**CSIR NET JUNE 2023**

**Part-A**

Q1. Which of the numbers $A = 162^3 + 327^3$ and $B = 612^3 - 123^3$ is divisible by 489?

(a) Both $A$ and $B$ (b) $A$ but not $B$

(c) $B$ but not $A$ (d) Neither $A$ nor $B$

Ans.: (a)

Q2. In a buffet, 4 curries A, B, C and D were served. A guest was to eat any one or more than one curry, but not the combinations having C and D together. The number of options available for the guest were

(a) 3 (b) 7 (c) 11 (d) 15

Ans.: (c)

Q3. Twenty litres of rainwater having a $2.0\,\mu\text{mol}/L$ concentration of sulfate ions is mixed with forty litres water having $4.0\mu$mol/L sulfate ions. If 50% of the total water evaporated, what would be sulfate concentration in the remaining water

(a) 3 $\mu$mol/L (b) 3.3 $\mu$mol/L (c) 4 $\mu$mol/L (d) 6.7 $\mu$mol/L

Ans.: (d)

Q4. Consider two datasets **A** and **B**, each with 3 observations, such that both the datasets have the same median. Which of the following MUST be true?

(a) Sum of the observations in **A** = Sum of the observations in **B**.

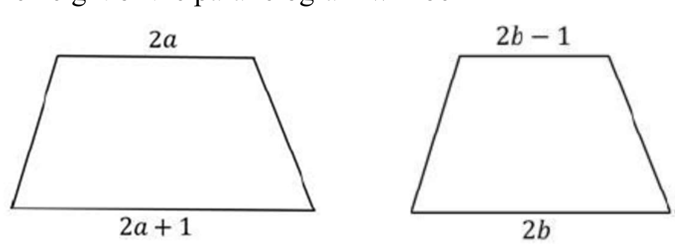
(b) Median of the squares of the observations in **A** = Median of the squares of the observations in B.

(c) The median of the combined dataset = median of **A** + median of **B**.

(d) The median of the combined dataset = median of **A**.

Ans.: (d)

Q5. If two trapeziums of the same height, as shown below, can be joined to form a parallelogram of area $2(a+b)$, then the height of the parallelogram will be




(a) 4 (b) 1 (c) 1/2 (d) 2

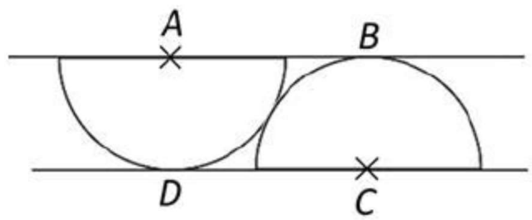
Ans.: (b)

Q6. Three consecutive integers $a, b, c,$ add to 15. Then the value of $(a-2)^2+(b-2)^2+(c-2)^2$ would be

(a) 25 (b) 27 (c) 29 (d) 31

Ans.: (c)

Q7. Two semicircles of same radii centred at A and C, touching each other, are placed between two parallel lines, as shown in the figure. The angle BAC is




(a) $30^0$ (b) $35^0$ (c) $45^0$ (d) $60^0$

Ans.: (a)

Q8. Three friends having a ball each stand at the three corners of a triangle. Each of them throws her ball independently at random to one of the others, once. The probability of two friends throwing balls at each other is

(a) 1/4 (b) 1/8 (c) 1/3 (d) ½

Ans.: (a)

Q9. A 50 litre mixture of paint is made of green, blue, and red colours in the ratio 5:3:2. If another 10 litre of red colour is added to the mixture, what will be the new ratio?

(a) 5:2:4 (b) 4:3:2 (c) 2:3:5 (d) 5:3:4

Ans.: (d)

Q10. A building has windows of sizes 2, 3 and 4 feet and their respective numbers are inversely proportional to their sizes. If the total number of windows is 26, then how many windows are there of the largest size?

(a) 4 (b) 6 (c) 12 (d) 9

Ans.: (b)

Q11. Given only one full 3 litre bottle and two empty ones of capacities 1 litre and 4 litres, all ungraduated, the minimum number of pourings required to ensure 1 litre in each bottle is

(a) 2 (b) 3 (c) 4 (d) 5

Ans.: (b)

Q12. At a spot S en-route, the speed of a bus was reduced by 20% resulting in a delay of 45 minutes. Instead, if the speed were reduced at 60 km after S, it would have been delayed by 30 minutes. The original speed, in km/h, was

(a) 90 (b) 80 (c) 70 (d) 60

Ans.: (d)

Q13. Three fair cubical dice are thrown, independently. What is the probability that all the dice read the same?

