PREVIOUS YEAR'S SOLUTION

CSIR-NET DECEMBER 2023

Scan For Video Solutions

CSIR-NET Physics Course

CSIR-NET December 2023

Part A

Q1. All the four entries in column A must be matched with all those in column B. Each correctly matched option gets one mark and no mark is awarded otherwise. Which of the following mark(s) CANNOT be scored?

1. 3 2. 1 3. 2 4. 4

Ans.: 4

Q2. SCRIPT: DIRECTOR:: ??: CHEF

Choose the most appropriate option from the following to fill the blank

1. MENU 2. RECIPE 3. RESTAURANT 4. MEAL

Ans.: 1

Q3. A truck from a post office is sent to collect post from a plane as per schedule. The plane lands ahead of schedule, therefore its contents are transported by a rickshaw. The rickshaw meets the truck 30 minutes after the arrival of plane, and the post is transferred. The truck returns to the post office 20 minutes early. How early did the plane arrive? (Assume all transactions are instantaneous.)

1. 10 minutes 2. 20 minutes 3. 30 minutes 4. 40 minutes

Ans.: 4

Q4. Four children had 27 apples among them. No child had less than 5 apples. If no two children had the same number of apples, then which of the following could NOT be the number of apples a child had?

1. 5 2. 6 3. 8 4. 9

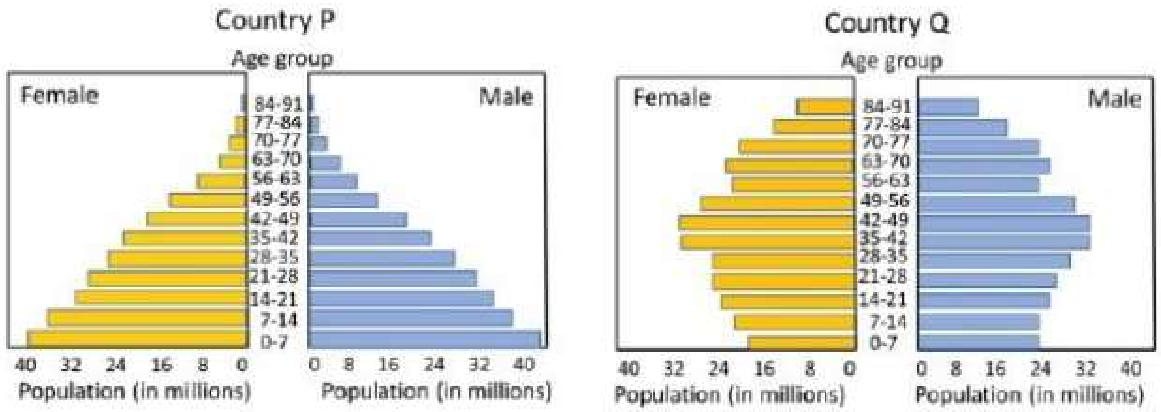
Ans.: 3

Q5. A bird keeps flying continuously between two trains, that are following each other on a straight track. The train behind is slower than the one ahead by 1.5 km/h. If the speed of the bird is 20 km/h, what distance would the bird cover in an hour?

1. 20 km 2. 30 km 3. 50 km 4. 60 km

Ans.: 3

Q6. The figure shows age-wise bar graph of male and female population of two countries. Which one



of the following is likely to be true?

1. Country Q has higher life expectancy

2. Country P has higher per-capita income

3. The population of country P is decreasing more rapidly than Q

4. Country P has better health facilities

Ans.: 3

Q7. Radius of a sphere is measured with 5% uncertainty. What is the uncertainty in the volume, determined from this radius?

1. 5% 2. 6.6% 3. 125% 4. 15%

Ans.: 4

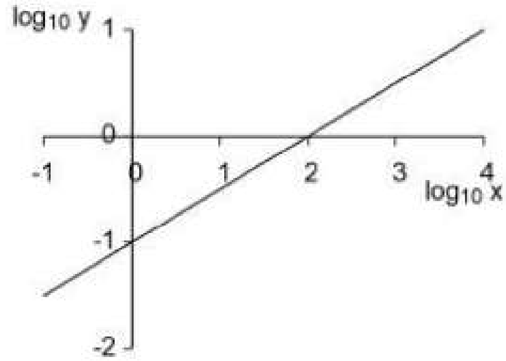
Q8. What is the value of x in the given magic square, (i.e, a square grid in which the sum of the numbers in rows, columns and diagonals is the same)?

x	$x-5$	8
$x+1$	y	$y-2$
2	9	4

1. 6 2. 4 3. 3 4. 1

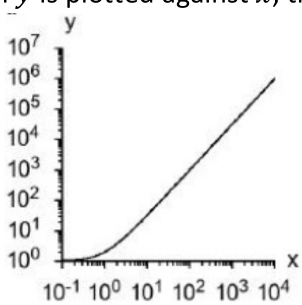
Ans.: 4

Q9. In the figure $\log_{10} y$ is plotted against $\log_{10} x$

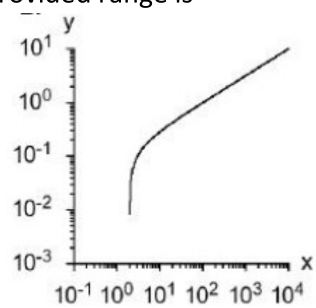


When y is plotted against x, then the plot in the provided range is

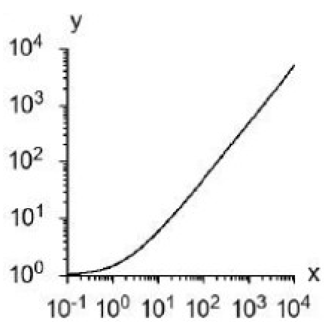
(a)



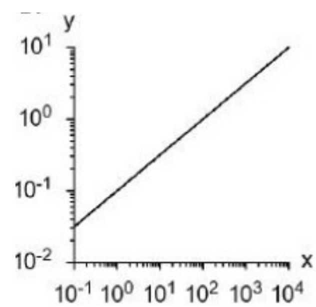
(b)



(c)



(d)



1. A 2. B 3. C 4. D

Ans.: 1

Q10. In a market, you can buy a mango for Rs. 10, a lemon for Re 1 and 8 chillies for Re 1. How many of these items do you need to buy to get a mix of 100 items for exactly Rs. 100?

1. 6 mangoes, 22 lemons, 72 chillis
2. 7 mangoes, 21 lemons, 72 chillis
3. 1 mango, 9 lemons, 80 chillis
4. 8 mangoes, 12 lemons, 80 chillis

Ans.: 3

Q11. In how many ways can a menu be made from 5 dishes, if the menu contains either 3 or 4 dishes?

1. 2 2. 3 3. 7 4. 15

Ans.: 3

Q12. The sum of the two positive integers is 14. Then their product CANNOT be divisible by

1. 12 2. 13 3. 14 4. 49

Ans.: 2

Q13. If $a < x < b$, then for which of the following relations does $0 < y < 1$ always hold?

1. $y = \frac{a-x}{b+a}$ 2. $y = \frac{x-a}{b-a}$ 3. $y = \frac{x-b}{b-a}$ 4. $y = \frac{b-x}{a+b}$

Ans.: 3

Q14. A letter is drawn at random from the following string of letters.

RAMUKYAJNAS

What is the probability that it is NOT a vowel?

1. 1/2 2. $6/11$ 3. $7/11$ 4. $8/11$

Ans.: 1

Q15. For every 5 chocolates that Ramesh gets, Suresh gets 3 chocolates. Geeta gets 3 chocolates for every 2 chocolates that Suresh gets. If Geeta has 18 chocolates, then the sum of chocolates with Ramesh and Suresh is

1. 16 2. 30 3. 32 4. 38

Ans.: 2

Q16. A person's viral load measured in some unit was 15,25,50,200,300,150 and 30 on days 1 to 7, respectively. The maximum relative change took place between

1. day 3 to day 4 2. day 4 to day 5 3. day 5 to day 6 4. day 6 to day 7

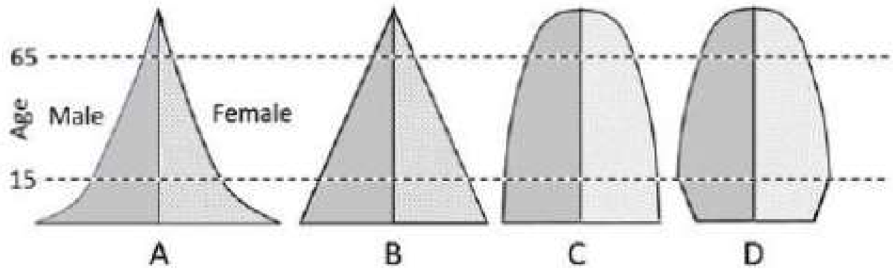
Ans.: 1

Q17. What is the minimum number of pourings needed to get 4 litre of milk from a fully filled 8 litre can, using ungraduated empty 5 and 3 litre cans? No milk should be wasted.

1. 4 2. 5 3. 6 4. 8

Ans.: 1

Q18.



The above figures show population pyramids to four countries A, B, C and D. The country showing the most stable population is

1. C 2. A 3. B 4. D

Ans.: 1

Q19. The time seen in a mirror placed opposite a numberless analog (with hands) wall clock is 4h 55 min. What approximately is the correct time?

1. 4h 55 min 2. 5h 05 min 3. 7h 05 min 4. 1h 35 min

Ans.: 2

Q20. In 1979, Ramesh's age was the sum of the digits of his year of birth. In 2017, on his birthday, what was his age?

1. 49 3. 57 3. 60 4. 64

Ans.: 3

PART B

Q1. A small bar magnet is placed in a magnetic field $B(\vec{r}) = B(x)\hat{z}$. The magnet is initially at rest with its magnetic moment along $\hat{y}$. At later times, it will undergo

1. angular motion in the yz plane and translational motion along $\hat{y}$
2. angular motion in the yz plane and translational motion along $\hat{x}$
3. angular motion in the zx plane and translational motion along $\hat{z}$
4. angular motion in the xy plane and translational motion along $\hat{z}$

QID: 705015

Topic: Electromagnetic Theory

Sub topic: Magnetism

Ans.: 2

Solution**:** Since we are interested to see the rotation,

The torque τ experienced by a magnetic dipole in a magnetic field is given by:

$$\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$$

Now, initially the magnetic moment of the bar magnet is initially along the $\hat{y}$ direction and the magnetic field is $B(x)\hat{z}$, the torque experienced by the magnet will be in the $\hat{y} \times \hat{z} = \hat{x}$ direction. This torque will cause the magnet to undergo angular motion in the yz plane.

Now, to see the translational motion, the force $\mathbf{F}$ experienced by the bar magnet in a magnetic field is given by:

$$\boldsymbol{F} = \nabla(\boldsymbol{m} \cdot \boldsymbol{B})$$

Since the gradient of $B(x)$ with respect to (x) is non-zero and the magnetic moment of the bar magnet is along $\hat{y}$, the force experienced by the magnet will have a component along the $\hat{x}$ direction. This will cause translational motion along the $\hat{x}$ direction.

So, the correct option is: 2.

Q2. Each allowed energy level of a system of non-interacting fermions has a degeneracy M. If there are N fermions and R is the remainder upon dividing N by M, then the degeneracy of the ground state is

1. R^M 2. 1 3. M 4. ${}^M C_R$

QID: 705019

Topic: Thermodynamics and Statistical Mechanics

Sub topic: Identical Particle

Ans.: 4

Solution: If there are N fermions and M degenerate states of each energy level for ground state of the system (not lowest energy level), all levels with degeneracy will be filled. Uppermost level will have R fermions to be filled in M degenerate levels and that will be filled in ${}^{M}C_{R}$ ways. That is the degeneracy of the ground state.

Q3. The Beta function is defined as $B(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}dt$.

