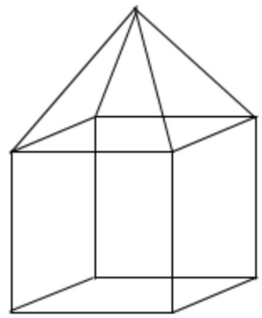
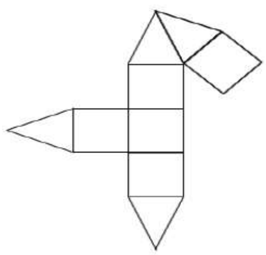
TIFR 2021

Q1. A 3-dimensional view of a polygon, whose faces are either squares or isosceles triangles, is sketched below.

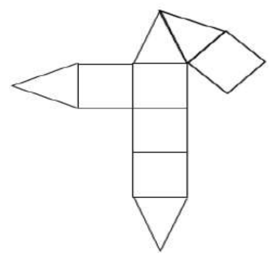


Which of the following 2-dimensional figures represents it after flattening?

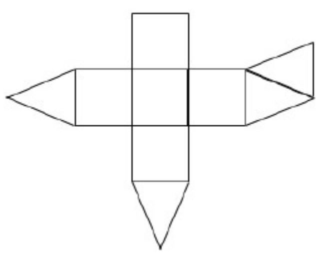
(a)



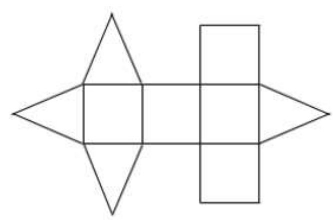
(b)



(c)



(d)



Q2. The intergral

$$I = \frac{3/4}{1/2} dx \exp\left\{-\exp\left(\frac{1}{x}\right)\right\}$$

Eveluates to $I =$

(a) 0.00215 (b) $\exp\sqrt{2}$ (c) 1.762633 (d) $-\exp(-1)$

Q3. A unitary matrix U is expressed in terms of a Hermitian matrix H, such that

$$U = e^{i\pi H/2}$$

If the matrix H is given by

$$H = \sqrt{3}\begin{pmatrix} 1/3 & 0 & \sqrt{2}/3 \\ 0 & 1/\sqrt{3} & 0 \\ 2/3 & 0 & -1/3 \end{pmatrix}$$

Then U will have the form

(a) $\begin{pmatrix} 1/\sqrt{3} & 0 & i\sqrt{2}/\sqrt{3} \\ 0 & i & 0 \\ i\sqrt{2} & 0 & -i/\sqrt{3} \end{pmatrix}$ (b) $\begin{pmatrix} \sqrt{3} & 0 & \sqrt{6} \\ 0 & 3\sqrt{3} & 0 \\ \sqrt{6} & 0 & -\sqrt{3} \end{pmatrix}$

(c) $\begin{pmatrix} i\sqrt{3} & 1/\sqrt{3} & \sqrt{2}\sqrt{3} \\ 1/\sqrt{3} & i & 1/\sqrt{3} \\ \sqrt{2}\sqrt{3} & 1/\sqrt{3} & i\sqrt{3} \end{pmatrix}$ (d) $\begin{pmatrix} 3\sqrt{3}i & \sqrt{3} & 3/\sqrt{2} \\ \sqrt{3} & i & 0 \\ \sqrt{2}/\sqrt{3} & 0 & 3\sqrt{3}i \end{pmatrix}$

Q4. In a country, the fraction of population infected with Covid-19 is 0.2. It is also known that out of the people who are infected with Covid-19, only a fraction 0.3 show symptoms of the disease, while the rest do not show any symptoms.

If you randomly select a citizen of this country, the probability that this person will **NOT** show symptoms of Covid-19 is

(a) 0.94 (b) 0.56 (c) 0.86 (d) 0.80

Q5. In a futuristic scenario, an inter-planetary meeting is arranged on planet Pegasus XIV, which is 10 light years away from the Earth. The team of representatives from Earth would like to take a spaceship to Pegasus XIV that has a one-hour flight time according to the watches of the passengers. The Team Leader from Earth would like to leave from the Earth half-an-hour later, but taking a spaceship that has a half-anhour flight time according to the watches of the passengers. If the time taken for acceleration may be neglected for both spaceships, which of the following statements is correct?

(a) The Team Leader would reach Pegasus XIV before the committee members.

(b) The Team Leader would reach Pegasus XIV at the same time as the committee members.

(c) The Team Leader would reach Pegasus XIV after the committee members.

(d) The situation described in the problem is not possible by the laws of physics.

Q6. A planet is moving around a star of mass M_0 in a circular orbit of radius R. The star starts to lose its mass very slowly (adiabatically), and after some time, it reaches a mass $M\left(M < M_0\right)$. If the motion of the planet is still circular at that time, the radius of its orbit will become

(a) $R\left(\frac{M_0}{M}\right)^2$ (b) $R\left(\frac{M}{M_0}\right)^2$ (c) $R\left(\frac{M_0}{M}\right)^{1/2}$ (d) $R\left(\frac{M}{M_0}\right)$

Q7. A star moves in an orbit under the influence of massive but invisible object with the effective one-dimensional potential

$$V\left(r\right) = -\frac{1}{r} + \frac{L^2}{2r^2} - \frac{L^2}{r^3}$$

Where L is the angular momentum of the star. There would be two possible circular orbits of the star if

(a) $L^2 > 12$ (b) $L^2 > 6$ (c) $L^2 > 3$ (d) $L^2 > 9$

Q8. A particle of mass m moves in a plane $\left(r, \theta\right)$ under the influence of a force

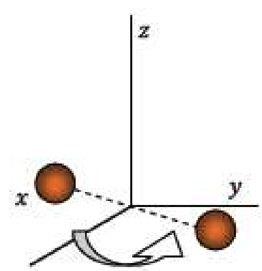
$$\vec{F} = \frac{mk}{r^3}\left(x\hat{r} + y\hat{\theta}\right)$$