(a) 1/6 (b) 1/36 (c) 1/216 (d) 13/216

Ans.: (b)

Q14. Sum of all the internal angels of a regular octagon is _______ degrees.

(a) 360 (b) 1080 (c) 1260 (d) 900

Ans.: (b)

Q15. Persons A and B have 73 secrets each. On some day, exactly one of them discloses his secret to the other. For each secret A discloses to B in a given day, B discloses two secrets to A on the next day. For each secret B discloses to A in a given day, A discloses four secrets to Bon the next day. The one who starts, starts by disclosing exactly one secret. What is the smallest possible number of days it takes for B to disclose all his secrets?

(a) 5 (b) 6 (c) 7 (d)8

Ans.: (a)

Q16. When a student in Section A who scored 100 marks in a subject is exchanged for a student in Section B who scored 0 marks, the average marks of the Section A falls by 4 , while that of Section B increases by 5 . Which of the following statements is true?

(a) A has the same strength as B

(b) A has 5 more strength than B

(c) B has 5 more strength than A

(d) The relative strengths of the classes cannot be assessed from the data

Ans.: (b)

Q17. What is the largest number of father-son pairs that can exist in a group of four men?

(a) 3 (b) 2 (c) 4 (d) 6

Ans.: (a)

Q18. Price of an item is increased by 20% of its cost price and is then sold at 10% discount for Rs. 2160. What is its cost price?

(a) 1680 (b) 1700 (c) 1980 (d) 2000

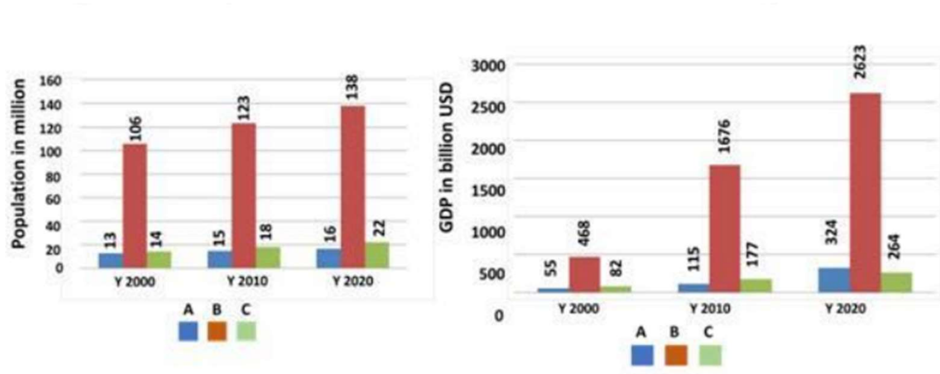
Ans.: (d)

Q19. If the sound of its thunder is heard 1s after a lightning was observed, how far away (in $\mathrm{m}$ ) was the source of thunder/lightning from the observer (given, speed of sound = $x\ ms^{-1}$ speed of light = $y\ ms^{-1}$)?

(a) $x^2/y$ (b) $xy/(y-x)$ (c) $xy/(x-y)$ (d) $y^2/x$

Ans.: (b)

Q20. The populations and gross domestic products (GDP) in billion USD of three countries A, B and C in the years 2000,2010 and 2020 are shown in the two figures below.




The decreasing order of per capita GDP of these countries in the year 2020 is

(a) $A, B, C$ (b) $A, C, B$ (c) $B, C, A$ (d) $C, A, B$

Ans.: (a)

**Part-B**

Q1. A uniform circular diss on the $xy$ plane with its canter at the origin has a moment of inertia $I_0$ about the $x$ - axis. If the disc is set in rotation about the origin with an angular velocity $\omega = \omega_0(\hat{j}+\hat{k})$ the direction of its angular momentum is along

(a) $-\hat{i}+\hat{j}+\hat{k}$ (b) $-\hat{i}+\hat{j}+2\hat{k}$ (c) $\hat{j}+2\hat{k}$ (d) $\hat{j}+\hat{k}$

Ans.: (c)

Solution: The moment of inertia of disc about an $z$ axis is $\frac{MR^2}{2}$.MI of disc about $x$ and $y$ axis is $\frac{MR^2}{4}=I_0$ the product of inertia is given by $0$ . the moment of inertia tensor is given by $\begin{bmatrix} I_0 & 0 & 0 \\ 0 & I_0 & 0 \\ 0 & 0 & 2I_0 \end{bmatrix}$ the angular velocity is given by $\vec{\omega}=\omega_0\begin{pmatrix}0\\1\\1\end{pmatrix}$ so angular momentum is

$\vec{L}=I\vec{\omega} \Rightarrow L = I_0\omega_0\begin{bmatrix}0\\1\\2\end{bmatrix}$ so angular momentum is $\hat{j}+2\hat{k}$