Then $B(x,y+1)+B(x+1,y)$ can be expressed as

1. $B(x,y-1)$ 2. $B(x+y,1)$ 3. $B(x+y,x-y)$ 4. $B(x,y)$

QID: 705011

Topic: Mathematical Physics

Sub topic: Special Function

Ans.: 4

Solution: The Beta function is defined as, $B(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}dt$

$$\mathrm{B(x,y+1)}=\int_0^1 \mathrm{t^{x-1}(1-t)^{(y+1)-1}\ dt}=\int_0^1 \mathrm{t^{x-1}(1-t)^{y}\ dt}$$

$$\mathrm{B(x+1,y)}=\int_0^1 \mathrm{t^{(x+1)-1}(1-t)^{y-1}dt}=\int_0^1 \mathrm{t^{x}(1-t)^{y-1}\ dt}$$

Now, let′s add these two expressions together:

$$\mathrm{B(x,y+1)+B(x+1,y)}=\int_0^1 \mathrm{t^{x-1}(1-t)^{y}\ dt}+\int_0^1 \mathrm{t^{x}(1-t)^{y-1}\ dt}$$

$$=\int_0^1 [t^{x-1}(1-t)^{y}+t^{x}(1-t)^{y-1}]\,dt=\int_0^1 t^{x-1}(1-t)^{y-1}[(1-t)+t]\,dt=B(x,y)$$

Q4. A particle moves in a circular orbit under a force field given by $\vec{F}(\vec{r})=-\frac{k}{r^2}\hat{r}$, where k is a positive constant. If the force changes suddenly to $\vec{F}(\vec{r})=-\frac{k}{2r^2}\hat{r}$, the shape of the new orbit would be

1. parabolic 2. Circular 3. elliptical 4. hyperbolic

QID: 705005

Topic: Classical Mechanics

Sub topic: Central Force Problem

Ans.: 1

Solution: If force is $F=-\frac{k}{r^2}\hat{r}$ and orbit is circular then $\frac{J^2}{mr^3}=\frac{k}{r^2}$ (condition for circular orbit)

$J^2=mkr$ if $r=r_0$ is radius of circle then $J^2=mkr_0$.

Now if $\vec{F}(\vec{r}) = -\frac{k}{2r^2}\hat{r}$ is force then potential is is $V(r) = -\dfrac{k}{2r}$

Total energy in new orbit is given by $E = \dfrac{1}{2}m\dot{r}^2 + \dfrac{J^2}{2mr^2} - \dfrac{k}{2r}$ hence energy changes suddenly state of system is not change $r = r_0$ $\dot{r} = 0$ so $E = \dfrac{J^2}{2mr_0^2} - \dfrac{k}{2r_0}$

Put $J^2 = mkr_0$ $E = \dfrac{mkr_0}{2mr_0^2} - \dfrac{k}{2r_0} = 0$ so new orbit is parabola

Q5. The 1-dimensional Hamiltonian of a classical particle of mass m is

$$H = \frac{p^2}{2m}e^{-x/a} + V(x)$$

where a is a constant with appropriate dimensions. The corresponding Lagrangian is,

1. $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 e^{x/a} - V(x)$
2. $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 e^{-x/a} - V(x)$
3. $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2 e^{x/a} - V(x)$
4. $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2 e^{-x/a} - V(x)$

QID: 705002

Topic: Classical Mechanics

Sub topic: Hamiltonian

Ans.: 1

Solution: $H = \dfrac{p^2}{2m}e^{-x/a} - V(x)$

$$L = \dot{x}p - H = \dot{x}p - \frac{p^2}{2m}e^{-x/a} + V(x)$$

$\dfrac{\partial H}{\partial p} = \dot{x} \Rightarrow \dfrac{p}{m}\mathrm{e}^{-x/a} = \dot{x} \Rightarrow p = m\dot{x}\mathrm{e}^{x/a}$ so Lagrangian is $\dfrac{1}{2}m\dot{x}^2 e^{x/a} - V(x)$

Q6. A one-dimensional infinite long wire with uniform linear charge density λ, is placed along the z-axis. The potential difference $\delta V = V(\rho + a) - V(\rho)$, between two points at radial distances $\rho + a$ and ρ from the z-axis, where $a \ll \rho$, is closest to

1. $-\frac{\lambda}{2\pi\varepsilon_0}\frac{a^2}{\rho^2}$
2. $-\frac{\lambda}{2\pi\varepsilon_0}\frac{a}{\rho}$
3. $\frac{\lambda}{2\pi\varepsilon_0}\frac{a}{\rho}$
4. $\frac{\lambda}{2\pi\varepsilon_0}\frac{a^2}{\rho^2}$

QID 705017:

Topic: Electromagnetic Theory

Sub topic: Electrostatic

Ans.: 2

Solution: Using Gauss's theorem electric field due to infinite wire is given by,

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

Potential Difference in terms of Electric Field is given by:

$V_2 - V_1 = -\int_{r_1}^{r_2} \vec{E} \cdot d\vec{r}$

$V_{\rho+a} - V_\rho = -\int_{\rho}^{\rho+a} \frac{\lambda}{2\pi\epsilon_0 r} dr = -\frac{\lambda}{2\pi\epsilon_0} \ln\frac{(\rho+a)}{\rho} = -\frac{\lambda}{2\pi\epsilon_0} ln(1+\frac{a}{\rho}) = -\frac{\lambda}{2\pi\epsilon_0}\frac{a}{\rho}$

Approximation used $ln(1+x) \approx x$ For small value of x

Q7. A classical ideal gas is subjected to a reversible process in which its molar specific heat changes with temperature T as $C(T) = C_V + R\frac{T}{T_0}$. If the initial temperature and volume are T_0 and V_0, respectively, and the final volume is $2V_0$, then the final temperature is

1. $T_0/\ln 2$ 2. $2T_0$ 3. $T_0/[1-\ln 2]$ 4. $T_0[1+\ln 2]$

QID 705018:

Topic: Thermodynamics and Statistical Mechanics

Sub topic: Laws of thermodynamics

Ans.: 1

Solution: From the first law of thermodynamics,

$$dQ = dU + pdV$$

$$dQ = C_v dT + pdV$$

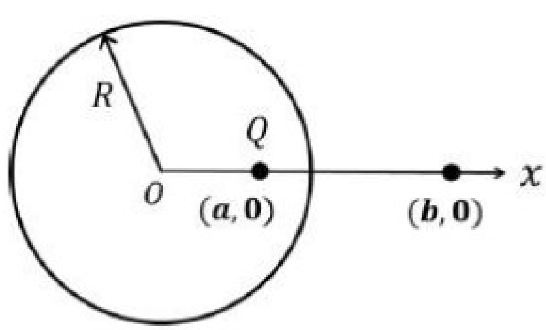
$$\left(C_V + R\frac{T}{T_0}\right) dT = C_v dT + pdV$$

$$\left(R\frac{T}{T_0}\right) dT = RT\frac{dV}{V}$$

$$\int_{T_0}^{T} \left(R\frac{1}{T_0}\right) dT = \int_{V_0}^{2V_0} R\frac{dV}{V}$$

$$T = T_0[1+\ln 2]$$

Q8. A conducting shell of radius R is placed with its centre at the origin as shown below. A point charge Q is placed inside the shell at a distance a along the x-axis from the centre.



The electric field at a distance $b > R$ along the x-axis from the centre is

1. $\frac{Q}{4\pi\varepsilon_0 b^2}\hat{x}$
2. $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(b-a)^2} - \frac{aR}{(ab-R^2)^2}\right]\hat{x}$
3. $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(b-a)^2} + \frac{aR}{(ab-R^2)^2}\right]\hat{x}$
4. $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{b^2} - \frac{R^2}{a^2b^2}\right]\hat{x}$

QID 705016:

Topic: EMT

Sub topic: Electrostatic

Ans.: 1

Solution: The inner shell will have $-Q$ charge due to induction and outer surface will have Q charge uniformly distributed on the inner and outer surface respectively.

Draw a gaussian surface passing through (b,0) and apply gauss theorem:

$$\oint \boldsymbol{E}\,.\,d\boldsymbol{a} = \frac{Q_{en}}{\varepsilon_0} = \frac{Q-Q+Q}{4\pi\varepsilon_0 b^2}\hat{x} = \frac{Q}{4\pi\varepsilon_0 b^2}\hat{x}$$

Q9. A particle of mass m is moving in a stable circular orbit of radius r_0 with angular momentum L. For a potential energy $V(r) = \beta r^k (\beta > 0$ and $k > 0)$, which of the following options is correct?

1. $k = 3, r_0 = \left(\frac{3L^2}{5m\beta}\right)^{1/5}$
2. $k = 2, r_0 = \left(\frac{L^2}{2m}\right)^{1/4}$
3. $k = 2, r_0 = \left(\frac{L^2}{4m\beta}\right)^{1/4}$
4. $k = 3, r_0 = \left(\frac{5L^2}{3m}\right)^{1/5}$

QID 705001:

Topic: Classical Mechanics

Sub topic: Central Force Problem

Ans.: 2

Solution: $V(r) = \beta r^k$ force $V_{effcective} = \frac{L^2}{2mr^2} + \beta r^k$

For circular orbit $\frac{\partial V_{effective}}{\partial r} = 0 \Rightarrow \frac{-L^2}{mr^3} + \beta k r^{k-1} = 0 \Rightarrow r = r_0 = \left(\frac{L^2}{m\beta k}\right)^{1/k+2}$

For stable orbit $\left.\frac{\partial^2 V_{effective}}{\partial r^2}\right|_{r=r_0} > 0$

$$\frac{3L^2}{mr^4} + \beta k(k-1)r^{k-2} > 0 \Rightarrow 3\beta k r^{k-2} + \beta k(k-1)r^{k-2} > 0$$

$$3 + k - 1 > 0 \Rightarrow k + 2 > 0 \Rightarrow k > -2$$

So best answer is 2

Q10. The light incident on a solar cell has a uniform photon flux in the energy range of $1\mathrm{eV}$ to $2\mathrm{eV}$ and is zero elsewhere. The active layer of the cell has a bandgap of $1.5\mathrm{eV}$ and absorbs 80% of the photons with energies above the bandgap. Ignoring non-radiative losses, the power conversion efficiency (ratio of the output power to the input power) is closest to

1. 47% 2. 70% 3. 23% 4. 35%

QID 705025:

Topic: Electronics and Experimental Method

Sub topic: Solar-Cell

Ans.: 1

Solution: Solar Cell active layer has a band gap of 1.5 eV so below that nothing will be absorbed.

If I consider total of 100 photons only 50 will be absorbed with 80% efficiency.

Power conversion efficiency = $\frac{(1.5+2)/2\times 50\times 0.80}{(1+2)/2\times 100}=0.4666$

Q11. The Schrödinger wave function for a stationary state of an atom in spherical polar coordinates (r,θ,ϕ) is

$$\psi = Af(r)\sin\,\theta\cos\,\theta e^{i\phi}$$

where A is the normalization constant. The eigenvalue of $\widehat{L_z}$ for this state is

1. $2\hbar$ 2. $\hbar$ 3. $-2\hbar$ 4. $-\hbar$

QID 705006:

Topic: Quantum Mechanics

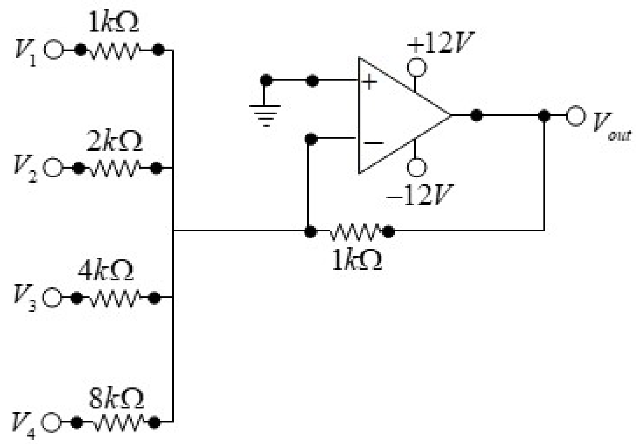
Sub topic: Angular Momentum

Ans.: 2

Solution The eigen value of $m\hbar$ with eigen function $f_1(r,\theta).\exp im\phi$

$\psi = Af(r)\sin\,\theta\cos\,\theta e^{i\phi}$ the value of $m=1$ so eigen value is $1\hbar$

Q12. In the circuit shown below using an ideal op-amp, inputs $V_j(j = 1,2,3,4)$ may either be open or connected to a -5 V battery.



The minimum measurement range of a voltmeter to measure all possible values of V_{out} is

1. 10V 2. 30V 3. 3V 4. 1V

QID 705024:

Topic: Electronics and Experimental Method

Sub topic: OP-AMP

Ans.: 1

Solution: $V_0 = -\frac{R_f}{R_1}V_1 - \frac{R_f}{R_2} - \frac{R_f V_3}{R_3} - \frac{R_f}{R_4}V_4$

$R_f = 1\text{k}\Omega,\ R_1 = 1\text{k}\Omega,\ R_2 = 2\text{k}\Omega$

$R_3 = 4\text{k}\Omega,\ R_4 = 8\text{k}\Omega$

If all input V_j are open, then

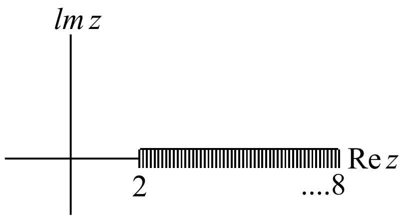
$$V_0 = 0$$

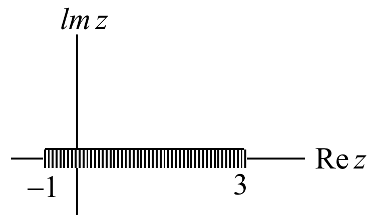
if all input V_j ore connected to $-5\ V$ buttery, then

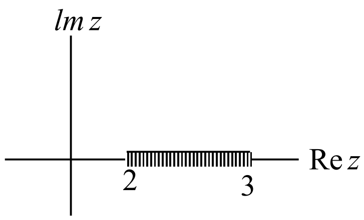
$$\begin{aligned} V_0 = &\ \frac{-1\times 10^3}{1\times 10^3}(-5) - \frac{1\times 10^3}{2\times 11^3}(-5) \\ &-\frac{1\times 10^3}{4\times 10^3}(-5) - \frac{1\times 10^3}{8\times 10^3}(-5) \\ = &\ 5 + 2.5 + 1.25 + 0.625 \\ = &\ 9.375\ \text{V} \end{aligned}$$

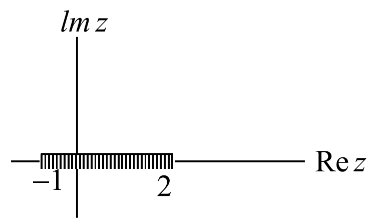
So minimum measurement range of a voltmeter to measure all possible value of $V_0 = 10\ \text{V}$

Q13. The branch line for the function $f(z) = \sqrt{((z^2 - 5z + 6)/(z^2 + 2z + 1))}$ is

1. 