Where $x = r\cos\theta$ and $y = r\sin\theta$ while k is a constant. The Lagrangian for this system is

(a) $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 - \frac{kx}{r^2}\right)$ (b) $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 - \frac{ky}{r^2}\right)$

(c) $L = \frac{1}{2}m\left(\frac{\dot{r}^2}{r} + r\dot{\theta}^2 - \frac{kxy}{r}\right)$ (d) $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 \frac{kx}{r^2}\right)$

Q9. Consider a diatomic molecule of oxygen which is rotating in the xy-plane about the z axis. The z axis passes through the centre of the molecule and is perpendicular to its length. At room temperature, the average separation between the two oxygen atoms is $1.21\times10^{-10}m$ (the atoms are treated as point masses). The molar mass of oxygen is $16gm/mol$.



If the angular velocity of the molecule about the z axis is $2\times10^{12}rad/s$, its rotational kinetic energy will be closest to

(a) $3.89\times10^{-22}Joule$ (b) $7.78\times10^{-22}Joule$

(c) $15.56\times10^{-22}Joule$ (d) $1.95\times10^{-22}Joule$

Q10. A hydrogen atom in its ground state collides with an electron of energy 13.377 eV, absorbs most of the energy of the electron, and goes into an excited state. The maximum possible fraction

$$f \equiv \frac{R_{final} - R_{initial}}{R_{initial}}$$

By which its radius R would increase will be

(a) $f = 0.63$ (b) $f = 0.48$ (c) $f = 0.60$ (d) $f = 0.07$

Q11. What are the energy eigenvalues for relative motion in one-dimension of a two-body simple quantum harmonic oscillator (each body having mass m) with the following Hamiltonian?

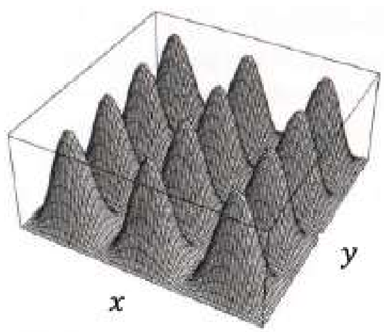
$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2\left(x_1 - x_2\right)^2$$

(a) $\sqrt{2}\left(n+\frac{1}{2}\right)\hbar\omega$ (b) $\left(n+\frac{1}{2}\right)\hbar\omega$ (c) $\frac{1}{\sqrt{2}}\left(n+\frac{1}{2}\right)\hbar\omega$ (d) $\sqrt{\frac{3}{2}}\left(n+\frac{1}{2}\right)\hbar\omega$

Q12. An electron is confined to a two-dimensional square box with the following potential

$$V = \begin{cases} 0 & \text{for } 0 < x < L \text{ and } < 0 < y < L \\ \infty, & \text{otherwise} \end{cases}$$

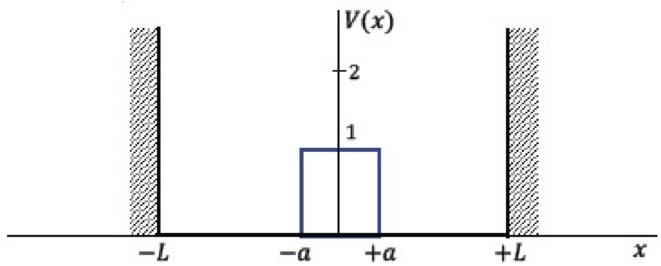
The probability distribution of the electron in one of its eigenstates is shown below



How many total different eigenstates of the electron have the same energy as this state?

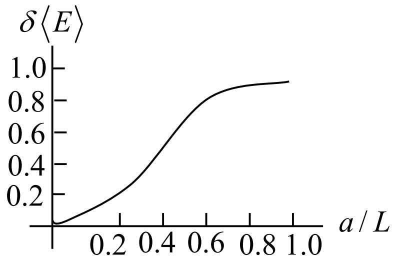
(a) 4 (b) 2 (c) 6 (d) 1

Q13. The particle of mass m, confined in a one-dimensional box between $x=-L$ and $x=L$, is in its first excited quantum state. Now, a rectangular potential barrier of height $V(x)=1$ and extending from $x=-a$ to $x=a$ is suddenly switched on, as shown in the figure below.

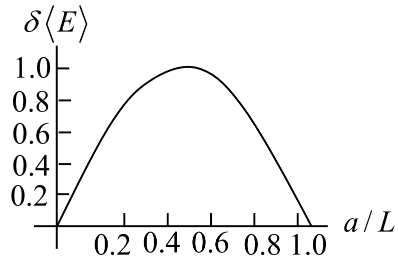


Which of the following curves most closely represents the resulting change in average energy $\delta\langle E\rangle$ of the system when plotted as a function of a/L, immediately after the barrier is created?

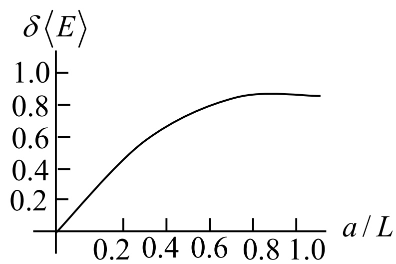
(a)



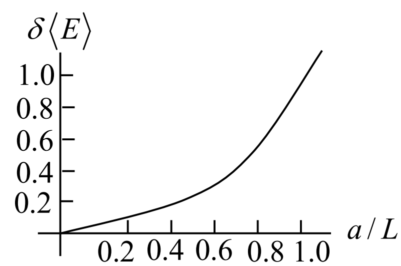
(b)



(c)

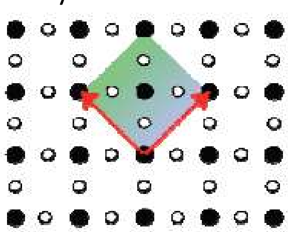


(d)

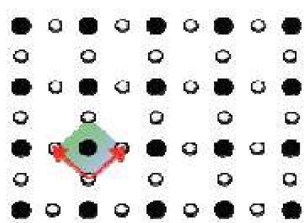


Q14. The pictures below are intended to a two-dimensioanl lattice with permitive vectors indicated by arrows and with the unit cell shaded. Which of the following pictures is correct?