**Topic-Classical mechanics**

**Sub topic: moment of inertia tensor**

Q2. The locus of the curve $\text{Im}\left(\frac{\pi(z-1)-1}{z-1}\right)=1$ in the complex $z-$plane is a circle centered at ( $x_0\, y_0$ ) and $R-$respectively are

(a) $(1,\frac{1}{2})$ and $\frac{1}{2}$ (b) $(1,-\frac{1}{2})$ and $\frac{1}{2}$ (c) $(1,1)$ and $1$ (d) $(1,-1)$ and $1$

**Topic-Mathematical Physics**

**Sub topic: Complex analysis**

Ans. : (b)

Solution: $\frac{\pi(x+iy-1)+1}{x+iy-1}=\frac{(\pi x-\pi-1)+i\pi y}{(x-1)+iy}=\frac{((\pi x-\pi-1)+i\pi y)((x-1)-iy)}{(x-1)^2+y^2}$

$$\frac{\pi y(x-1)-y(\pi x-\pi-1)}{(x-1)^2+y^2}=\frac{\pi yx-\pi y-\pi yx+\pi y+y}{(x-1)^2+y^2}$$

$$\frac{y}{(x-1)^2+y^2}=1,\ (x-1)^2+y^2=y,(x-1)^2+y^2-y=0$$

$(x-1)^2+\left(y+\frac{1}{2}\right)^2=\frac{1}{4};$ Centre $\left(1,-\frac{1}{2}\right)$, Radius $=\frac{1}{2}$

Q3. The value of $\langle L_x^2 \rangle$ in the state $|\varphi\rangle$ for which $L_x^2|\varphi\rangle = 6\hbar^2|\varphi\rangle$ and $L_z|\varphi\rangle = 2\hbar|\varphi\rangle$ is

(a) 0 (b) $4\hbar^2$ (c) $2\hbar^2$ (d) $\hbar^2$

**Topic-Quantum Mechanics**

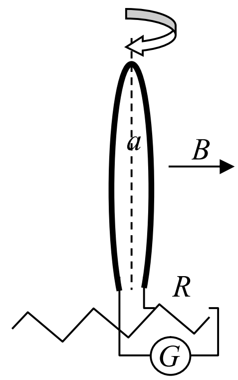
**Sub topic: Angular momentum**

Ans.: (d)

Solution: $L^2|\phi\rangle = 6\hbar^2|\phi\rangle, L_z|\phi\rangle = 2\hbar|\phi\rangle$ so $|\phi\rangle = Y_2^2$ , so $l = 2, m = 2$

$\langle L_x^2 \rangle$ on any $Y_l^m$ is $\dfrac{l(l+1)\hbar^2 - m^2\hbar^2}{2} = \dfrac{6\hbar^2 - 4\hbar^2}{2} = \hbar^2$

Q4. A small circular wire loop of radius a and number of turns $N$, is oriented with its axis parallel to the direction of the local magnetic field $B$. A resistance and Galvano meter are connected to the coil as shown in then figure




When the coil is flipped (i.e. the direction of its axis is reversed) the galvanometer measures the total charge Q that flow through it. If the induce emf through the coil $E_F = IR$ then $Q$ is

(a) $\pi Na^2B/2R$ (b) $\pi Na^2B/R$ (c) $\sqrt{2}\pi Na^2B/R$ (d) $2\pi Na^2B/R$

**Topic-EMT**

**Sub topic: Faraday's law**

Ans.: (d)

Solution: Change in flux $BA - (-BA) = 2BA$

$$\varepsilon = N\frac{d\phi_B}{dt} \Rightarrow IR = N\frac{d\phi_B}{dt}$$

$$\Delta QR = N\Delta\phi_B \Rightarrow \Delta Q = \frac{N\Delta\phi_B}{R} \Rightarrow \Delta Q = \frac{2NB\pi a^2}{R}$$

Q5. The dispersion relation of a gas of non-interacting bosons in two dimensions is $E(k) = c\sqrt{k}$ where c is a positive constant. At low temperatures, the leading dependence of the specific heat on temperature T is

(a) $T^4$ (b) $T^3$ (c) $T^2$ (d) $T^{3/2}$

**Topic-Solid state**

**Sub topic: Heat capacity**

Ans.: (a)