2. 

3. 

4. 

QID 705010:

Topic: Mathematical Physics

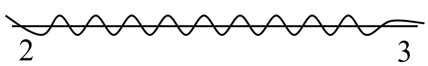
Sub topic: Complex Analysis

Ans.: 3

Solution: $f(z) = \sqrt{\dfrac{z^2 - 5z + 6}{z^2 + 2z + 1}}$

$$= \sqrt{\frac{z^2 - 2z - 3z + 6}{(z+1)^2}} = \frac{1}{(z+1)}\sqrt{z(z-2) - 3(z-2)} = \frac{1}{z+1}\sqrt{(z-3)(z-2)}$$

branch point $z = 3, 2$ are branch points and the curve connecting them is branch cut.



Q14. The coordinates of the following events in an observer's inertial frame of reference are as follows:

Event 1: $t_1 = 0, x_1 = 0$: A rocket with uniform velocity $0.5c$ crosses the observer at origin along x axis

Event 2: $t_2 = T, x_2 = 0$: The observer sends a light pulse towards the rocket

Event 3: t_3, x_3 : The rocket receives the light pulse

The values of t_3, x_3 respectively are

1. $2T, cT$
2. $2T, \frac{c}{2}T$
3. $\frac{\sqrt{3}}{2}T, \frac{2}{\sqrt{3}}cT$
4. $\frac{2}{\sqrt{3}}T, \frac{\sqrt{3}}{2}cT$

QID 705004:

Topic: Classical Mechanics

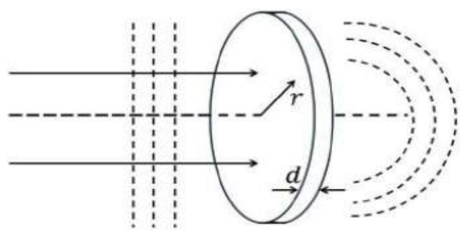
Sub topic: STR

Ans.: 1

Solution: Use kinematics equation $c\left(t_3-T\right)=0.5c\left(t_3-0\right)\Rightarrow t_3=2T$

$$x_3=0.5c\times t_3=cT$$

Q15. For a flat circular glass plate of thickness d, the refractive index $n(r)$ varies radially, where r is the radial distance from the centre of the plate. A coherent plane wavefront is normally incident on this plate as shown in the figure below.



If the emergent wavefront is spherical and centered on the axis of the plate, then $n(r)-n(0)$ should be proportional to

1. $r^{1/2}$ 2. r 3. r^2 4. $r^{3/2}$

QID 705014:

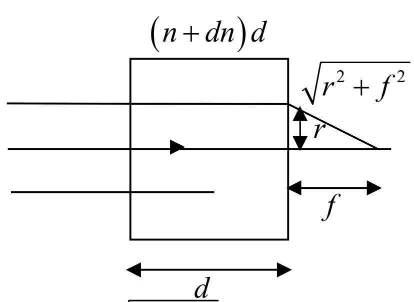
Topic: EMT

Sub topic: Optics

Ans.: 3

Solution: Here refractive index is varying with r i.e $n\left(r\right)$

This is spherical wave front so optical path different will same for any two point



(Behaving like lens)

$$\eta d+f=\left(x+dx\right)d+\sqrt{r^2+f^2}$$

$$f=dd\eta+\sqrt{r^2+f^2}$$

$$d\eta d=f-\sqrt{r^2+f^2}\text{ , }d\eta=\frac{f}{d}\left[1-\left(1+r^2/f^2\right)^{1/2}\right]$$

Refractive index is gradually decreasing $f>>r$

So, $\int_{\eta(0)}^{\eta(r)} d\eta = \frac{f}{d}\left[1-\left(1+r^2/2\right)2f^2\right]$

$\eta(r)-(0)=\frac{f}{d}\frac{r^2}{2f^2}$, $\eta(r)-(0)=-\frac{r^2}{2fd}$

Q16. The Hamiltonian for two particles with angular momentum quantum numbers $l_1 = l_2 = 1$, is

$$\hat{H} = \frac{\epsilon}{\hbar^2}\left[(\hat{L}_1+\hat{L}_2)\cdot\hat{L}_2 - \left(\hat{L}_{1z}+\hat{L}_{2z}\right)^2\right].$$

If the operator for the total angular momentum is given by $\hat{L} = \hat{L}_1 + \hat{L}_2$, then the possible energy eigenvalues for states with $l = 2$, (where the eigenvalues of $\widehat{L^2}$ are $l(l+1)\hbar^2$) are

1. $3\epsilon, 2\epsilon, -\epsilon$
2. $6\epsilon, 5\epsilon, 2\epsilon$
3. $3\epsilon, 2\epsilon, \epsilon$
4. $-3\epsilon, -2\epsilon, \epsilon$

QID 705007:

Topic: Quantum Mechanics

Sub topic: Angular Momentum

Ans.: 1

Solution: $H = \frac{\varepsilon}{\hbar^2}\left(\left(\vec{L}_1+\vec{L}_2\right).\vec{L}_2-\left(L_{1z}+L_{2z}\right)^2\right) = \frac{\varepsilon}{\hbar^2}\left(\vec{L}_1.\vec{L}_2+\left|L_2\right|^2-\left(L_{1z}+L_{2z}\right)^2\right)$

$\frac{\varepsilon}{\hbar^2}\left(\left(\frac{L^2-L_1^2-L_2^2}{2}\right)+L_2^2-\left(L_{1z}+L_{2z}\right)^2\right)$

$l_1=1, l_2=1, l=2,1,0$

For $l=2, l_1=1, l_2=1 \;\; l_{1z}=1, l_{2z}=1 l_{1z}=-1, l_{2z}=-1$

$E=\frac{\varepsilon}{\hbar^2}\left(\left(\frac{6\hbar^2-2\hbar^2-2\hbar^2}{2}\right)+2\hbar^2-\left(\hbar+\hbar\right)^2\right)=\varepsilon$

$l=2, l_1=1, l_2=1 \;\; l_{1z}=1, l_{2z}=0 \;\; or l_{1z}=0, l_{2z}=1, l_{1z}=-1, l_{2z}=0, l_{1z}=0, l_{2z}=-1$

$E=\frac{\varepsilon}{\hbar^2}\left(\left(\frac{6\hbar^2-2\hbar^2-2\hbar^2}{2}\right)+2\hbar^2-\left(\hbar\right)^2\right)=2\varepsilon$

$l=2, l_1=1, l_2=1 \;\; l_{1z}=1, l_{2z}=0 \;\; or l_{1z}=0, l_{2z}=0$

$E=\frac{\varepsilon}{\hbar^2}\left(\left(\frac{6\hbar^2-2\hbar^2-2\hbar^2}{2}\right)+2\hbar^2-\left(0\right)^2\right)=3\varepsilon$

Q17. A system of N non-interacting classical spins, where each spin can take values $\sigma = -1, 0, 1$, is placed in a magnetic field h. The single spin Hamiltonian is given by

$$H = -\mu_B h\sigma + \Delta(1-\sigma^2),$$

where μ_B, Δ are positive constants with appropriate dimensions.

If M is the magnetization, the zero-field magnetic susceptibility per spin $\frac{1}{N}\frac{\partial M}{\partial h}\Big|_{h\to 0}$, at a temperature $T = 1/\beta k_B$ is given by

1. $\beta\mu_B^2$ 2. $\frac{2\beta\mu_B^2}{2+e^{-\beta\Delta}}$ 3. $\beta\mu_B^2 e^{-\beta\Delta}$ 4. $\frac{\beta\mu_B^2}{1+e^{-\beta\Delta}}$

QID 705021:

Topic: Thermodynamics and Statistical Mechanics

Sub topic: Canonical Ensemble

Ans.: 2

Solution: $\sigma = -1, 0, 1$

$H = -\mu_B h\sigma + \Delta\left(1-\sigma^2\right)$. Draw energy levels using $\sigma = -1, 0, 1$ and $H = -\mu_B h\sigma + \Delta\left(1-\sigma^2\right)$

———————— $-\mu_B h$

———————— Δ

———————— $\mu_B h$

$$Z = e^{\beta\mu_B h} + e^{-\beta\mu_B h} + e^{-\beta\Delta}$$

$$F = -k_B T \ln\left(e^{\beta\mu_B h} + e^{-\beta\mu_B h} + e^{-\beta\Delta}\right)$$

$$dF = -SdT - PdV - \mu dh$$

$$\mu = -\left(\frac{\partial F}{\partial h}\right)_{T,V} = k_B T\frac{1}{e^{\beta\mu_B \hbar} + e^{-\beta\mu_B \hbar} + e^{-\beta\Delta}}(\beta\mu_B e^{\beta\mu_B h} - \beta\mu_B e^{-\beta\mu_B h})$$

$$= k_B T \quad \beta\mu_B\frac{\left(e^{\beta\mu_B h} - e^{-\beta\mu_B h}\right)}{e^{\beta\mu_B h} + e^{-\beta\mu_0 h} + e^{-\beta\Delta}}.$$

In the limit where h is small

$$M = \frac{\left(1+\beta\mu_B h - 1 + \beta\mu_B h\right)\mu_B}{2+e^{-\beta\Delta}} = \frac{\beta\mu_B{}^2}{2+e^{-\beta\Delta}}$$

Q18. The normalized wave function of an electron is

$$\psi(\vec{r}) = R(r)\left[\sqrt{\frac{3}{8}}Y_1^0(\theta,\varphi)\chi_- + \sqrt{\frac{5}{8}}Y_1^1(\theta,\varphi)\chi_+\right],$$

where Y_l^m are the normalized spherical harmonics and $\chi_\pm$ denote the wavefunction for the two spin states with eigenvalues $\pm\frac{1}{2}\hbar$. The expectation value of the z component of the total angular momentum in the above state is

1. $-\frac{3}{4}\hbar$ 2. $\frac{3}{4}\hbar$ 3. $-\frac{9}{8}\hbar$ 4. $\frac{9}{8}\hbar$

QID 705008:

Topic: Quantum Mechanics

Sub topic: Angular Momentum

Ans.: 2

Solution: The possible measurement of z component of total angular momentum $-\frac{\hbar}{2}$ with probability $\frac{3}{8}$ and $\frac{3}{2}\hbar$ so average

momentum $\langle J_z\rangle = -\frac{\hbar}{2}\times\frac{3}{8}+\frac{3}{2}\hbar\times\frac{5}{8} = \frac{-3\hbar+15\hbar}{16} = \frac{12\hbar}{16} = \frac{3}{4}\hbar$

Q19. Four distinguishable particles fill up energy levels $0, \epsilon, 2\epsilon$. The number of available microstates for the total energy 4ϵ is

1. 20 2. 24 3. 11 4. 19

QID 705020:

Topic: Thermodynamics and Statistical Mechanics

Sub topic: Identical Particles

Ans.: 4

Solution: Let us consider those microstates where total energy is 4ϵ. Three cases that may occur where energy is 4ϵ have been categorized in the following table. We must see in how many ways these cases may occur.

	A	AB
$ABCD$	BC	
	D	CD
${}^4C_4 = 1$	${}^4C_1\times{}^3C_2\times{}^1C_1 = 12$	${}^4C_2\times{}^2C_2 = 6$

So, the total number of microstates is 1+12+6 =19.

Q20. If z is a complex number, which among the following sets is neither open nor closed?