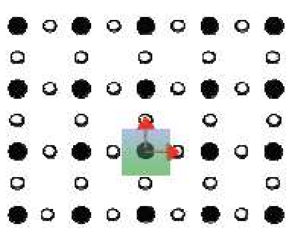
(a)



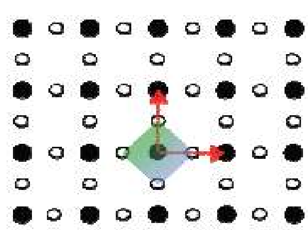
(b)



(c)



(d)



Q15. A boiler of volume $1.7m^3$, when filled with $1.0kg$ of steam at 100^0C, has a pressure of $1.0\,atm$. What will be the boiling point of water in this boiler when the pressure is $2.0\,atm$.

[The latent heat of vaporization of water is $2250\times10^3 J/kg$; $1atm=10^5 N/m^2$]

(a) 128^0C (b) 118^0C (c) 78^0C (d) 88^0C

Q16. Which of the following is the entropy generated when two identical blocks at temperatures $2T$ and T are brought into thermal contact and allowed to reach equilibrium?

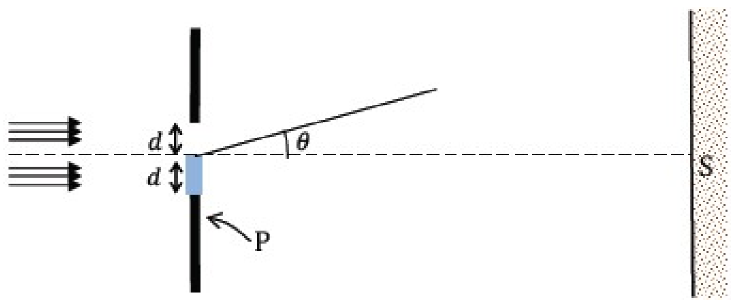
[Assume that the heat capacity of each block is C]

(a) $C(2\ln3-3\ln2)$ (b) Zero (c) $2C\ln\frac{3}{2}$ (d) $C(2\ln2-3\ln3)$

Q17. In a Universe with only two spatial dimensions, the total energy radiated by a perfect blackbody across all wavelengths per unit surface area per unit time is proportional to

(a) T^3 (b) T^4 (c) T^2 (d) $T^{3/2}$

Q18. The lower half of a single slit of width d is covered with a half-wave plate P as shown in the figure below.



As a result, a beam of coherent monochromatic light of wave vector $k=2\pi/\lambda$ incident on the single slit, transmits with an amplitude and a Fraunhofer diffraction pattern is formed on a screen S placed parallel to the slit.

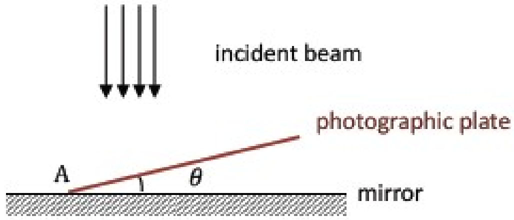
The intensity at a point on the screen at an angle θ measured from the centre of the slit (see figure), is proportional to

$$T(x)=\begin{cases}1 & \text{for } 0<x\le d/2\\ -1 & \text{for } -d/2<x\le 0\\ 0 & \text{otherwise}\end{cases}$$

(a) $\frac{1}{\phi^2}\sin^2\phi$ where $\phi=\frac{1}{2}kd\sin\theta$ (b) $\frac{1}{\phi^2}\sin^4\phi$ where $\phi=kd\sin\theta$

(c) $\frac{1}{\phi}\sin\phi$ where $\phi=kd\cos\theta$ (d) $\frac{1}{\phi^2}\cos^2\phi$ where $\phi=\frac{1}{2}kd\sin\theta$

Q19. A collimated coherent light beam of wavelength λ is incident normally on an assembly of a mirror and a photographic plate as shown below. The photographic plate is placed at the position A with a small angle θ with respect to the mirror surface as shown in the figure below. Assume that the photographic plate is almost transparent to the incident light and has a negligible thickness. After sufficient exposure, the plate is developed.



Which of the following statements is true for the above experimental setup?

(a) The plate will show dark strips separated by distances $\frac{\lambda}{2\sin\theta}$ with the first strip at a distance $\frac{\lambda}{4\sin\theta}$ from the point of contact

(b) The plate will show dark strips separated by distances $\frac{\lambda}{\sin\theta}$ with the first strip at the point of contact.

(c) The plate will show dark strips separated by distances $\frac{\lambda}{\sin\theta}$ with the first strip at a distance $\frac{\lambda}{2\sin\theta}$ from the point of contact

(d) The plate will show dark strips separated by distances $\frac{\lambda}{\sin\theta}$ with the first strip at the point of contact.