Solution: From the given equation

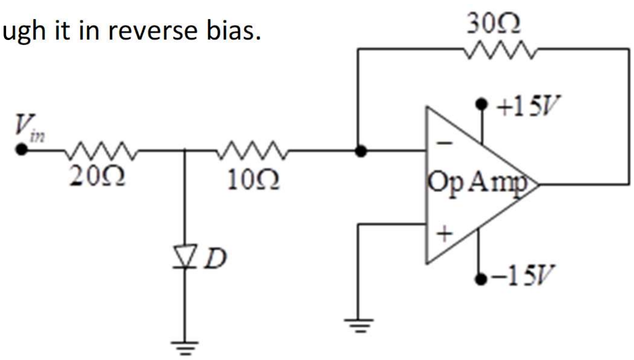
$$E(k) = c\sqrt{k}$$

We can say $\omega \alpha \sqrt{k} \Rightarrow n = \frac{1}{2}$

Now the specific heat in d dimension d can be written as $C = T^{\frac{d}{n}}$

In $2-d$, $C = T^{\frac{2}{1/2}} = T^4$

Q6. In the circuit below, there is a voltage drop of 0.7 V across the diode in forward bias while no current flows through it in reverse bias.




In $V_{in}$ is a sinusoidal signal of frequency $50\,Hz$ with rms value of $1V$ the maximum current that flows through the diode is closest to

(a) $1A$ (b) $0.14\ A$ (c) $0\ A$ (d) $0.07\ A$

**Topic-Electronics**

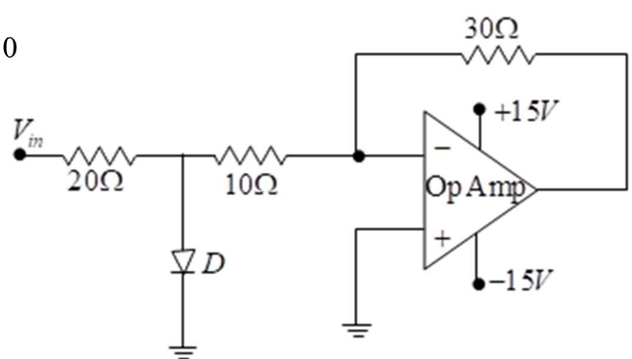
**Sub topic: Op-amp**

Ans.: (c)

Solution: Given that,

$$V_{in} = V_o \sin(\omega t),\quad V_{rms} = 1V,\quad V_o = \sqrt{2}\,V = 1.44V,\ f = 50\ Hz,\ \frac{V_{in} - V_D}{20} = \frac{V_D - 0}{10}$$

$$V_D = \frac{V_{in}}{3} = \frac{1.44}{3} < 0.7V \Rightarrow I = 0$$



Q7. The trajectory of a particle moving in a plane is expressed in polar coordinates $(r,\theta)$ by the equation $r = r_0 e^{\beta t}$ and $\frac{d\theta}{dt} = \omega$ where the parameters $r_0$, $\beta$ and $\omega$ are positive. Let $v_r$ and $a_r$ denote the velocity and acceleration, respectively, in the radial direction. For this trajectory

(a) $a_r < 0$ at all times irrespective of the values of the parameters

(b) $a_r > 0$ at all times irrespective of the values of the parameters

(c) $\frac{dv_r}{dt} > 0$ and $a_r > 0$ for all choices of parameters

(d) $\frac{dv_r}{dt} > 0$ however, $a_r = 0$ for some choices of parameters

**Topic- Classical mechanics**

**Sub topic: newton's law in polar coordinate**

Ans.: (d)

Solution: $r = r_0 \exp \beta t$ and $\frac{d\theta}{dt} = \omega$

$$v_r = \frac{dr}{dt} = r_0 \beta \exp \beta t \Rightarrow \frac{dv_r}{dt} = r_0 \beta^2 \exp \beta t > 0$$

$a_r = \ddot{r} - \dot{\theta} r = r_0 \beta^2 \exp \beta t - \omega^2 r_0 \exp \beta t \Rightarrow \left(\beta^2 - \omega^2\right) r_0 \exp \beta t = \left(\beta^2 - \omega^2\right) r$ which will zero for $\omega = \beta$

Q8. A long cylindrical wire of radius $R$ and conductivity $\sigma$, lying along the $z$-axis, carries a uniform axial current density $I$. The Poynting vector on the surface of the wire is (in the following $\hat{\rho}$ and $\hat{\varphi}$ denote the unit vectors

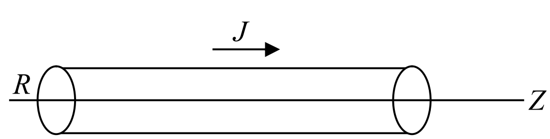
(a) $\frac{I^2 R}{2\sigma}\hat{\rho}$ (b) $-\frac{I^2 R}{2\sigma}\hat{\rho}$ (c) $-\frac{I^2 \pi R}{4\sigma}\hat{\varphi}$ (d) $\frac{I^2 \pi R}{4\sigma}\hat{\varphi}$