1. $\{z|0 \le |z-1| \le 2\}$
2. $\{z||z| \le 1\}$
3. $\{z \mid z \in (\mathbb{C}-\{3\}) \text{ and } |z| \le 100\}$
4. $\left\{z \mid z = re^{i\theta}, 0 \le \theta \le \frac{\pi}{4}\right\}$

QID 705012:

Topic: Mathematical

Sub topic: Complex Number

Ans.: 3

Solution: 1. $\{z|0 \le |z-1| \le 2\}$ Closed circle center 1 radius 2

2. $\{z||z| \le 1\}$ closed circle radius less than equal to 1

3. $\{z \mid z \in (\mathbb{C}-\{3\}) \text{ and } |z| \le 100\}$

Neither open nor closed. As it it closed on the side of 100 but we can go as close as possible to three but cannot touch it.

4. $\left\{z \mid z = re^{i\theta}, 0 \le \theta \le \frac{\pi}{4}\right\}$ Open

Q21. A particle of unit mass subjected to the 1-dimensional potential

$$V(x) = \frac{2\alpha}{x^3} - \frac{3\beta}{x^2}$$

executes small oscillations about its equilibrium position, where α and β are positive constants with appropriate dimensions. The time period of small oscillations is

1. $\frac{\pi\alpha^2}{\sqrt{6\beta^5}}$
2. $\frac{\pi\alpha^2}{\sqrt{3\beta^5}}$
3. $\frac{2\pi\alpha^2}{\sqrt{3\beta^5}}$
4. $\frac{2\pi\alpha^2}{\sqrt{6\beta^5}}$

QID 705003:

Topic: Classical Mechanics

Sub topic: Stability Analysis

Ans.: 4

Solution: $V(x) = \frac{2\alpha}{x^3} - \frac{3\beta}{x^2}$ for equilibrium point $\frac{\partial V}{\partial x} = 0 \Rightarrow -\frac{6\alpha}{x^4} + \frac{6\beta}{x^3} = 0 \Rightarrow x_0 = \frac{\alpha}{\beta}$

$$\omega = \sqrt{\frac{\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=x_0}}{m}} \qquad \frac{\partial^2 V}{\partial x^2} = \frac{24\alpha}{x^5} - \frac{18\beta}{x^4} \text{ put } x_0 = \frac{\alpha}{\beta} \quad \frac{\partial^2 V}{\partial x^2} = \frac{24\alpha\beta^5}{\alpha^5} - \frac{18\beta^5}{\alpha^4} = \frac{6\beta^5}{\alpha^4}$$

$$\frac{2\pi}{T} = \left(\frac{6\beta^5}{\alpha^4}\right)^{1/2} \Rightarrow T = \frac{2\pi\alpha^2}{(6\beta^5)^{1/2}}$$

Q22. For three inputs A, B and C, the minimum number of 2-input NAND gates required to generate the output $Y = \overline{A + B} + \bar{C}$ is

1. 3 2. 4 3. 7 4. 6

QID 705023:

Topic: Electronics and Experimental Method

Sub topic: Digital

Ans.: 2

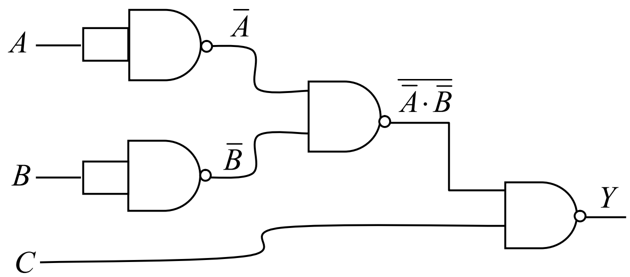
Solution:

$$\mathrm{Y} = \overline{A + B} + \bar{C}$$

$$\mathrm{Y} = \overline{\overline{\overline{\bar{A} \,.\, \bar{B} + \bar{C}}}}$$

$$\mathrm{Y} = \overline{\overline{\bar{A} \,.\, \bar{B}} \,,\, \bar{\bar{C}}}$$

$$\mathrm{Y} = \overline{\overline{\bar{A} \,.\, \bar{B}} \,,\, C}$$



Q23. A quantum system is described by the Hamiltonian

$$H = JS_z + \lambda S_x$$

where $S_i = \frac{\hbar}{2}\sigma_i$ and $\sigma_i (i = x, y, z)$ are the Pauli matrices. If $0 < \lambda \ll J$, then the leading correction in λ to the partition function of the system at temperature T is

1. $\frac{\hbar\lambda^2}{2Jk_BT}\coth\left(\frac{J\hbar}{2k_BT}\right)$ 2. $\frac{\hbar\lambda^2}{2Jk_BT}\tanh\left(\frac{J\hbar}{2k_BT}\right)$ 3. $\frac{\hbar\lambda^2}{2Jk_BT}\cosh\left(\frac{J\hbar}{2k_BT}\right)$ 4. $\frac{\hbar\lambda^2}{2Jk_BT}\sinh\left(\frac{J\hbar}{2k_BT}\right)$

QID 705009:

Topic: Thermodynamics and Statistical Mechanics

Sub topic: Canonical Ensemble

Ans.: 4

Solution: write the Hamiltonian using Pauli spin matrices,

$$H = \begin{pmatrix} \frac{J\hbar}{2} & \frac{\lambda\hbar}{2} \\ \frac{\lambda\hbar}{2} & \frac{-j\hbar}{2} \end{pmatrix}$$

Eigen values are $= \pm\sqrt{J^2 + \lambda^2}\,\frac{\hbar}{2}$

$$z = e^{-\beta\sqrt{j^2+\lambda^2}\frac{\hbar}{2}} + e^{\beta\sqrt{j^2+\lambda^2}\hbar/2}$$

$$= e^{-\frac{\beta\hbar J}{2}\left(1+\frac{\lambda^2}{J^2}\right)^{1/2}} + e^{\frac{\beta\hbar J}{2}\left(1+\frac{\lambda^2}{J^2}\right)^{1/2}} = e^{-\frac{\beta\hbar j}{2}\left(1+\frac{\lambda^2}{2j^2}\right)} + e^{\frac{\beta\hbar \mathrm{j}}{2}\left(1+\frac{\lambda^2}{2j^2}\right)}$$

$$= e^{-\frac{\beta\hbar j}{2}} \cdot e^{\frac{-\beta\hbar\lambda^2}{4j}} + e^{\frac{\beta\hbar j}{2}} \cdot e^{\frac{\beta\hbar\lambda^2}{4J}} = e^{-\frac{\beta\hbar y}{2}}\left(1 - \frac{\beta\hbar\lambda^2}{4j}\right) + e^{\frac{\beta\hbar j}{2}}\left(1 + \frac{\beta\hbar\lambda^2}{4y}\right)$$

$$= \left(e^{\frac{\beta\hbar j}{2}} + e^{\frac{-\beta\hbar j}{2}}\right) + \frac{\beta\hbar\lambda^2}{4j}\left(e^{\frac{\beta\hbar J}{2}} - e^{-\frac{\beta\hbar J}{2}}\right)$$

$$\text{correction term} = \frac{\beta\hbar\lambda^2}{4j}\left(e^{\frac{B\hbar J}{2}} - e^{-\frac{\beta\hbar J}{2}}\right)$$

$$= \frac{\hbar\lambda^2}{2k_BTJ}\sin\, h\left(\frac{\hbar J}{2k_BT}\right)$$

Q24. Let M be a 3×3 real matrix such that

$$e^{M\theta} = \begin{bmatrix} \cos\,\theta & \sin\,\theta & 0 \\ -\sin\,\theta & \cos\,\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where θ is a real parameter. Then M is given by

1. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 2. $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 3. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 4. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

QID 705013:

Topic: Mathematical Physics

Sub topic: Matrices

Ans.: 2

Solution: $e^M = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\det e^M = \cos^2\theta + \sin^2\theta = 1$$

$$\det e^M = e^{TraceM}$$

Option 3 and 4 are wrong

$$1.\ M = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix} -1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = 0$$

$$-(1-\lambda)(1-\lambda)(-\lambda) = 0, \qquad (1+\lambda)(1-\lambda)\lambda = 0$$

$$\lambda = 0, \lambda = 1, -1, e^0, e^1, \quad \det = e^2$$

2. $M=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ -1 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix}=0$$

$$-\alpha\left(\lambda^2-0\right)+1(\lambda)=0, e^0, e^{-i}, e^i$$

$$\lambda^3+\lambda=0 \qquad \det=1$$

$$\lambda-(2+1)=0,\ \lambda=0,-i,i$$

Q25. In the measurement of a radioactive sample, the measured counts with and without the sample for equal time intervals are $C=500$ and $B=100$, respectively. The errors in the measurements of C and B are $|\Delta C|=20$ and $|\Delta B|=10$, respectively. The net error $|\Delta Y|$ in the measured counts from the sample $Y=C-B$, is closest to

1. 22 2. 10 3. 30 4. 43

QID 705022:

Topic: Electronics and Experimental Method

Sub topic: Error Analysis

Ans.: 1

Solution: $Mean\ Count\ C\ = 500$

$Mean\ Background\ noise\ \ B=100,$

$|\Delta C|=20$ and $|\Delta B|=10$

Measurement is done in equal time intervals. Assuming, mean values are for Poisson distribution.

The net error $|\Delta Y|$ in the measured counts from the sample $Y=C-B$, is

$$|\Delta Y|=\sqrt{(|\Delta C|)^2+(|\Delta B|)^2}=\sqrt{(20)^2+(10)^2}=\sqrt{500}=22.3\ \approx 22$$

PART - C

Q1. A canonical transformation from the phase space coordinates (q, p) to (Q, P) is generated by the function

$$\psi(p, Q) = \frac{p^2}{2\omega}\tan\ 2\pi Q,$$

where ω is a positive constant. The function $\psi(p, Q)$ is related to $F(q, Q)$ by the Legendre transform $\psi = pq - F$, where F is defined by $dF = pdq - PdQ$. If the solution for (P, Q) is

$$P(t) = \frac{\omega}{4\pi}t^2, Q(t) = Q_0 = \text{ constant},$$

where t is time, then the solution for (p, q) variables can be written as

1. $p = \frac{\omega t}{2\pi}\cos\ 2\pi Q_0, q = \frac{t}{2\pi}\sin\ 2\pi Q_0$
2. $p = -\frac{\omega t}{2\pi}\cos\ 2\pi Q_0, q = \frac{t}{2\pi}\sin\ 2\pi Q_0$
3. $p = \frac{\omega t}{2\pi}\sin\ 2\pi Q_0, q = \frac{t}{2\pi}\cos\ 2\pi Q_0$
4. $p = -\frac{\omega t}{2\pi}\sin\ 2\pi Q_0, q = \frac{t}{2\pi}\cos\ 2\pi Q_0$

QID 705027:

Topic: Classical Mechanics

Sub topic: Generating Function

Ans.: 1

Solution: $\psi = pq - F \Rightarrow d\psi = pdq + qdp - dF$ where $dF = pdq - PdQ$

$$pdq + qdp - pdq + PdQ \Rightarrow d\psi = qdp + PdQ$$

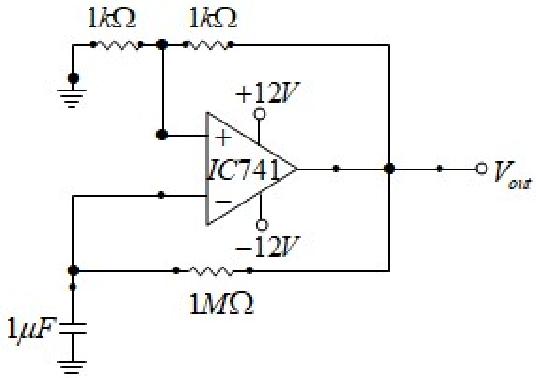
$\frac{\partial \psi}{\partial p} = q$ and $\frac{\partial \psi}{\partial Q} = P$ where $\psi(p, Q) = \frac{p^2}{2\omega}\tan 2\pi Q$, $\frac{\partial \psi}{\partial p} = q \Rightarrow q = \frac{p}{\omega}\tan(2\pi Q)$

and $\frac{\partial \psi}{\partial Q} = P \Rightarrow \frac{p^2}{2\omega}.2\pi \sec^2 2\pi Q \Rightarrow p = \sqrt{\frac{\omega P}{\pi}}\cos(2\pi Q) = \sqrt{\frac{\omega}{\pi}.\frac{\omega t^2}{4\pi}}\cos(2\pi Q_0)$

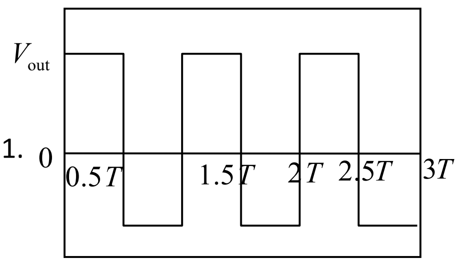
$= p = \frac{\omega t}{2\pi}\cos(2\pi Q_0) \qquad q = \frac{p}{\omega}\tan(2\pi Q_0) = q = \frac{\omega t}{2\pi}\cos(2\pi Q_0).\frac{1}{\omega}\tan(2\pi Q_0)$

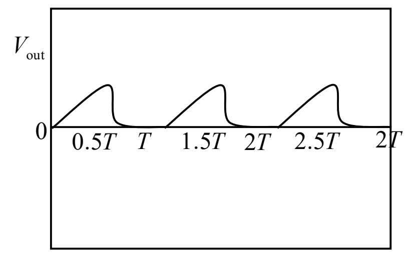
$$q = \frac{t}{2\pi}\sin(2\pi Q_0)$$

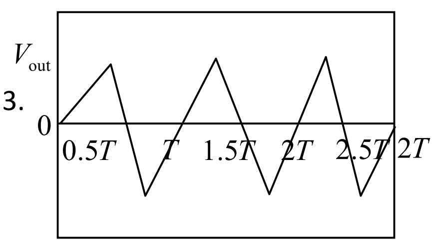
Q2. A circuit with operational amplifier is shown in the figure below.