Q20. In an experiment that measures the resistivity ρ of a substance it was observed that ρ varies with temperature T and a parameter Δ, as

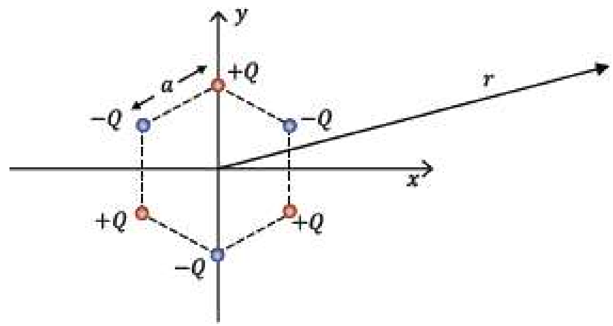
$$\rho = \rho_0 e^{\Delta/T}$$

Where ρ_0 is a constant

In one measurement, made at $T = 100K$ and $\Delta = 50$, the percentage error in Δ is found to be 2% while the percentage error in T was 3%. What was the approximate percentage error for the resistivity ρ?

(a) 1.8% (b) 3.6% (c) 9% (d) 18%

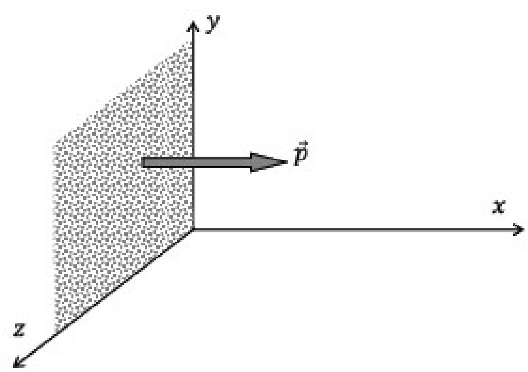
Q21. Consider 6 charges fixed at the vertices of a regular hexagon of side a, as shown in the figure below.



The behaviour of the electrostatic potential at distance $r \to \infty$ in the xy plane is proportional to

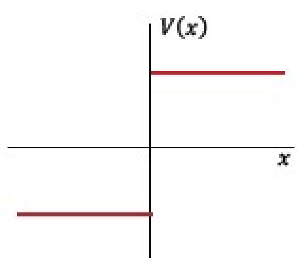
(a) $1/r^4$ (b) $1/r^5$ (c) $1/r^3$ (d) $1/r^2$

Q22. Consider an infinite uniform layer of point-like dipoles, placed in the $y-z$ plane, with a constant dipole strength $\vec{p} = p\hat{x}$. per unit area, as shown in figure below.

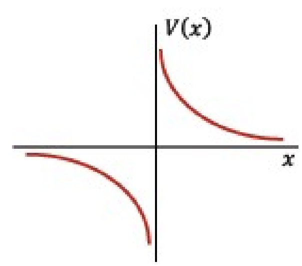


Which graph best represents the variation of potential along the x direction?

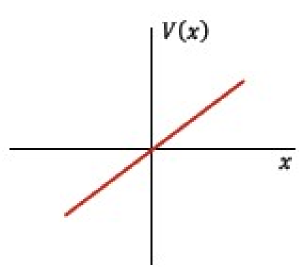
(a)



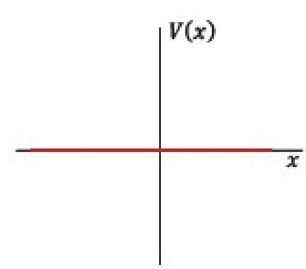
(b)



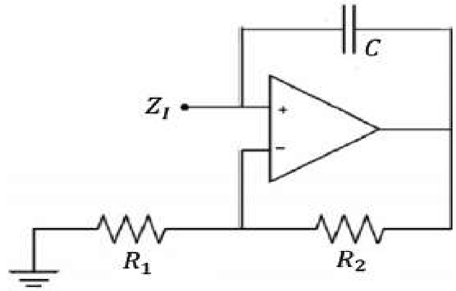
(c)



(d)



Q23. An operational amplifier is configured as shown in the figure below. For an AC input this circuit behaves effectively as



(a) a capacitor with a negative capacitance.

(b) an inductor with a negative inductance.

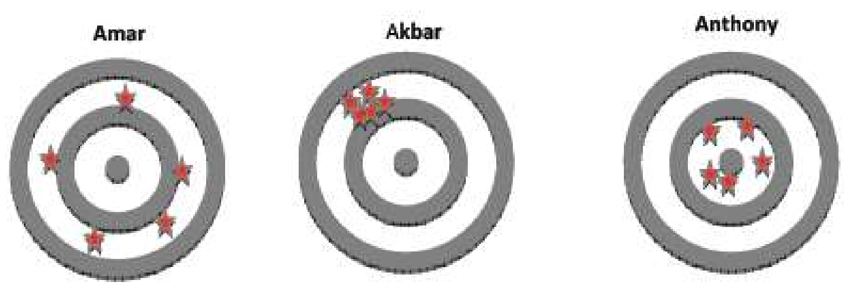
(c) a resistor with a negative resistance.

(d) an inductor with a positive inductance

Q24. A three variable (A, B, C) truth table has a high output for the input conditions 000, 010, 100, and 110 and low otherwise. This effectively means the circuit following this truth table is the equivalent of

(a) $\overline{C}$ (b) $A+\overline{A}$ (c) $A+B$ (d) $\overline{C}\left(A+B\right)$

Q25. In an archery contest, the aim is to shoot arrows at the center of a board. Three archers, Amar, Akbar and Anthony each shot 5 arrows at the board. The locations of their arrow hits are shown in the figures with red stars. Which of the following statements are true?



(a) Akbar has more precision than Anthony
(b) Amar has more precision than Akbar
(c) Akbar has more accuracy than Anthony
(d) Amar has more accuracy than Anthony

Section B

(Only for Integrated M.Sc.-Ph.D. candidates)

This Section consists of 15 questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +5 marks, an incorrect answer will get 0 mark.