**Topic-EMT**

**Sub topic: Poynting vector**

Ans.: (b)

Solution: $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B};$ $J = \sigma E$




$$= \frac{1}{\mu_0}\frac{J}{\sigma}\hat{z}\times\frac{\mu_0 I}{2\pi R}\hat{\varphi},\ J = \frac{I}{\pi R^2}$$

$$= \frac{I}{\sigma\pi R^2}\cdot\frac{I}{2\pi R}\left(-\hat{\rho}\right),\ = \frac{I^2 R}{2\sigma}\left(-\hat{\rho}\right)$$

Q9. A charged particle moves uniformly on the $xy$-plane along a circle of radius a centered at the origin. A detector is put at a distance d on the x axis is to detect the electromagnetic wave radiated by the particle along the x direction. If $d >> a$, the wave received by detector is

(a) unpolarized

(b) circularly polarized with the plane of polarization being the yz-plane

(c) linearly polarized along the y-direction

(d) linearly polarized along the $z$ −direction

**Topic-EMT**

**Sub topic: Polarization**

Ans.: (c)

Solution: It will be plane polarized light along y direction.

Q10. The single particle energies of a system of $N$ non-interacting fermions of spin $s$ ($at\, T = 0$) are

$E_n = n^2 E_0,\ n = 1, 2, 3, \ldots\ldots$. The ratio of $\dfrac{\varepsilon_F(\frac{3}{2})}{\varepsilon_F(\frac{1}{2})}$ the Fermi energy of Fermions of spin $\frac{3}{2}$ and $\frac{1}{2}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 2 (d) 1

**Topic-Solid state Physics**

**Sub topic: Free electron theory**

Ans.: (b)

Solution: There will be $4$ spin $\frac{3}{2}$ can stay in each level

Thus, $\varepsilon_F(\frac{3}{2}) = \left(\frac{N}{4}\right)^2 E_0$

There will be $2$ spin $\frac{1}{2}$ can stay in each level

$$\varepsilon_F(\frac{1}{2}) = \left(\frac{N}{2}\right)^2 E_0$$

$$\frac{\varepsilon_F(\frac{3}{2})}{\varepsilon_F(\frac{1}{2})} = \frac{1}{4}$$

Q11. The Hamiltonian of a two-dimensional quantum harmonic oscillator is $H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 x^2 + 2m\omega^2 y^2$ where $m$ and $\omega$ are positive constants. The degeneracy of the energy level $\frac{27}{2}\hbar\omega$ is

(a) 14 (b) 13 (c) 8 (d) 7

**Topic-Quantum Mechanics**

**Sub topic: 2D Harmonic oscillator**

Ans.: (d)

Solution: $H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 x^2 + 2m\omega^2 y^2 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m(2\omega)^2 y^2$

$\omega_x = \omega, \omega_y = 2\omega$

For two-dimensional harmonic oscillator energy

$E_{n_x,n_y} = \left(n_x + \frac{1}{2}\right)\hbar\omega_x + \left(n_y + \frac{1}{2}\right)\hbar\omega_y = \left(n_x + \frac{1}{2}\right)\hbar\omega + \left(n_y + \frac{1}{2}\right)\hbar 2\omega$

where $n_x = 0.1.2...,\ n_y = 0,1,2...$

For given state $E_{n_x,n_y} = \frac{27}{2}\hbar\omega \left(n_x + \frac{1}{2}\right)\hbar\omega + \left(n_y + \frac{1}{2}\right)\hbar 2\omega = \frac{27}{2}\hbar\omega$

$\left(n_x + 2n_y + \frac{3}{2}\right)\hbar\omega = 27\frac{\hbar\omega}{2} \Rightarrow (2n_x + 4n_y + 3) = 27 \Rightarrow (2n_x + 4n_y) = 24 \Rightarrow n_x + 2n_y = 12$

The combination of $(n_x, n_y)$ which will satisfy the constrain $n_x + 2n_y = 12$ with $n_x = 0.1.2...,$ $n_y = 0,1,2...$ is $(0,6),(2,5),(4,4),(6,3),(8,2),(10,1),(12,0)$ so there is seven fold degeneracy

Q12. The minor axis of Earth's elliptical orbit divides the area within it into two halves. The eccentricity of the orbit is 0.0167. The difference in time spent by Earth in the two halves is closest to