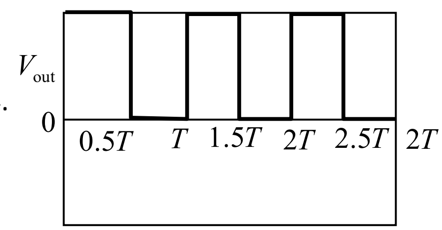


The output voltage waveform V_{out} will be closest to

1.


2.


3.


4.


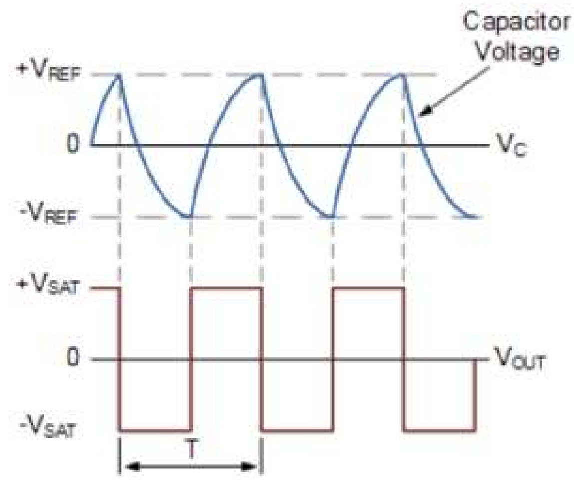
QID 705045:

Topic: Electronics

Sub topic: Op-Amp

Ans.: 1

Solution: The circuit is standard op-amp multivibrator circuit. Let's assume that the capacitor is initially uncharged and output of the op-amp is saturated at the positive saturation voltage. The capacitor, C starts to charge up from the output voltage, Vout through resistor, R. As soon as the capacitors charging voltage at the op-amps inverting (-) terminal is equal to or greater than the voltage at the non-inverting terminal (the op-amps output change state and be driven to the opposing negative saturation voltage. Once the op-amps inverting terminal reaches the new negative reference voltage, -Vref at the non-inverting terminal, the op-amp once again changes state and the output is driven to the opposing supply voltage, +V(sat).



Q3. In the rotational-vibrational spectrum of an idealized carbon monoxide (CO) molecule, ignoring rotational-vibrational coupling, two transitions between adjacent vibrational levels with wavelength λ_1 and λ_2, correspond to the rotational transition from $J' = 0$ to $J'' = 1$, and $J' = 1$ to $J'' = 0$, respectively. Given that the reduced mass of CO is $1.2x^{-26}$ kg, equilibrium bond length of CO is 0.12 nm and vibrational frequency is 5×10^{13} Hz, the ratio of $\frac{\lambda_1}{\lambda_2}$ is closest to

1. 0.9963 2. 0.0963 3. 1.002 4. 1.203

QID 705052:

Topic: Atomic and Molecular

Sub topic: Rotational and Vibrational Spectra

Ans.: 3

Solution: Since there is no coupling between rotational and vibrational spectra. I will ignore that. During transition we will see the change in rotational energy.

As we know, Rotational energy in terms of wave no.

$$\varepsilon_J = BJ(J+1)$$

The change in wave no. during $J' = 0 \ to \ J'' = 1$ is

$$v'_1 = \varepsilon_{J''} - \varepsilon_{J'} = 2B'' = \frac{1}{\lambda_1} ....(1)$$

The change in wave no. during $J' = 1 \ to \ J'' = 0$ is

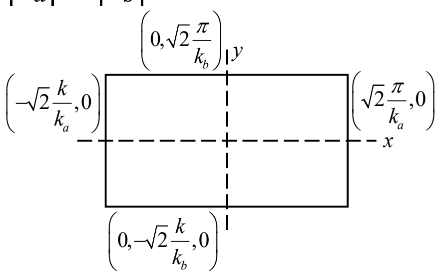
$$v'_2 = \varepsilon_{J'} - \varepsilon_{J''} = 2B' = \frac{1}{\lambda_2} \;....(2)$$

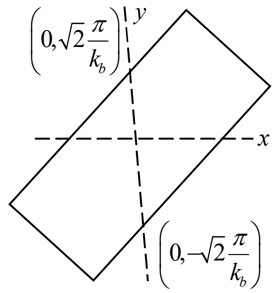
Divide (2) by (1) $\dfrac{\lambda_1}{\lambda_2} = \dfrac{B'}{B''} < 1 \qquad \because B \;\alpha\; \dfrac{1}{r_o^2}$

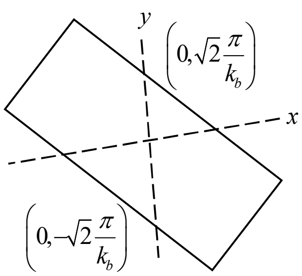
Q4. A 2-dimensional resonant cavity supports a TM mode built from a function

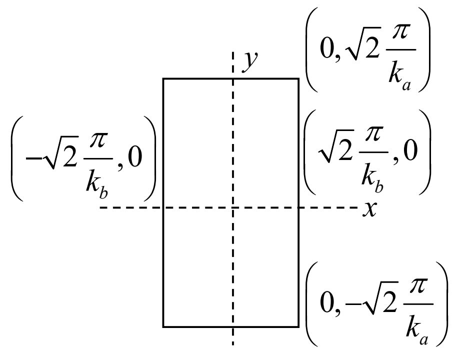
$$\psi(x,y,t) = \sin\left(\vec{k}_a \cdot \vec{r} - \omega t\right) + \sin\left(\vec{k}_b \cdot \vec{r} - \omega t\right) + \sin\left(\vec{k}_a \cdot \vec{r} + \omega t\right) + \sin\left(\vec{k}_b \cdot \vec{r} + \omega t\right)$$

where $\vec{k}_a$ and $\vec{k}_b$ lie in the xy-plane and make angles $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ with the x-axis, respectively. If $0 < |\vec{k}_a| < |\vec{k}_b|$, then which of the following closely describes the outline of the cavity?

1. 

2. 

3. 

4. 

QID 705040:

Topic: EMT

Sub topic: Wave-Guide

Ans. : 2

Q5. A quantum particle of mass m is moving in a one-dimensional potential

$$V(x) = V_0\theta(x) - \lambda\delta(x),$$

where V_0 and λ are positive constants, $\theta(x)$ is the Heaviside step function and $\delta(x)$ is the Dirac delta function. The leading contribution to the reflection coefficient for the particle incident from the left with energy $E \gg V_0 > \lambda$ and $\sqrt{2mE} \gg \frac{V_0 h}{\lambda}$ is

1. $\frac{V_0^2}{4E^2}$ 2. $\frac{V_0^2}{8E^2}$ 3. $\frac{m\lambda^2}{2Eh^2}$ 4. $\frac{m\lambda^2}{4Eh^2}$

QID 705031:

Topic: Quantum Mechanics

Sub topic: Dirac Delta Potential

Ans.: 3

Solution: $V(x)=V_0\theta(x)-\lambda\delta(x)$

$$V(x)=\begin{cases}0, x<0\\ V_0-\lambda\delta(x), x\geq 0\end{cases}$$

$$\psi(x)=\begin{cases}A\exp(ik_1x)+B\exp(-ik_1x), & x\leq 0\\ C\exp ik_2x, & x\geq 0\end{cases}$$

Where $k_1=\sqrt{\dfrac{2mE}{\hbar^2}}, k_2=\sqrt{\dfrac{2m(E-V_0)}{\hbar^2}}$

The wave function is continuous $A+B=C$

$\dfrac{-\hbar^2}{2m}\dfrac{\partial^2\psi}{\partial x^2}+(V_0-\lambda\delta(x))\psi(x)=E\psi(x)$ integrating both sides

$$\frac{-\hbar^2}{2m}\int_{-\varepsilon}^{\varepsilon}\frac{\partial^2\psi}{\partial x^2}dx+\int_{-\varepsilon}^{\varepsilon}(V_0-\lambda\delta(x))\psi(x)dx=\int_{-\varepsilon}^{\varepsilon}E\psi(x)dx$$

$$-\frac{\hbar^2}{2m}\frac{d\psi}{dx}\bigg|_{-\varepsilon}^{\varepsilon}+\int_{-\varepsilon}^{\varepsilon}V_0\psi(x)dx-\int_{-\varepsilon}^{\varepsilon}\lambda\psi(x)\delta(x)dx=\int_{-\varepsilon}^{\varepsilon}E\psi(x)dx$$

$$-\frac{\hbar^2}{2m}\frac{d\psi}{dx}\bigg|_{-\varepsilon}^{\varepsilon}+0-\lambda\psi(0)=0=-\frac{\hbar^2}{2m}\left(Cik_2-(Aik_1-Bik_1)\right)-\lambda(A+B)=0$$

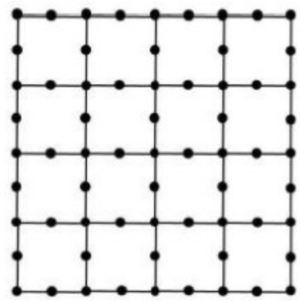
$$-\frac{\hbar^2}{2m}\left((A+B)ik_2-(Aik_1-Bik_1)\right)-\lambda(A+B)=0$$

$$A\left(i(k_2-k_1)+\frac{2m\lambda}{\hbar^2}\right)=B\left(-\frac{2m\lambda}{\hbar^2}-i(k_1+k_2)\right)\Rightarrow\frac{B}{A}=\frac{\left(i(k_2-k_1)+\frac{2m\lambda}{\hbar^2}\right)}{\left(-\frac{2m\lambda}{\hbar^2}-i(k_1+k_2)\right)}$$

$$R=\left|\frac{B}{A}\right|^2=\frac{\frac{4m^2\lambda^2}{\hbar^4}+(k_1-k_2)^2}{\frac{4m^2\lambda^2}{\hbar^4}+(k_1+k_2)^2}=\frac{\frac{4m^2\lambda^2}{\hbar^4}+\left(\sqrt{\frac{2mE}{\hbar^2}}-\sqrt{\frac{2m(E-V_0)}{\hbar^2}}\right)^2}{\frac{4m^2\lambda^2}{\hbar^4}+\left(\sqrt{\frac{2mE}{\hbar^2}}+\sqrt{\frac{2m(E-V_0)}{\hbar^2}}\right)^2}$$

$$\frac{\frac{4m^2\lambda^2}{\hbar^4}+\frac{2mE}{\hbar^2}\left(1-\sqrt{1-\frac{V_0}{E}}\right)^2}{\frac{4m^2\lambda^2}{\hbar^4}+\frac{2mE}{\hbar^2}\left(1+\sqrt{1-\frac{V_0}{E}}\right)^2}=\frac{\frac{4m^2\lambda^2}{\hbar^4}}{\frac{4m^2\lambda^2}{\hbar^4}+\frac{8mE}{\hbar^2}}=\frac{1}{1+\frac{2E\hbar^2}{m\lambda^2}}=\frac{m\lambda^2}{2E\hbar^2}$$

Q6. In the section of an infinite lattice shown in the figure below, all sites are occupied by identical hard circular discs so that the resulting structure is tightly packed.