Q26. A differentiable function $f(x)$ obeys

$$x\int_0^x \frac{f(y)}{y^2}dy = f(x)$$

If $f(1)=1$. It follows that $f(2)=$

(a) 4 (b) $3/4$ (c) 1 (d) 6

Q27. If $y(x)$ satisfies the following differential equation

$$x\frac{dy}{dx} = \cot y - \cos\sec y \cos x$$

and we have

$$\lim_{x\to\infty} y(x) = 0$$

Then $y(\pi/2)=$

(a) $-\cos^{-1}(2/\pi - 2)$ (b) $\sin^{-1}(2/\pi)$ (c) $\pi/2$ (d) 0

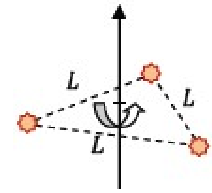
Q28. A body is dropped from rest at a height h above the surface of the Earth at a latitude λ_N. in the northern hemisphere. If the angular velocity of rotation of the Earth is ω the lateral displacement of the body at its point of impact on the Earth's surface will be

(a) $\left(\frac{8h^3\omega^2}{9g}\right)^{1/2}\cos\lambda_N$ (b) $\left(\frac{8h^3\omega^2}{9g}\right)^{1/2}\sin\lambda_N$

(c) $\left(\frac{2h^3\omega^2}{9g}\right)^{1/2}\cos\lambda_N$ (d) $\left(\frac{2h^3\omega^2}{9g}\right)^{1/2}\sin\lambda_N$

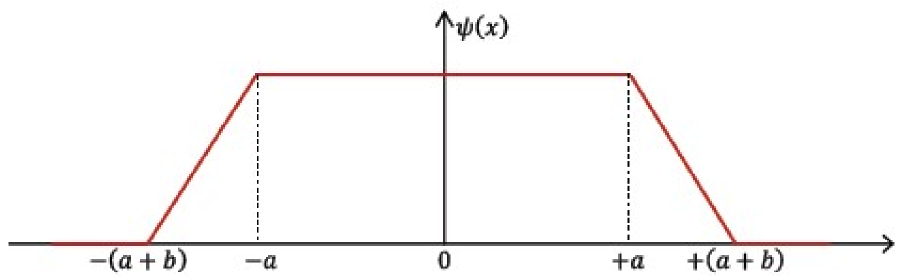
Q29. Three stars, each of mass M, are rotating under gravity around a fixed common axis such that they are always at the vertices of an equilateral triangle of side L (see figure).

The time period of rotation of this triple star system is



(a) $\frac{2\pi L^{3/2}}{\sqrt{3G_N M}}$ (b) $\frac{2\pi L^{3/2}}{3\sqrt{3G_N M}}$ (c) $\frac{\pi L^{3/2}}{\sqrt{3G_{NM}}}$ (d) $\frac{\pi L^{3/2}}{3\sqrt{G_N M}}$

Q30. The wave function of a one-dimensional particle of mass m is shown below. The average kinetic energy of the particle can be written as



(a) $\frac{3\hbar^2}{2mb(3a+b)}$ (b) 0 (c) $\frac{\hbar^2}{2mb^2}$ (d) $\frac{\hbar^2}{2mb(a+b)}$

Q31. Suppose a system is in a normalised state $|\Psi\rangle$. such that

$$|\Psi\rangle = c\left(|\varphi_0\rangle + e^{i\theta}|\varphi_1\rangle\right)$$

where $|\varphi_0\rangle$ and $|\varphi_1\rangle$ are the first two normalised eigenstates of a one-dimensional simple harmonic oscillator of frequency ω and $c > 0$ is a real constant. If the expectation value of the position operator $\hat{x}$. is given by

$$\langle\Psi|\hat{x}|\Psi\rangle = \frac{1}{2}\sqrt{\frac{\hbar}{m\omega}}$$

The value of θ must be

(a) $\pi/4$ (b) $\pi/2$ (c) $3\pi/2$ (d) π

Q32. A very sensitive spring balance with spring constant $k = 2\times10^{8}\,Nm^{-1}$ is operating at a temperature of $300K$. The thermal fluctuations can lead to an error in the measurement of mass. If you are trying to measure a mass of $1mg$, the relative error in the measurement is closest to

(a) 0.9% (b) 10.0% (c) 20.0% (d) 0.01%

Q33. A pulsed laser beam has photon number density n in each pulse. The photon number density n' inside each pulse, when measured from a frame moving in a direction perpendicular to the beam with velocity v is given by

[Assume the usual notation $\beta = v/c, \gamma = \left(1-\beta^2\right)^{-1/2}$]

(a) $\frac{n}{\gamma}$ (b) $\gamma(1-\beta)n$ (c) n (d) $\frac{\beta n}{\gamma}$

Q34. A spherical balloon of radius R is made of a material with surface tension γ and filled with N particles of an ideal gas. If the outside air pressure is P, the pressure P_b inside the balloon is given by

(a) $P_b = P + 2\gamma/R$ (b) $P_b = P$ (c) $P_b = P - 2\gamma/R$ (d) $P_b = P + 3\gamma/R$

Q35. For a pure germanium semiconductor, cooled in liquid nitrogen, the average density of conduction electrons is about $n = 10^{12}\,cm^{-3}$. At this temperature, the electron and hole mobilities are equal and have the common value $\mu = 5.0\times10^{3}\,cm^{2}V^{-1}s^{-1}$. If a potential of $100V$ is applied across opposite faces of a cube of this cooled germanium sample having side 1 cm, the current through the sample can be estimated as

(a) $160mA$ (b) $16mA$ (c) $8mA$ (d) $80mA$

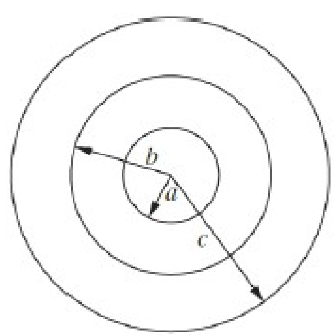
Q36. Three concentric spherical metallic shells with radii $c > b > a$ (see figure) are charged with charges e_c, e_b and e_a respectively. The outermost shell (of radius c) is at a potential V_c^0. Now, the innermost shell (of radius a) is grounded, and the potential of the outermost shell becomes V_c^9. The difference $V_c^g - V_c^0$ will be

