decay $n \rightarrow p + e^-$ of a neutron (n) into a proton (p) and an electron (e^-) is forbidden due to the violation of conservation of:

- (a) Angular momentum and baryon number
 (b) Energy and lepton number
 (c) Angular momentum and lepton number
 (d) Electric charge and baryon number

Ans. (c)

Consider a crystalline material which, under ambient conditions, is given to have the FCC (face-centered cubic) lattice structure with monoatomic basis. Answer the following three questions for this system.

Q32. A primitive unit cell of the monoatomic FCC crystal contains:

- (a) 1 atom (b) 2 atom (c) 3 atom (d) 4 atom

Ans. (a)

Q33. The photon dispersion of a monoatomic FCC crystal has:

- (a) 3 branches of acoustic photons only.
(b) 3 branches of acoustic phonons, and 9 branches of optical phonons.
(c) 1 branches of acoustic phonons, and 2 branches of optical phonons.
(d) 3 branches of optical phonons only

Ans. (a)

Q34. Suppose by changing the temperature, if the crystal structure of the material changes from the monoatomic FCC to monoatomic BCC (body-centered cubic), then the number of optical phonon branches will change by:

- (a) 0 (b) 2 (c) 3 (d) 6

Ans. (a)

Answer the following three questions on the semi-empirical formula for the binding energy of atomic nuclei in terms of the nuclear mass number A and the proton number Z

Q35. In the formula for binding energy per nucleon, the volume energy term is

- (a) a constant (b) proportional to Z
(c) proportional to A (d) Proportional to $A^{1/3}$

Ans. (a)

Q36. In the formula for binding energy per nucleon, the contribution from the Coulomb repulsion between protons is:

- (a) proportional to Z only (b) proportional to $Z(Z-1)$ only
(c) proportional to $Z(Z-1)A^{-1/3}$ (d) proportional to $Z(Z-1)A^{-4/3}$

Ans. (d)

Q37. In the formula for binding energy per nucleon, the pairing energy term is:

- (a) always zero
(b) zero only when A is an odd integer
(c) non-zero when A is an odd integer
(d) always non-zero

Ans. (b)

Q38. If the scalar and vector potentials are given by $\phi(\vec{r}, t) = 0$ and $\vec{A}(\vec{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$, the corresponding electric field (\vec{E}) is:

- (a) 0 (b) $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$ (c) $\frac{1}{4\pi\epsilon_0} \frac{q}{r} \hat{r}$ (d) $-\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$

Ans. (b)

A body of mass m is thrown up vertically with an initial speed u . The air exerts a drag force $-kv$ upon it, where v is the instantaneous velocity of the body and k is a constant. The body also experiences gravitational acceleration g .

Answer the following questions on this problem.

Q39. What is the terminal speed attained by the body?

- (a) $\frac{mg}{k}$ (b) $\frac{g}{k}$ (c) $\frac{k}{mg}$ (d) u

Ans. (a)

Q40. What is the time it will to attain the maximum height?

- (a) $\ln\left(1 + \frac{mg}{ku}\right)$ (b) $\frac{k}{m} \ln\left(1 + \frac{ku}{mg}\right)$
 (c) $\frac{m}{k} \ln\left(1 + \frac{ku}{mg}\right)$ (d) $\frac{m}{k} \ln\left(1 + \frac{mg}{ku}\right)$

Ans. (c)

Q41. What is the maximum height attained by the body?

- (a) $\frac{mu}{k} + g\left(\frac{m}{k}\right)^2 \ln\left(1 + \frac{ku}{mg}\right)$ (b) $\frac{mu}{k} - g\left(\frac{m}{k}\right)^2 \ln\left(1 + \frac{ku}{mg}\right)$
 (c) $\frac{mu}{k} - g\left(\frac{m}{k}\right)^2 \ln\left(1 - \frac{ku}{mg}\right)$ (d) $\frac{mu}{k} + g\left(\frac{m}{k}\right)^2 \ln\left(1 - \frac{ku}{mg}\right)$

Ans. (b)

Q42. The Fourier transformation for a function $f(x)$ of a real variable x can be defined as:

$f(x) = \int_{-\infty}^{+\infty} dk e^{ikx} g(k)$, where $g(k)$ is a function of another real variable k . If $g(k) = e^{iky}$ for a given y , then what is $f(x)$?

- (a) $\delta(x+y)$ (b) $\delta(x-y)$ (c) $2\pi\delta(x+y)$ (d) $2\pi\delta(x-y)$

Ans. (d)

Consider the one-dimensional simple harmonic oscillator of mass m and frequency ω described by the Hamiltonian, $H = \frac{1}{2m} p^2 + \frac{1}{2} m\omega^2 x^2 = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right)$, with eigenvalues $E_n = \hbar\omega \left(n + \frac{1}{2} \right)$ and eigenstates $|n\rangle$. The creation and annihilation operators a^\dagger and a are related to the coordinate x and momentum p as: $x = \sqrt{\frac{\hbar}{2m\omega}} (a^\dagger + a)$ and $p = i\sqrt{\frac{m\hbar\omega}{2}} (a^\dagger - a)$. Answer the following three questions on this problem.

Q47. The commutator $(a^\dagger a, a^\dagger a^\dagger)$ is equal to:

- (a) $-2a^\dagger a^\dagger$ (b) $2a^\dagger a$ (c) $2a^\dagger a^\dagger$ (d) $-2a^\dagger a$

Ans. (c)

Q48. What is the uncertainty in position, $\sqrt{\langle x^2 \rangle - \langle x \rangle^2}$, in the eigenstate $|n\rangle$?

- (a) $\sqrt{\frac{\hbar}{m\omega}} (2n+1)$ (b) $\sqrt{\frac{\hbar}{m\omega}} \left(n + \frac{1}{2} \right)$
 (c) 0 (d) $\sqrt{\frac{\hbar}{2}}$

Ans. (b)

Q49. Which of the following is the correct expression for the creation operator?

- (a) $\sqrt{n+1} |n\rangle \langle n+1|$ (b) $\sum_{n=0}^{\infty} \sqrt{n+1} |n+1\rangle \langle n|$
 (c) $\sum_{n=0}^{\infty} \sqrt{n} |n\rangle \langle n+1|$ (d) $\sqrt{n} |n\rangle \langle n-1|$

Ans. : (b)

Q50. Consider a rectangular waveguide with a cross-section a dimension $2\text{ cm} \times 1\text{ cm}$. If the driving frequency is $1.7 \times 10^{10} \text{ Hz}$, the transverse Electric (TE) mode that will propagate in this wave guide is:

- (a) $0.53 \times 10^{10} \text{ Hz}$ (b) $0.75 \times 10^{10} \text{ Hz}$
 (c) $1.9 \times 10^{10} \text{ Hz}$ (d) $1.4 \times 10^9 \text{ Hz}$

Ans. (b)