The packing fraction is

1. $\frac{3\pi}{4}$ 2. $\frac{\pi}{4}$ 3. $\frac{3\pi}{16}$ 4. $\frac{9\pi}{16}$

QID 705049:

Topic: Condensed matter physics

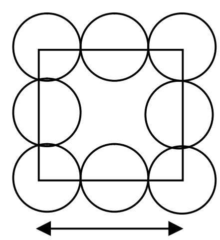
Sub topic: Crystallography

Ans.: 3

Solution: $a=4r$

$n_{eff}=3$

$$APF=\frac{3\times\pi r^2}{a^2}=\frac{3\pi a^2}{16a^2}=\frac{3\pi}{16}$$



Q7. Gauge factor of a strain gauge is defined as the ratio of the fractional change in resistance $\left(\frac{\Delta R}{R}\right)$ to the fractional change in length $\left(\frac{\Delta L}{L}\right)$. A metallic strain gauge with a gauge factor 2 has a resistance of 100Ω under unstrained condition. An aluminum foil with Young's modulus $Y=70\mathrm{GN/m^2}$ is installed on the metallic gauge. Keeping the foil within its elastic limit, a stress of $0.2\mathrm{GN/m^2}$ is applied on the foil. The change in the resistance of the gauge will be closest to

1. 0.14Ω 2. 1.23Ω 3. 0.28Ω 4. 0.56Ω

QID 705044:

Topic: Electronics and Experimental Method

Sub topic: Error Analysis

Ans.: 4

Solution: The gauge factor is defined as the ratio of per unit change in resistance to per unit change in length.

Gauge factor $G_f = \frac{\Delta R/R}{\Delta L/L}$

$$\frac{\Delta R}{R} = G_f \frac{\Delta L}{L} = G_f \varepsilon$$

Where ε = strain = $\Delta L/L$

$$\text{Young modulus} = \frac{stress}{strain}$$

Given that, gauge factor $(G_f) = 2$
Stress $(\varepsilon) = 0.2(\mathrm{GN/m^2})$

$\mathrm{Y} = 70(\mathrm{GN/m^2})$

$$\text{Strain } (\varepsilon) = \frac{0.2 \times 10^9}{70 \times 10^9} = 2.857 \times 10^{-3}$$

$$\frac{\Delta R}{R} = G_f \varepsilon$$
$$\Delta R = 2 \times 2.857 \times 10^{-3} \times 100 = 0.57\Omega$$

Q8. In a quantum harmonic oscillator problem, $\hat{a}$ and $\widehat{N}$ are the annihilation operator and the number operator, respectively. The operator $e^{\hat{N}} \hat{a} e^{-\hat{N}}$ is

1. $\hat{a}$ 2. $e^{-1}\hat{a}$ 3. $e^{-(\hat{I}+\hat{a})}$ 4. $e^{\hat{a}}$

(where $\hat{I}$ is the identity operator)

QID 705030:

Topic: Quantum Mechanics

Sub topic: Harmonic oscillator

Ans.: 2

Solution: $e^A B e^{-A} = B + [A,B] + \frac{1}{\lfloor 2}[A,[A,B]] + \frac{1}{\lfloor 3}[A,[A,[A,B]]].......$

Here $A = \hat{N}, B = \hat{a}$ and $[\hat{N},\hat{a}] = -\hat{a}$

$$e^{\hat{N}} \hat{a} e^{-\hat{N}} = \hat{a} + [\hat{N},\hat{a}] + \frac{1}{\lfloor 2}[\hat{N},[\hat{N},\hat{a}]] + \frac{1}{\lfloor 3}[N,[\hat{N},[\hat{N},\hat{a}]]].......$$

$$\hat{a} - \hat{a} + \frac{1}{\lfloor 2}[N,-\hat{a}] + \frac{1}{\lfloor 3}[N,[N,-\hat{a}]]...., \ \hat{a} - \hat{a} + \frac{\hat{a}}{\lfloor 2} - \frac{\hat{a}}{\lfloor 3}.... = \hat{a}\left[1 - 1 + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3}......\right] = \hat{a}e^{-1}$$

Q9. Atmospheric neutrinos are produced from the cascading decays of cosmic pions ($\pi^{\pm}$)to stable particles. Ignoring all other neutrino sources, the ratio of muon neutrino $(v_\mu + \bar{v}_\mu)$ flux to electron neutrino $(v_e + \bar{v}_e)$ flux in atmosphere is expected to be closest to

1. 2: 3 2. 1: 1 3. 1: 2 4. 2: 1

QID 705053:

Topic: Nuclear Physics

Sub topic: Particle Physics

Ans.: 4

Solution: Cosmic rays primarily consist of protons, which primarily decay into pions upon impact with the atmosphere of the Earth. The pions decay according to

$$\pi^+ \to \mu^+ + \nu_\mu$$

and

$$\pi^- \to \mu^- + \overline{\nu_\mu}$$

Then the most subsequent decay is the following

$$\mu^+ \to e^+ + \nu_e + \overline{\nu_\mu}$$

and $$\mu^- \to e^- + \overline{\nu_e} + \nu_\mu$$

The total no of muon neutrino $(v_\mu + \bar{v}_\mu)$ particles produced are 1+1+1+1=4

The total no of electron neutrino $(v_e + \bar{v}_e)$ particles produced are 0+0+1+1=2

This implies that the ratio of muon neutrino $(v_\mu + \bar{v}_\mu)$ flux to electron neutrino $(v_e + \bar{v}_e)$ flux in the atmosphere is 2:1

Q10. A system of non-relativistic and non-interacting bosons of mass m in two dimensions has a density n. The Bose-Einstein condensation temperature T_c is

1. $\frac{12n\hbar^2}{\pi m\ \ _B}$ 2. $\frac{3n\hbar^2}{\pi m k_B}$ 3. $\frac{6n\hbar^2}{\pi m k_B}$ 4. 0

QID 705041:

Topic: Thermodynamics and Statistical Mechanics

Sub topic: Bose-Einstein

Ans.: 4

Solution: For 2D System Bose einstein condensation can be only realized at theoretically $T = 0$

Q11. The lattice constant of the bcc structure of sodium metal is 4.22Å. Assuming the mass of the electron inside the metal to be the same as free electron mass, the free electron Fermi energy is closest to

1. 3.2eV
2. 2.9eV
3. 3.5eV
4. 2.5eV

QID 705047:

Topic: Condensed matter physics

Sub topic: Fermi energy

Ans.: 1

Solution: $a = 4.22\overset{0}{\text{A}}$ (Lattice constant)

Structure is BCC hence, effective number of atom is 2

$$\Rightarrow E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

$$\therefore n = \frac{N_{eff}}{a^3} = \frac{2}{\left(4.22\times 10^{-10}\right)^3} = \frac{2}{75.15\times 10^{-30}}$$

$$n = 0.0268\times 10^{30}$$

$$E_F = \frac{\left(6.67\times 10^{-34}\right)^2}{8\times(3.14)^2\times 9.1\times 10^{-31}}\left[3\times 3.14\times 3.14\times 0.0266\times 10^{30}\right]^{2/3}$$

$$E_F = \frac{44.48\times 10^{-68}}{717.77\times 10^{-31}}(0.8530)\times 10^{20}$$

$$E_F = \frac{37.94\times 10^{-48}\times 10^{31}}{717.77\times 1.6\times 10^{-19}}eV,\ E_F = \frac{3794}{1148.432}eV,\ E_F = 3.30eV$$

Q12. The regular representation of two nonidentity elements of the group of order 3 are given by

1. $\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix},\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$

2. $\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$

3. $\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix},\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$

4. $\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$

QID 705033:

Topic: Mathematical Physics

Sub topic: Group Theory

Ans.: 3

Solution: Check option 1

If two elements are nonidentity and they are elements of group of order 3 which means remaining elements must be identity. so, multiplication operation between two must be identity.

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$ Which is not Identity

Check option 2

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$ Which is not Identity

Check option 3

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$ Which is identity.

Check option 4

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ Which is not Identity

So, option 3 is correct.

Q13. A quantum system is described by the Hamiltonian

$$H = -J\sigma_z + \lambda(t)\sigma_x,$$

where $\sigma_i (i = x, y, z)$ are Pauli matrices, J and λ are positive constants ($J \gg \lambda$) and

$$\lambda(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ \lambda & \text{for} \quad 0 < t < T \\ 0 & \text{for} \quad t > T \end{cases}$$

At $t < 0$, the system is in the ground state. The probability of finding the system in the excited state at $t \gg T$, in the leading order in λ is

1. $\frac{\lambda^2}{8J^2}\sin^2 \frac{JT}{\hbar}$ 2. $\frac{\lambda^2}{J^2}\sin^2 \frac{JT}{\hbar}$ 3. $\frac{\lambda^2}{4J^2}\sin^2 \frac{JT}{\hbar}$ 4. $\frac{\lambda^2}{16J^2}\sin^2 \frac{JT}{\hbar}$

QID 705032:

Topic: Quantum Mechanics

Sub topic: Time Dependent Perturbation

Ans.: 2

Solution: $H = -J\sigma_z + \lambda(t)\sigma_x = \begin{pmatrix} -J & 0 \\ 0 & J \end{pmatrix} + \begin{pmatrix} 0 & \lambda(t) \\ \lambda(t) & 0 \end{pmatrix} = H_0 + H_P$

The ground state energy for $H_0 = E_i = -J$ with eigen state $|\phi_i\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

The first excited state energy for $H_0, E_f = J$ with eigen state $|\phi_f\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

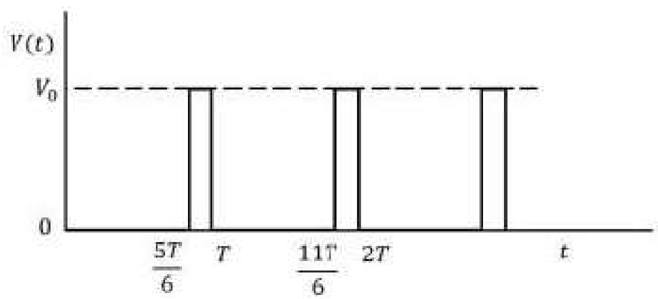
Transition probability is given by $P_{if} = \left| -\frac{1}{\hbar}\int_0^{\infty} \langle \phi_f | H_p | \phi_i \rangle \exp i\omega_0 t dt \right|^2$

It is given $\lambda(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ \lambda & \text{for} \quad 0 < t < T \\ 0 & \text{for} \quad t > T \end{cases}$ and $\omega_0 = \frac{E_f - E_i}{\hbar} = \frac{2J}{\hbar} =$

$$P_{if} = \left| -\frac{1}{\hbar}\int_0^{\infty} \langle \phi_f | H_p | \phi_i \rangle \exp i\omega_0 t dt \right|^2 = \frac{1}{\hbar^2}\left| \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2 \left| \int_0^T \exp i\omega_0 t dt \right|^2$$

$$\frac{\lambda^2}{\hbar^2} \frac{\sin^2 \frac{\omega_0 T}{2}}{\left(\frac{\omega_0}{2}\right)^2} = \frac{\lambda^2}{\hbar^2} \frac{\sin^2 \frac{2JT}{2\hbar}}{\left(\frac{2J}{2\hbar^2}\right)^2} = \frac{\lambda^2}{J^2} \sin^2 \frac{JT}{\hbar}$$

Q14. An infinite waveform $V(t)$ varies as shown in the figure below



The lowest harmonic that vanishes in the Fourier series of $V(t)$ is

1. 2
2. 3
3. 6
4. None

QID 705046:

Topic: Mathematical Physics

Sub topic: Fourier Series

Ans.: 3

Solution: The first step is to write the function seeing the graph,

$$f(t) = \begin{cases} 0 & t < 5T/6 \\ V_0 & 5T/6 < t < T \end{cases}$$

$$2L = T, \qquad L = T/2$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{T/2}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{T/2}\right)$$

$$a_n = \frac{1}{T/2}\int_{5T/6}^{T} V_0 \cos\left(\frac{2n\pi t}{T}\right) dt$$

$$= \frac{2V_0}{T}\left[\sin\left(\frac{2n\pi t}{T}\right)\right]_{\frac{5T}{6}}^{T} \times \frac{T}{2n\pi} = \frac{2V_0}{2n\pi}\left[\sin 2n\pi - \sin\frac{2n\pi 5}{6}\right] = \frac{2V_0}{2n\pi}\left[0 - \sin\frac{2n\pi 5}{6}\right]$$

For n=3 it will become zero.