(a) $3.9\ days$ (b) $4.8\ days$ (c) $12.3\ days$ (d) $0\ days$

**Topic-Classical Mechanics**

**Sub topic: Central force**

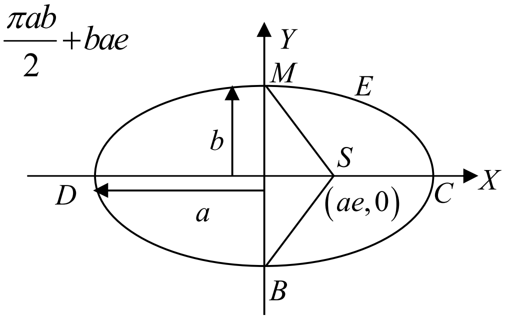
Ans.: (a)

Solution: Apply the concept of Kepler's law

The areal velocity is constant,

From above figure,

The area of MSBDM= $\dfrac{\pi ab}{2}+\dfrac{1}{2}2b\times ae=\dfrac{\pi ab}{2}+bae$




The area of MCBS= $\dfrac{\pi ab}{2}-bae$

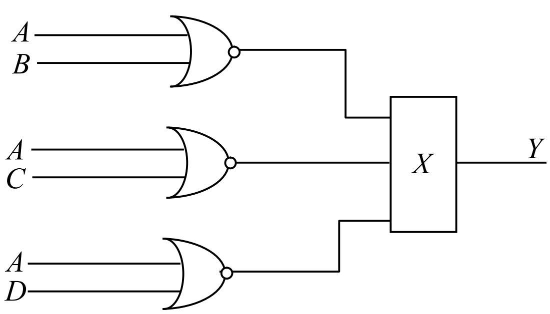
Now we know

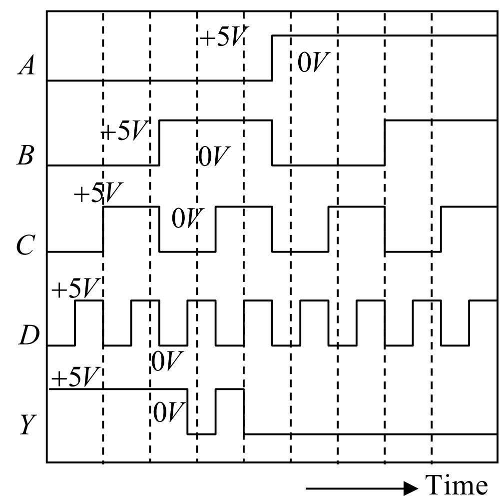
$\dfrac{dA}{dt}$ = constant

$$\frac{\frac{\pi ab}{2}+bae}{t_1}=\frac{\frac{\pi ab}{2}-bae}{t_2}\Rightarrow\frac{t_1}{t_2}=\frac{\frac{\pi}{2}+e}{\frac{\pi}{2}-e}\Rightarrow t_1=\frac{\frac{\pi}{2}+e}{\pi}\ \&\ t_2=\frac{\frac{\pi}{2}-e}{\pi}\Rightarrow t_1-t_2=\frac{2e}{\pi}\times 365\,days$$

$=\dfrac{2\times 0.0167}{\pi}\times 365\,days$ =3.88 days

Q13. For the given logic circuit, the input waveforms A, B, C and D are shown as a function of time.




To obtain the output Y as shown in the figure, the logic gate X should be

(a) 1 an AND Gate (b) an OR gate (c) a NAND gate (d) a NOR gate

**Topic-Electronics**

**Sub topic: Digital**

Ans. : (b)

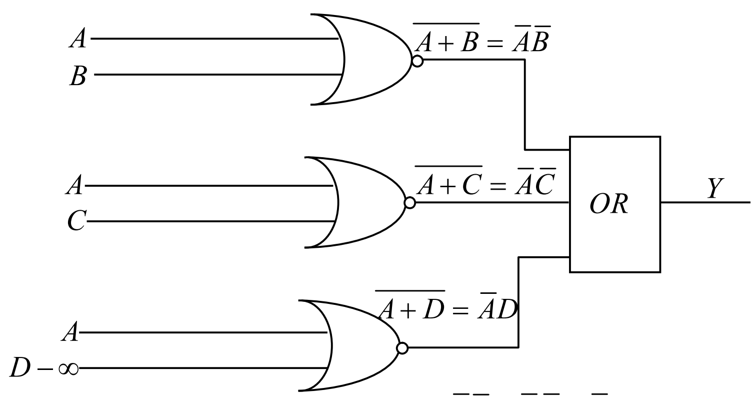
Solution: From the output, we can make k-map which clearly shows that the out put can be simplified

as

$y=\overline{A}\overline{B}+\overline{A}\overline{C}+\overline{A}D$

| AB \ CD | $\overline{C}\overline{D}$ 00 | $\overline{C}D$ 01 | $CD$ 11 | $CD$ 10 |
|---|---|---|---|---|
| $\overline{A}\overline{B}$ 10 | 1 | 1 | 1 | 1 |
| $\overline{A}B$ 00 | 1 | 1 | 1 | 0 |
| $AB$ 01 | 0 | 0 | 0 | 0 |
| $A\overline{B}$ 11 | 0 | 0 | 0 | 0 |

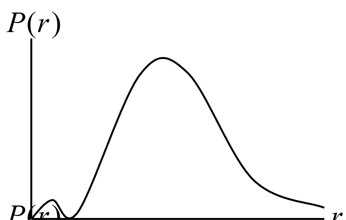
This output we can achieve if the unknown logic is OR gate.