(a) $-\frac{1}{4\pi\varepsilon_0}\frac{a}{c}\left(\frac{e_a}{a}+\frac{e_b}{b}+\frac{e_c}{c}\right)$

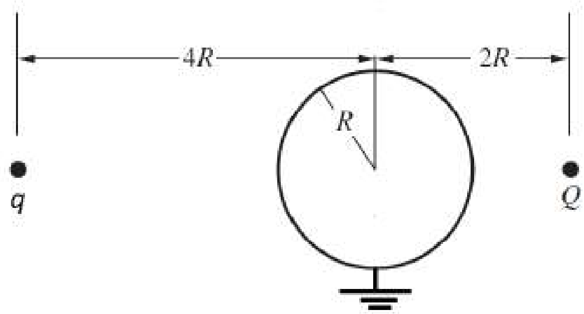
(b) $-\frac{1}{4\pi\varepsilon_0}\frac{c}{a}\left(\frac{e_a}{a}+\frac{e_b}{b}+\frac{e_c}{c}\right)$

(c) $-\frac{1}{4\pi\varepsilon_0}\frac{c}{a}\left(\frac{e_a}{c}+\frac{e_b}{b}+\frac{e_c}{a}\right)$

(d) $-\frac{1}{4\pi\varepsilon_0}\frac{c}{a}\left(\frac{e_c}{c-a}+\frac{e_b}{b}\right)$



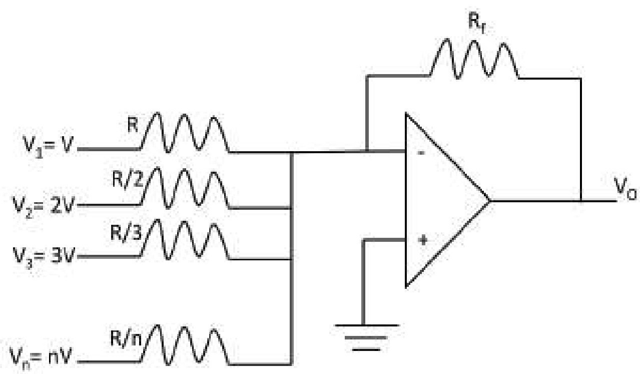
Q37. Two positive charges Q and q are placed on opposite sides of a grounded sphere of radius R at distances of $2R$ and $4R$ respectively, from the centre of the sphere, as shown in the diagram below.



The charge q feels a force AWAY from the centre of the sphere if

(a) $\frac{q}{Q} < \frac{25}{144}$ (b) $\frac{q}{Q} < \frac{25}{16}$ (c) $\frac{q}{Q} < \frac{25}{36}$ (d) $\frac{q}{Q} < \frac{49}{144}$

Q38. Consider the following circuit with an op-amp.



If the output voltage V_0 is measured to be $V_0 = -V$, then the value of the feedback Resistance R_f must be

(a) $R_f = \frac{6R}{n(n+1)(2n+1)}$ (b) $R_f = \frac{3R}{n(n+1)(2n+1)}$

(c) $R_f = nR$ (d) $R_f = R/n$

Q39. In an amplifier circuit, an input sine wave of amplitude $5V$ gives a sine wave of amplitude $25V$ as an output in an open load configuration. On applying a $20k\Omega$ load resistance, the output drops to $10V$. This implies that the output resistance of the amplifier must be

(a) $30k\Omega$ (b) $20k\Omega$ (c) $10k\Omega$ (d) $2k\Omega$

Q40. In an experiment, a counting device is used to record the number of charged particles passing through it. Once this counter records a charged particle, it does not respond for a short interval of time, called the 'dead time' of that counter. This device is used to count the charged particles emitted by a particular radioactive source. It is found that if the source emits 20,000 counts/second at random intervals, the counter records 19,000 particles per second on an average. It follows that the counter dead time must be

(a) 2.63 microseconds (b) 2.63 nanoseconds

(c) 50.0 milliseconds (d) 2.63 seconds

Section B

(only for Ph.D. candidates)

This Section consists of 15 questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +5 marks, an incorrect answer will get 0 mark.

Q41. How many distinct values can the following function take at a given value of z ?

$$f(z)=\sqrt{\frac{z^2-1}{\sqrt{z}}}(z-i)^{1/3}$$

(a) 12 (b) 3 (c) 4 (d) 24

Q42. Given the following $x-y$ data table

x	1.0	2.0	3.0	4.0	5.0	6.0
y	0.602	0.984	1.315	1.615	1.894	2.157

which would be the best-fit curve, where a and b are constant positive parameters?

(a) $y=bx^{1(1+a)}$ (b) $y=ax-b$ (b) $y=a+e^{bx}$ (b) $y=a\log_{10}bx$

Q43. A particle of mass m, moving in one dimension x satisfies the Lagrangian

$$L=\frac{1}{2}m\dot{x}^2e^{2kx}$$

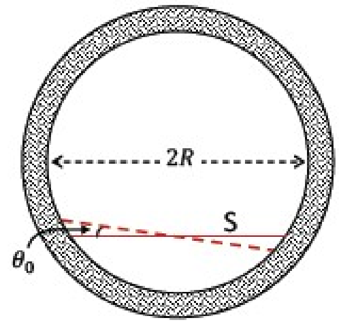
where k is a constant.

If H is the Hamiltonian of the system, the canonical equations of motion are

(a) $\dot{x}=\frac{p}{m}e^{-2kx},\dot{p}=-2kH$ (b) $\dot{x}=\frac{p}{m}e^{2kx},\dot{p}=-2H$

(c) $\dot{x}=\frac{p}{m}e^{-2kx},\dot{p}=-\frac{1}{2}kH$ (d) $\dot{x}=\frac{p}{m}e^{-2kx},\dot{p}=-2H$

Q44. A stick S of uniform density of mass M, length L and negligible width, is constrained to move such that its two ends always stay on the inside of a fixed vertical, circular ring of inner radius R, as shown below.