The lowest harmonic that vanishes in the Fourier series of $V(t)$ is 3

Q15. A transmission line has the characteristic impedance of $(50+1j)\Omega$ and is terminated in a load resistance of $(70-7j)\Omega$ (where $j^2=-1$). The magnitude of the reflection coefficient will be closest to

1. $\frac{5}{7}$ 2. $\frac{1}{2}$ 3. $\frac{1}{6}$ 4. $\frac{1}{7}$

QID 705037

Topic: EMT

Sub topic: EM-Waves

Ans.: 3

Solution: To find the magnitude of the reflection coefficient, you can use the formula:

$$|\Gamma| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|$$

Where:

- (Z_L) is the load impedance

- (Z_0) is the characteristic impedance

Given:

- $(Z_L = 70 - 7j)$ ohms

- $(Z_0 = 50 + 1j)$ ohms

Let's substitute the values into the formula:

$$|\Gamma| = \left|\frac{(70-7j)-(50+1j)}{(70-7j)+(50+1j)}\right| = \left|\frac{20-8j}{120-6j}\right| = \frac{1}{6}$$

Q16. The function $f(z)=\frac{1}{(z+1)(z+3)}$ is defined on the complex plane. The coefficient of the $(z-z_0)^2$ term of the Laurent series of $f(z)$ about $z_0=1$ is

1. $\frac{7}{64}$ 2. $\frac{7}{128}$ 3. $\frac{9}{64}$ 4. $\frac{9}{128}$

QID 705035

Topic: Mathematical Physics

Sub topic: Complex variable

Ans.: 2

Solution: Taylor expansion is possible and the coefficients of $(z-z_0)^2$ about $z_0=1$ is $\frac{7}{128}$

Q17. The radius of a sphere oscillates as a function of time as $R+a\cos\,\omega t$, with $a<R$. It carries a charge Q uniformly distributed on its surface at all times. If P is the time averaged radiated power through a sphere of radius r, such that $r\gg R+a$ and $r\gg\frac{c}{\omega}$, then

1. $P\propto\frac{Q^2\omega^4a^2}{c^3}$ 2. $P\propto\frac{Q^2\omega^2}{c}$ 3. $P=0$ 4. $P\propto\frac{Q^2\omega^6a^4}{c^5}$

QID 705039

Topic: EMT

Sub topic: Radiation

Ans.: 3

Solution: Let $R(t)=R+a\cos\,(\omega t)$ be the radius of the sphere as a function of time, where R is the average radius, a is the amplitude of oscillation, ω is the angular frequency, and t is time.

The electric field E due to a uniformly charged sphere can be expressed as:

$$E=\frac{Q}{4\pi\epsilon_0r^2}$$

Where, For $r>R+a$, the electric field will be constant over time, as the rate of change of the radius does not affect the electric field at points beyond the maximum radius attainable by the pulsating sphere.

Thus, the time-averaged radiated power P through a sphere of radius r, such that $r\gg R+a$ and $r\gg c/\omega$, will be zero beyond the maximum attainable radius. This implies that $P=0$.

So, the correct option is $\mathrm{P}=0$.

Q18. A Lagrangian is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}\dot{z} + \dot{z}^2) - \alpha(2x + 3y + z).$$

The conserved momentum is

1. $m[2\dot{x} + \dot{z}]$ 2. $m[2\dot{x} + \dot{y} + \dot{z}]$ 3. $m\left[\dot{x} + \frac{3}{2}\dot{y} + \frac{1}{2}\dot{z}\right]$ 4. $m[2\dot{x} + 3\dot{z}]$

QID 705028

Topic: Classical Mechanics

Sub topic: Lagrangian

Ans.: 2

Solution: $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}\dot{z} + \dot{z}^2\right) - \alpha\left(2x + 3y + z\right)$

Using equation of motion

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right) = 0 \Rightarrow m\ddot{x} + 2\alpha = 0$ .....equation A

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \left(\frac{\partial L}{\partial y}\right) = 0 \Rightarrow \frac{m\ddot{z}}{2} + 3\alpha = 0$ .....equation B

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \left(\frac{\partial L}{\partial z}\right) = 0 \Rightarrow \frac{m\ddot{y}}{2} + m\ddot{z} + \alpha = 0$ ....equation C

A+C-B=0 $m\ddot{x} + \frac{m\ddot{y}}{2} + m\ddot{z} - \frac{m\ddot{z}}{2} = 0 \Rightarrow m\ddot{x} + \frac{m\ddot{y}}{2} + \frac{m\ddot{z}}{2} = 0$

$\Rightarrow \frac{d}{dt}m\left(2\dot{x} + \dot{y} + \dot{z}\right) = 0 \Rightarrow m\left(2\dot{x} + \dot{y} + \dot{z}\right) = c$

Q19. The solution $y(x)$ of the differential equation $y'' + \frac{y}{4} = \frac{x}{2}$, where $0 \le x \le \pi$, together with the boundary conditions $y(0) = y(\pi) = 0$ is

1. $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^n \frac{\pi}{n}\frac{\sin nx}{\frac{1}{4}-n^2}$ 2. $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^n \frac{\pi}{2n}\frac{\sin nx}{\frac{1}{4}-n^2}$

3. $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\pi}{n}\frac{\sin nx}{\frac{1}{4}-n^2}$ 4. $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\pi}{2n}\frac{\sin nx}{\frac{1}{4}-n^2}$

QID 705036

Topic: Mathematical Physics

Sub topic: Differential equation

Ans.: 4

Solution: First of all, lets solve the DE,

$y'' + \frac{y}{4} = x/2, 0 \leq x \leq \pi/2$

$y(0) = y(\pi) = 0$

Complementary function:

$D^2 + \frac{1}{4} = 0, D^2 = \frac{-1}{4}, D = \pm\frac{i}{2}$

Roots are complex.

$\alpha + i\beta;\ \ \beta = 1/2, \alpha = 0$

$y_{CF} = e^{0x}\left[A\cos\frac{\mathrm{x}}{2} + B\sin\frac{\mathrm{x}}{2}\right]$

Now, Particular Integral, $y_p = \frac{1}{D^2+\frac{1}{4}}(x/2)\ = 4[1+4D^2]^{-1}x/2$

$= 4[1 - 4D^2 + \cdots]x/2 = 4\frac{x}{2} = 2x$

$y = y_{CF} + y_{PE} = A\cos\ x/2 + B\sin\ x/2 + 2x$

Applying Boundary conditions

$y(0) = 0; 0 = A; \therefore y = B\sin\ x/2 + 2x$

$y(\pi) = 0; 0 = B\sin\ \pi/2 + 2\pi; B = -2\pi$

$y = -2\pi\sin\frac{x}{2} + 2x = 2\left[x - \pi\sin\frac{x}{2}\right]$

Now write Fourier Series for this function as the answer is given in the form of Fourier half series.

Let us calculate the first term is coming to be 8/3, 4/3, -8/3 and -4/3 respectively for all four options. From the given function we are getting -4/3. So, (4) is correct option. For your practice you should write the complete Fourier Series.

Q20. An incident plane wave with wavenumber k is scattered by a spherically symmetric soft potential. The scattering occurs only in S - and P-waves. The approximate scattering amplitude at angles $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{2}$ are

$$f\left(\theta = \frac{\pi}{3}\right) \simeq \frac{1}{2k}\left(\frac{5}{2} + 3i\right) \text{ and } f\left(\theta = \frac{\pi}{2}\right) \simeq \frac{1}{2k}\left(1 + \frac{3i}{2}\right).$$

Then the total scattering cross-section is closest to

1. $\frac{37}{4k^2}$ 2. $\frac{10}{k^2}$ 3. $\frac{35\pi}{4k^2}$ 4. $\frac{9\pi}{k^2}$

QID 705029

Topic: Quantum Mechanics

Sub topic: Scattering

Ans.: 1

Solution of Schrodinger wave equation in radiation zone after scattering $\psi(r,\theta) \approx A\left\{e^{ikz} + f(\theta)\frac{e^{ikr}}{r}\right\}$

Where $f(\theta) = \sum_{l=0}^{\infty}(2l+1)a_l P_l(\cos\theta)$

The total scattering cross-section is given by $\sigma = 4\pi\sum_{l=0}^{\infty}(2l+1)|al|^2$

$f(\theta) = \sum_{l=0}^{\infty}(2l+1)a_l P_l(\cos\theta)$ from given condition for $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{2}$

$$\frac{1}{2k}\left(\frac{5}{2}+3i\right) = a_0 + 3a_1\cos\left(\frac{\pi}{3}\right),\ \frac{1}{2k}\left(1+\frac{3i}{2}\right) = a_0 + 3a_1\cos\frac{\pi}{2}$$

$$a_0 = \frac{1}{2k}\left(1+\frac{3i}{2}\right)$$

$$\frac{1}{2k}\left(\frac{5}{2}+3i\right) = \frac{1}{2k}\left(1+\frac{3i}{2}\right) = \frac{3}{2}a_1$$

$$\frac{1}{2k}\left(\frac{5}{2}+3i-1-\frac{3i}{2}\right) = \frac{3}{2}a_1 \quad \Rightarrow \frac{1}{2k}\left(\frac{3}{2}+\frac{3i}{2}\right) = \frac{3}{2}a_1 \Rightarrow a_1 = \frac{1}{2k}(1+i)$$

$$\sigma = 4\pi\sum_l(2l+1)(a_l)^2$$

$$= 4\pi\times\left(1\times\frac{1}{4k^2}\left(1+\frac{3i}{2}\right)^2 + 3\times\frac{1}{4k^2}(1+i)^2\right) = \frac{4\pi}{4k^2}\left[1+\frac{9}{4}+3\times 2\right] = \frac{4\pi}{4k^2}\left(\frac{4+9+24}{4}\right) = \frac{37\pi}{4k^2}$$

Q21. A solar probe mission detects a fractional wavelength shift $(\Delta\lambda/\lambda)$ of the spectral line $\lambda = 630\text{ nm}$ within a sunspot to be of the order of 10^{-5}. Assuming this shift is caused by the normal Zeeman effect (i.e., neglecting other physical effects), the estimated magnetic field (in tesla) within the observed sunspot is closest to

1. 3×10^{-5} 2. 300 3. 0.3 4. 3×10^5

QID 705051

Topic: Atomic and Molecular Physics

Sub topic: Zeeman-Effect

Ans.: 3

Solution: Zeeman shift for Normal Zeeman effect

$$\Delta E = \mu_B B \Rightarrow \Delta \nu = \frac{\mu_B B}{h} \Rightarrow |\Delta\lambda| = \frac{\mu_B B \lambda^2}{hc}$$

$$\Rightarrow |\Delta\lambda| = \frac{\mu_B B \lambda^2}{hc} \Rightarrow B = \frac{|\Delta\lambda|}{\lambda}\frac{hc}{\mu_B \lambda} = \frac{10^{-5}\times 3\times 10^{8}\times 6.6\times 10^{-34}}{9.27\times 10^{-24}\times 630\times 10^{-9}} = 0.33$$

Q22. A photon inside the sun executes a random walk process. Given the radius of the sun $\approx 7 \times 10^{8}$ km and mean free path of a photon $\approx 10^{-3}$ m, the time taken by the photon to travel from the centre to the surface of the sun is closest to

1. 10^{6}sec 2. 10^{24}sec 3. 10^{12}sec 4. 10^{18}sec

QID 705043

Topic: Thermodynamics and Statistical Mechanics

Sub topic: Random-Walk Problem

Ans.: 3

Solution: In a random walk, the distance traveled is proportional to the square root of the number of steps taken. So, the number of steps the photon takes to travel from the center to the surface of the Sun is:

$$N = \left(\frac{R}{\lambda}\right)^2$$

where R is the radius of the Sun and λ is the mean free path of the photo.

Given that $R = 7 \times 10^{8}$ km and $\lambda = 10^{-3}$ m, we can calculate N as:

$$N = \left(\frac{7\times 10^{8}\ \text{km}}{10^{-3}\ \text{m}}\right)^2$$
$$N = (7\times 10^{11})^2$$
$$N = 49 \times 10^{22}$$

Time per step $= \frac{\lambda}{c}$
where c is the speed of light.
Time per step $= \frac{10^{-3}\ \text{m}}{3\times 10^{8}\ \text{m/s}}$

Time per step $= \frac{1}{3\times10^{11}}$ s

Now, to find the total time, we multiply the time per step by the number of steps:

Total time = Time per step $\times N$

Total time $= \frac{1}{3\times10^{11}} \times 49 \times 10^{22}$ s

Total time $\approx 1.633 \times 10^{11}$ s

Now, comparing this to the provided options:

1 10^{6}sec

2 10^{24}sec

3 10^{12}sec

4 10^{18}sec

The closest option is 10^{12}sec (option 3).