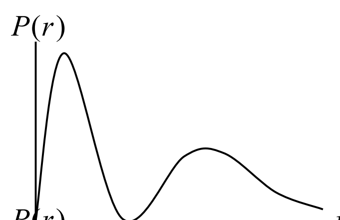

$Y=\overline{A}\overline{B}+\overline{A}\overline{C}+\overline{A}D$

Q14. The radial wavefunction of hydrogen atom with the principal quantum number $n=2$ and the orbital quantum $R_{20}=N(1-\frac{r}{2a})e^{-\frac{r}{2a}}$ where $N$ is the normalized constant. The best schematic representation of the probability density $p(r)$ for the electron to be between $r$ and $r+dr$ is

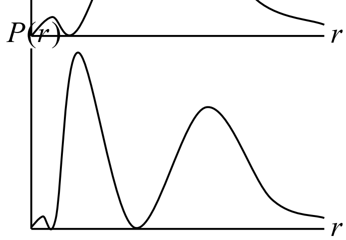
(a)



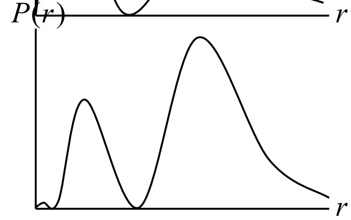

(b)




(c)




(d)




Topic-Quantum Mechanics
Sub topic: Hydrogen Atom

Ans.: (a)

Solution: $R_{20} = N\left(1-\frac{r}{2a_0}\right)\exp-\frac{r}{2a_0}$

The radial probability density $\rho(r)=|R_{2,0}|^2 r^2 \Rightarrow \left(1-\frac{r}{2a_0}\right)^2 r^2 \exp\left(-\frac{r}{a_0}\right)$

The correct plot is option 1 fig

Q15. A one-dimensional rigid rod is constrained to move inside a sphere such that its two ends are always in contact with the surface. The number of constraints on the Cartesian coordinates of the endpoints of the rod is

(a) 3 (b) 5 (c) 2 (d) 4

**Topic-Classical Mechanics**

**Sub topic: DOF**

Ans.: (a)

Solution: The equation of constrain is $x_1^2+y_1^2+z_1^2=R^2, x_2^2+y_2^2+z_2^2=R^2$,

And $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}=l$ where $R$ is radius of sphere and $l$ is length of rod

So number of holonomic constrain is 3 $k=3$, $N=2$ so $DOF=3N-K=3\times 2-3=2$

Q16. A DC motor is used to lift a mass M to a height H from the ground. The electric energy delivered to the motor is VIt, where $V$ is the applied voltage, I is the current and t the time for which the motor runs. The efficiency e of the motor is the ratio between the work done by the motor and the energy delivered to it. If $M=2.0\pm 0.02$ kg, $h=1.00\pm 0.01$ m, $V=10.0\pm 0.1$ V, $I=2.00\pm 0.02$ A and $t=300\pm 15$ s, then the fractional error $|\delta e/e|$ in the efficiency of the motor is closest to

(a) 0.05 (b) 0.09 (c) 0.12 (d) 0.15

**Topic-Experimental Technique**

**Sub topic: Error Analysis**

Ans. (a)

Q17. A particle in one dimension is in an infinite potential well between $-\frac{L}{2} \le x \le \frac{L}{2}$. For a perturbation $\varepsilon \cos(\frac{\pi x}{L})$ where $\varepsilon$ is a small constant, the change in the energy of the ground state, to first order in $\varepsilon$ is $-\frac{L}{2} \le x \le \frac{L}{2}$

(a) $\frac{5\varepsilon}{\pi}$ (b) $\frac{10\varepsilon}{3\pi}$ (c) $\frac{8\varepsilon}{3\pi}$ (d) 4 $\frac{4\varepsilon}{\pi}$

**Topic-Quantum Mechanics**

**Sub topic: Infinite potential well**

Ans.: (a)

Solution: $V(x) = \begin{cases} 0, -\frac{L}{2} \le x \le \frac{L}{2} \\ \infty, otherwise \end{cases}$