If the stick S is displaced by a small angle θ_0 from its equilibrium position and then allowed to oscillate freely, the angular frequency ω of oscillations will be

[Ignore the friction between the stick and the ring.]

(a) $\left(\frac{6g}{6R^2-L^2}\right)^{1/2}\left(R^2-\frac{L^2}{4}\right)^{1/4}$ (b) $\left(\frac{4g}{6R^2-L^2}\right)^{1/2}\left(R^2-\frac{L^2}{4}\right)^{1/4}$

(c) $\left(\frac{6g}{3R^2-L^2}\right)^{1/2}\left(R^2-\frac{L^2}{4}\right)^{1/4}$ (d) $\left(\frac{4g}{2R^2-L^2}\right)^{1/2}\left(R^2-\frac{L^2}{4}\right)^{1/4}$

Q45. The Hamiltonian of a spin $-1/2$ particle in a magnetic field $\vec{B}$ is given by $H=-\mu\vec{S}\cdot\vec{B}$, where the components of the spin operator $\vec{S}$ have eigenvalues $\pm\hbar/2$. The spin is pointing in the $+\hat{x}$ direction, when a magnetic field $\vec{B}=B\hat{y}$ is turned on. After a time $t=\pi/2\mu B$, the spin will be pointing along the direction

(a) $+\hat{z}$ (b) $-\hat{z}$ (c) $-\hat{x}$ (d) $\hat{x}+\hat{z}$

Q46. An electron moves in a hydrogen atom potential in a state ψ that has the wave function

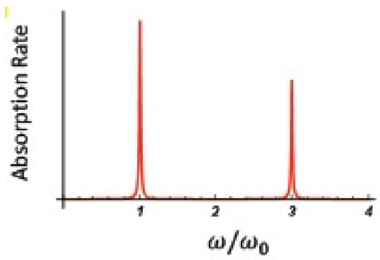
$$\psi(r,\theta,\varphi)=NR_{21}(r)\left[2iY_1^{-1}(\theta,\varphi)+(2+i)Y_1^0(\theta,\varphi)+3iY_1^1(\theta,\varphi)\right]$$

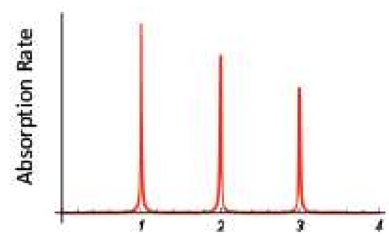
where N is a normalization constant, $R_{nl}(r)$ is the radial wave function and the $Y_l^m(\theta,\varphi)$ are spherical harmonics.

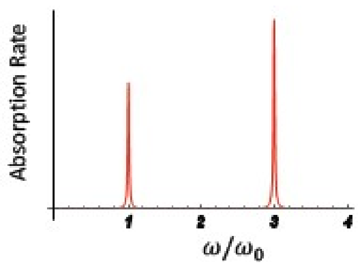
The expectation value of $\hat{L}_z$, i.e. the $\hat{z}$-component of the angular momentum operator is

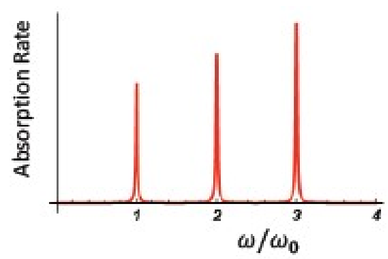
(a) $\frac{5}{18}\hbar$ (b) $\frac{4}{18}\hbar$ (c) $\frac{9}{18}\hbar$ (d) $\frac{13}{18}\hbar$

Q47. Consider a one-dimensional simple harmonic oscillator with frequency ω_0 in its ground state. An external wave passes through this system, creating a small time-dependent potential of the form $V(x,t) = Ax^3 \cos\omega t$ where A and ω are constants. If the absorption rate of the wave is measured as a function of ω, which of the following graphs is the likely result of such a measurement?

(a) 

(b) 

(c) 

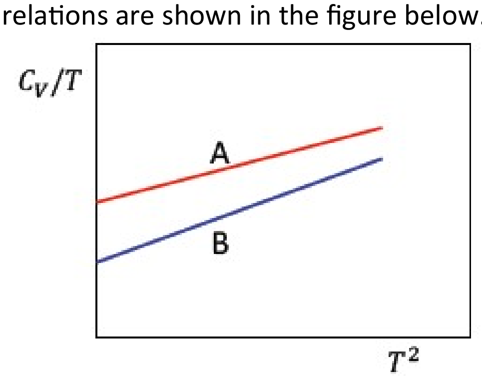
(d) 

Q48. A certain system has one state with energy E, two states with energy $2E$, three states with energy $3E$ and so on, where $E > 0$. The partition function Z. of the system at temperature T is given by

(a) $\frac{1}{Z} = 4\sinh^2\frac{E}{2T}$ (b) $\frac{1}{Z} = 2\cosh^2\frac{E}{4T}$

(c) $\frac{1}{Z} = 4\coth^2\frac{E}{2T}$ (d) $\frac{1}{Z} = 2\tanh^2\frac{E}{4T}$

Q49. The temperature dependence of specific heat of two metals A and B, both with quadratic dispersion relations are shown in the figure below.



Which of the following statements is necessarily false?

(a) The density of states at Fermi energy of A is smaller than that of B.

(b) The effective mass of B is larger than that of A.

(c) The effective mass of A is smaller than that of B.

(d) The density of states at Fermi energy of B is smaller than that of A.