Q23. The ionization potential of hydrogen atom is 13.6eV, and λ_H and λ_D denote longest wavelengths in Balmer spectrum of hydrogen and deuterium atoms, respectively. Ignoring the fine and hyperfine structures, the percentage difference $y = \frac{\lambda_H - \lambda_D}{\lambda_H} \times 100$, is closest to

1. 1.0003% 2. −0.03% 3. 0.03% 4. −1.0003%

QID 705050

Topic: Atomic and Molecular Physics

Sub topic: Hydrogen Atom

Ans. : 3

Solution: Form Balmer series

$$\frac{1}{\lambda} = R^M\left[\frac{1}{2^2} - \frac{1}{n^2}\right]; \qquad R^M = \frac{\mu}{m_e} R^\infty$$

Form longest wavelength

$$\frac{1}{\lambda} = R^M\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

For hydrogen atoms, the longest Balmer Wavelength can be written as follows

$$\frac{1}{\lambda_H} = \frac{\mu_H}{m_e} R^\infty \frac{5}{36} = \frac{m_p}{m_p + m_e} R^\infty \frac{5}{36} \Rightarrow \lambda_H = \frac{m_p + m_e}{m_p} \frac{36}{5R^\infty}$$

Similarly, for deuterium, the longest Balmer Wavelength can be written as follows

$$\frac{1}{\lambda_D} = \frac{\mu_D}{m_e} R^\infty \frac{5}{36} = \frac{2m_p}{2m_p + m_e} R^\infty \frac{5}{36} = \lambda_D = \frac{2m_p + m_e}{2m_p} \frac{36}{5R^\infty}$$

$$\frac{\lambda_H-\lambda_D}{\lambda_H}\times 100=\frac{\dfrac{m_p+m_e}{m_p}-\dfrac{2m_p+m_e}{2m_p}}{\dfrac{m_p+m_e}{m_p}}\times 100=\left(1-\frac{1}{2}\frac{2\times 1836m_e+m_e}{1836m_e+m_e}\right)\times 100=0.03\%$$

Q24. The collision time of the electrons in a metal in the Drude model is τ and their plasma frequency is ω_p. If this metal is placed between the plates of a capacitor, the time constant associated with the decay of the electric field inside the metal is

1. $\tau+\frac{1}{\omega_p}$ 2. $\omega_p\tau^2$ 3. $\frac{1}{\omega_p^2\tau}$ 4. $\frac{\tau}{1+\omega_p\tau}$

QID 705048

Topic: Condensed matter physics

Sub topic: Electric Property of Metal

Ans.: 3

Solution: $\omega_p^2=\dfrac{ne^2}{\varepsilon_0 m}$

$$\because \mu=\frac{ne\tau}{m}$$

$$\omega_p^2=\frac{e}{\varepsilon_0}\cdot\frac{\mu}{\tau},\ \omega_p^2=\frac{e}{\varepsilon_0}\cdot\frac{1}{\tau}\cdot\frac{v_d}{E}\ (\mu=v/E)$$

$$E=\frac{ev_\lambda}{\omega_p^2\tau\varepsilon_0}$$

$$=\frac{1}{\omega_p^2\tau}\frac{1}{\varepsilon_0}ev_d \qquad \therefore I=neAv_d,\quad ev_d=\frac{I}{\pi A}$$

$$E=\frac{1}{\omega_p^2\tau}\cdot\frac{1}{\varepsilon_0}\cdot\frac{I}{nA},\ E=\frac{1}{\omega_p^2}\times\frac{1}{\varepsilon_0}\times\frac{ne}{n\cdot\pi r^2}=\frac{E_0}{\omega_p^2\tau},\ E\propto\frac{1}{\omega_p^2\tau},\ E_0=\frac{e}{\pi\varepsilon_0 r^2}$$

Q25. Given the data points

x	1	3	5
y	4	28	92

using Lagrange's method of interpolation, the value of y at $x=4$ is closest to

1. 54   2. 55   3. 53   4. 56

QID 705034

Topic: Mathematical Physics

Sub topic: Numerical Technique

Ans.: 2

Solution:

X	Y
1	4
3	28
5	92

$y(x=4)=?$

LaGrange's method gives

$$y(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y(x_0)+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y(x_1)+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y(x_2)$$

$x=4,\ x_0=1,\qquad y(x_0)=4$

$x_1=3,\qquad y(x_1)=28$

$x_2=5\qquad y(x_2)=92$

$$y(x)=\frac{(4-3)(4-5)(4)}{(1-3)(1-5)}+\frac{(4-1)(4-5)\times 28}{(3-1)(3-5)}+\frac{(4-1)(4-3)}{(5-4)(5-3)}\times 92=55$$

Q26. A particle of mass m is moving in a 3-dimensional potential

$$\phi(r)=-\frac{k}{r}-\frac{k'}{3r^3}\ k,k'>0.$$

For the particle with angular momentum l, the necessary condition to have a stable circular orbit is

1. $kk'<\frac{l^4}{4m^2}$
2. $kk'>\frac{l^4}{4m^2}$
3. $kk'<\frac{l^4}{m^2}$
4. $kk'>\frac{l^4}{m^2}$

QID 705026

Topic: Classical Mechanics

Sub topic: Central Force Problem

Ans.: 1

Solution: $V_{eff}=\frac{l^2}{2mr^2}-\frac{k}{r}-\frac{k'}{3r^3}$, where $k,k'>0$ $\frac{l^4}{m^2}=4kk'$

For circular orbit

$$\frac{\partial V_{eff}}{\partial r}=0,\qquad r_0=\frac{l^2}{m}$$

$$-\frac{l^2}{mr^3}+\frac{k}{r^2}+\frac{3k'}{3r^4}=0$$

$$\frac{1}{r^2}\left(-\frac{l^2}{mr}+k+\frac{3k'}{3r^2}\right)=0$$

$$kr^2-\frac{l^2}{m}r+k'=0$$

$$r=r_0=\frac{\frac{l^2}{m}\pm\sqrt{\frac{l^4}{m^2}-4kk}}{2k}$$ r_0 must be real so $\frac{l^4}{m^2}>4kk', \frac{l^4}{4m^2}>kk'$

For stable equilibrium $\left.\frac{\partial^2 V_{eff}}{\partial r^2}\right|_{r=r_0}>0$

$\frac{3l^2}{mr^4}-\frac{2k}{r^3}-\frac{4k'}{r^5}>0$ at $r=r_0$

$\frac{2l^2}{mr_0^4}-\frac{2k}{r_0^3}-\frac{2k'}{r_0^5}+\frac{l^2}{mr_0^4}-\frac{2k'}{r_0^5}>0$ we have $-\frac{l^2}{mr_0^3}+\frac{k}{r_0^2}+\frac{k'}{r_0^4}=0$

$$\frac{l^2}{mr_0^4}-\frac{2k'}{r_0^5}>0$$

$$\frac{l^2}{m}>\frac{2k'}{l^2/2mk'}, \frac{l^2}{4m^2}>kk'$$

Q27. The work done on a material to change its magnetization M in an external field H is $dW=HdM$. Its Gibbs free energy is

$$G(T,H)=-\left(\gamma T+\frac{aH^2}{2T}\right),$$

where $\gamma, a>0$ are constants. The material is in equilibrium at a temperature $T=T_0$ and in an external field $H=H_0$. If the field is decreased to $\frac{H_0}{2}$ adiabatically and reversibly, the temperature changes to

1. $2T_0$
2. $\frac{T_0}{2}$
3. $\left(\frac{a}{2\gamma}\right)^{\frac{1}{4}}\sqrt{H_0T_0}$
4. $\left(\frac{a}{\gamma}\right)^{\frac{1}{4}}\sqrt{H_0T_0}$

QID 705042

Topic: Thermodynamics and Statistical Mechanics

Sub topic: Thermal Potential

Ans.: 2

Solution: $G(T,H)=-\left(rT+\frac{aH^2}{2T}\right)$

at $H=H_0, T=T_0$

at $H=\frac{H_0}{2}, T=??$

$dG = -SdT + VdP$

First Case $S_1|_{H=H_0} = -\left(\frac{\partial G}{\partial T}\right)_p = -\left(r - \frac{aH^2}{2T^2}\right),\ S_1 = -\left(r - \frac{aH_0^2}{2T_0^2}\right)$

$$S_2|_{H=\frac{H_0}{2}} = -\left(r - \frac{aH_0}{4 \times (2T^2)}\right)$$

Since process is adiabatic as reversible

$\Rightarrow s_1 = s_2 \Rightarrow \frac{aH_0^2}{2T_0^2} = \frac{aH_0^2}{8T^2}; T^2 = \frac{T_0^2}{4}; T = \frac{T_0}{2}$

option (2)

Q28. The ground state of ${}^{207}_{82}$Pb nucleus has spin-parity $J^\pi = \left(\frac{1}{2}\right)^-$, while the first excited state has $J^\pi = \left(\frac{5}{2}\right)^-$. For the transition from the first excited state to the ground state, possible multipolarities of emitted electromagnetic radiation are

1. E2, E3 2. M2, M3 3. M2, E3 4. E2, M3

QID 705054

Topic: Nuclear Physics

Sub topic: Shell Model

Ans.:

Solution: $J_i = 5/2, \quad J_f = 1/2$

$|J_i - J_f| \le L \le |J_i + J_f|$

$\left|\frac{5}{2} - \frac{1}{2}\right| \le L \le \left|\frac{5}{2} + \frac{1}{2}\right|$

$2 \le L \le 3$

So, L will be 2 or 3

Parity, $\pi_f = \pi_i(-1)^L$ Electric multipole

$\pi_f = \pi_i(-1)^{L+1}$ Magnetic multipole

Since parity is not changing put $L = 2$ in electric multipole and $L = 3$ in magnetic multipole so answer will be E_2 or M_3

Q29. In a shell model description, neglecting Coulomb effects, which of the following statements for the energy and spin-parity is correct for the first excited state of $A = 12$ isobars ${}^{12}_{5}\mathrm{B}$, ${}^{12}_{6}\mathrm{C}$, ${}^{12}_{7}\mathrm{N}$?

1. same for ${}^{12}_{5}\mathrm{B}$, ${}^{12}_{6}\mathrm{C}$ and ${}^{12}_{7}\mathrm{N}$
2. different for each ${}^{12}_{5}\mathrm{B}$, ${}^{12}_{6}\mathrm{C}$ and ${}^{12}_{7}\mathrm{N}$
3. same for ${}^{12}_{6}\mathrm{C}$ and ${}^{12}_{7}\mathrm{N}$, but different for ${}^{12}_{5}\mathrm{B}$
4. same for ${}^{12}_{5}\mathrm{B}$ and ${}^{12}_{7}\mathrm{N}$, but different for ${}^{12}_{6}\mathrm{C}$

QID 705055

Topic: Nuclear Physics

Sub topic: Nuclear Property

Ans.: 4

Solution: $5^{B^{12}}$: First excited state

$1s_{1/2}^2,\quad 1p_{3/2}^3,\quad 1p_{1/2},\quad 1d_{5/2}$

$P \to$ excited state $1s_{1/2}^2,\quad 1p_{3/2}^3,\quad 1p_{1/2}^1$

$l_p = 1$

$J_P = 1/2$

$N \to$ Excited state $1s_{1/2}^2,\quad 1p_{3/2}^3,\quad 1p_{1/2}^2$

$\ln = 1$

$Jn = 3/2$

$|l_n + l_p + j_n + j_p|$

$= \left|\frac{1}{2} + \frac{3}{2} + 1 + 1\right|$

$J = 4 \qquad$ i.e., $4^+ = J^\pi$

Parity $= +1$

${}^{12}_{6}C : {}^{P-6}_{N-6}$ even-even nuclei

Spin parity $J^\pi = 0^+$

$P = 7$: First excited state

$1s_{1/2}^2,\quad 1p_{3/2}^3,\quad 1p_{1/2}^2$

$l_p = 1$

$J_p = 3/2$

$n = 5$: First excited state

$1s_{1/2}^2, \quad 1p_{3/2}^3, \quad 1p_{1/2}^1$

$l_n = 1$

$J_n = 1/2$

$J = |l_n + l_p + J_n + J_p|$

Parity $J = 4$, $= +1 \qquad J^\pi = 4^+$

Hence option 4 is correct.

Same for ${}_5^{12}B$ and ${}_7^{12}N$ but differ for ${}_6^{12}C$

Q30. The permittivity of a medium $\varepsilon(\vec{k}, \omega)$, where ω and $\vec{k}$ are the frequency and wavevector, respectively, has no imaginary part. For a longitudinal wave, $\vec{k}$ is parallel to the electric field such that $\vec{k} \times \vec{E} = 0$, while for a transverse wave $\vec{k} \cdot \vec{E} = 0$. In the absence of free charges and free currents, the medium can sustain

1. Longitudinal waves with $\vec{k}$ and ω when $\varepsilon(\vec{k}, \omega) > 0$
2. Transverse waves with $\vec{k}$ and ω when $\varepsilon(\vec{k}, \omega) < 0$
3. Longitudinal waves with $\vec{k}$ and ω when $\varepsilon(\vec{k}, \omega) = 0$
4. Both longitudinal and transverse waves with $\vec{k}$ and ω when $\varepsilon(\vec{k}, \omega) > 0$

QID 705038

Topic: EMT

Sub topic: EM-Wave

Ans.: 3

Solution: For a charge free medium $\vec{\nabla} \cdot \vec{D} = 0$, since $D = \varepsilon E$, $\vec{\nabla} \cdot \varepsilon \vec{E} = 0, \varepsilon \vec{\nabla} \cdot \vec{E} = 0, \varepsilon \vec{k} \cdot \vec{E} = 0$

$\vec{k} \cdot \vec{E} \neq 0$ (for longitudinal wave). So $\varepsilon = 0$

Hence option 3 is correct.

We can also take an example of plasma oscillation

$\varepsilon = 1 - \dfrac{\omega_p^2}{\omega^2}$ at $\omega = \omega_p, \varepsilon = 0$ (for longitudinal propagation)