Q50. The Cartesian components of the electric field $\vec{E} = \{E_i | i = 1,2,3\}$ in a charge-free region of space are

$$E_i = C_i + \sum_j r_j D_{ji}$$

where C_i and D_{ji}'s are constant. The matrix of constants D_{ji} is

(a) Symmetric and traceless (b) Symmetric but not traceless

(c) Anti-symmetric and traceless (d) Anti-symmetric but not traceless

Q51. An oscillating point dipole of moment $\vec{p}(t) = \hat{z}p_0 \cos\omega t$, generates time-dependent electric and magnetic fields. At distances r far away from the dipole, the vector potential due to this dipole, in SI units, is

$$\vec{A} = \hat{z}\frac{\mu_0 p_0 \omega}{4\pi r}\sin\omega\left(t - \frac{r}{c}\right)$$

The total power radiated from this dipole is

(a) $\frac{\mu_0 p_0^2 \omega^4}{12\pi c}$ (b) $\frac{\mu_0 p_0^2 \omega^4}{8\pi c}$ (c) $\frac{\mu_0 p_0^2 \omega^4}{16\pi^2 c}$ (d) $\frac{\mu_0 p_0^2 \omega^4}{24\pi c}$

Q52. A radioactive tritium atom in its ground state undergoes a beta decay

$${}_1^3H \rightarrow {}_2^3He^+ + e^-$$

where the ${}_2^3He$ nucleus is stable. The probability that this beta decay will be followed immediately by emission of a photon is

(a) 0.3 (b) Zero (c) 0.7 (d) 0.5

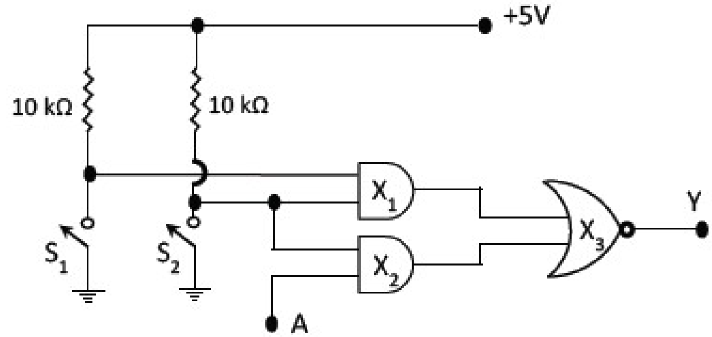
Q53. Consider the s-wave capture of a pion π^- by a deuteron d in its ground state, which then produces two neutrons, i.e.

$$\pi^- + d \rightarrow n + n$$

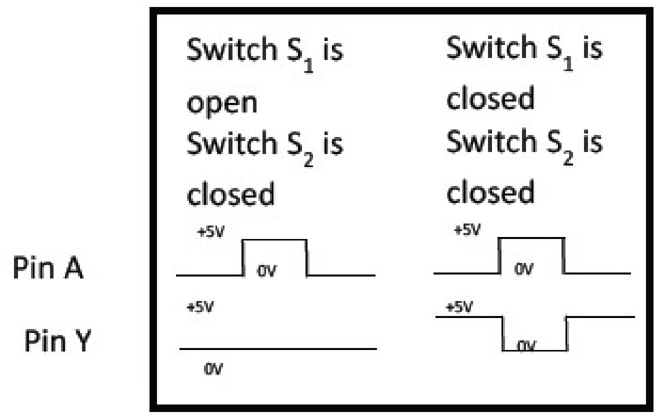
If we consider the two neutrons in the final state, they will satisfy

(a) $L = 1, S = 1$ (b) $L = 0, S = 1$ (c) $L = 1, S = 0$ (d) $L = 0, S = 1$

Q54. A sealed box containing a digital circuit has a circuit diagram pasted on its lid as shown below.



However, the output of the circuit is not as per this diagram. Some of the outputs actually obtained were as shown below



Based on this we can conclude that the actual circuit inside has

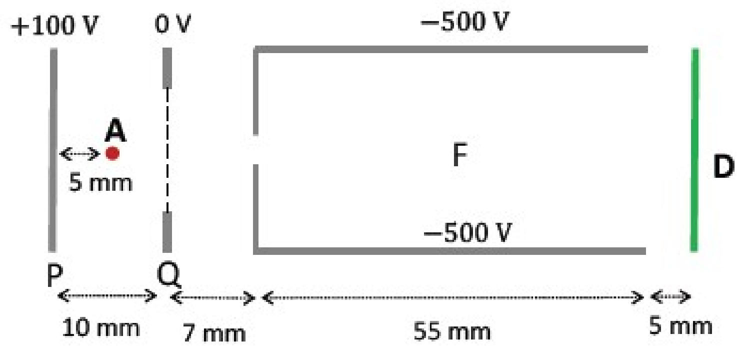
(a) OR gates instead of AND gates (X_1 and X_2)

(b) NAND gate instead of NOR gate (X_3)

(c) OR gate instead of NOR gate (X_3)

(d) AND gate instead of NOR gate (X_3)

Q55. The following picture shows the cross-sectional view of an apparatus used to detect O^+ ions produced in an experiment. The entire apparatus is kept in vacuum.



F is a conducting cylinder with a small aperture at its entrance. The electric potentials between various electrodes (except the detector D) are shown in the figure. The O^+ ions are generated in its ground state at point A with negligible kinetic energy. The wire mesh on the electrode Q allows ions to pass through, and also maintains a uniform electric field in the PQ region. The dimensions of the spectrometer are shown in the figure.

The ions are detected by the detector D situated at the end of the flight tube. If the detector can only detect ions with kinetic energy more than $550eV$, the potential on the detector to detect the O^+ ions must be

(a) more negative than $500V$

(b) more positive than $500V$

(c) zero, as the ions already have enough energy to be detected

(d) more positive than $-500V$