The ground state wave function is $\phi_1 = \begin{cases} \sqrt{\frac{2}{L}} \cos\frac{\pi x}{L}, -\frac{L}{2} \le x \le \frac{L}{2} \\ 0, otherwise \end{cases}$

$$E_1^1 = \int_{-L/2}^{L/2} \phi_1^* W \phi_1 dx = \frac{2}{L}\int_{-L/2}^{L/2} \cos\frac{\pi x}{L} \cos^2\frac{\pi x}{L} dx = \frac{2}{L}\int_{-L/2}^{L/2} \cos\frac{\pi x}{L}\left(1 - \sin^2\frac{\pi x}{L}\right) dx$$

$$= \frac{2}{L}.2\left(\int_0^{L/2} \cos\frac{\pi x}{L} - \int_0^{L/2} \cos\frac{\pi x}{L} \sin^2\frac{\pi x}{L}\right) = \frac{4}{L}\left(\left(\frac{\sin\frac{\pi}{2}}{\frac{\pi}{L}}\right) - \frac{\sin^3\frac{\pi}{2}}{3\frac{\pi}{L}}\right) = \frac{4}{\pi}\left(1 - \frac{1}{3}\right) = \frac{8}{3\pi}$$

Q18. The Hamiltonian of a two particle system is $H = p_1 p_2 + q_1 q_2$ where $q_1$ and $q_2$ are generalized coordinates and $p_1$ and $p_2$ are the respective canonical momenta. The Lagrangian of this system is

(a) $\dot{q}_1 \dot{q}_2 + q_1 q_2$ (b) $-\dot{q}_1 \dot{q}_2 + q_1 q_2$ (c) $-\dot{q}_1 \dot{q}_2 - q_1 q_2$ (d) $\dot{q}_1 \dot{q}_2 - q_1 q_2$

**Topic-Classical Mechanics**

**Sub topic: Hamiltonian**

Ans.: (d)

Solution: $H = p_1 p_2 + q_1 q_2$

$$L = p_1\dot{q}_1 + p_2\dot{q}_2 - H \Rightarrow p_1\dot{q}_1 + p_2\dot{q}_2 - p_1 p_1 - q_1 q_1$$

$$\frac{\partial H}{\partial p_1}=\dot{q}_1 \Rightarrow p_2=\dot{q}_1, \frac{\partial H}{\partial p_2}=\dot{q}_2 \Rightarrow p_1=\dot{q}_2$$

$$H=\dot{q}_2\dot{q}_1+\dot{q}_1\dot{q}_2-\dot{q}_1\dot{q}_2-q_1q_2 \Rightarrow H=\dot{q}_2\dot{q}_1-q_1q_2$$

Q19. The value of the integral $I=\int_0^\infty e^{-x}x\sin(x)dx$

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Ans. : (c)

Solution: We know from the Laplace transformation

$$L(Sinx)=\int_0^\infty e^{-sx}\sin(ax)dx=\frac{a}{s^2+1}=f(s)$$

Also, we know $L(x^n Sinx)=\int_0^\infty e^{-sx}x^n\sin(ax)dx=(-1)^n\frac{d^n}{ds^n}f(s)$

Now for $I=\int_0^\infty e^{-x}x\sin(x)dx, \quad f(s)=\frac{1}{s^2+1}, \; a=1, \; s=1$

$$I=\int_0^\infty e^{-x}x\sin(x)dx=-1\frac{d}{ds}\frac{1}{s^2+1}=\frac{2s}{\left(s^2+1\right)^2}=\frac{2}{4}=\frac{1}{2} \quad [\because s=1]$$

Q20. The energy levels available to each electron in a system of $N$ non-interacting electrons are $E_n=nE_0$ $n=0,1,2,\cdots$. A magnetic field, which does not affect the energy spectrum, but completely polarizes the electron spins, is applied to the system. The change in the ground state energy of the system is

(a) $\frac{n^2E_0}{2}$ (b) $n^2E_0$ (c) $\frac{n^2E_0}{8}$ (d) $\frac{n^2E_0}{4}$

**Topic- Statistical Mechanics**

**Sub topic: Distribution of spin half particle**

Ans. : (d)

Solution: The energy levels are given by,

$$E_n=nE_0$$

Case I For electrons Unpolarized both states available

$$E_1=2E_0\left(0+1+2+....+\left(\frac{N}{2}-1\right)\right)=2E_0\frac{\left(\frac{N}{2}+1\right)\frac{N}{2}}{2}$$

Case 2 For electrons polarized one state available so Paulis exclusion principle will be applied.

One particle one quantum state.

$$E_2 = E_0\left(0+1+....+\left(N-1\right)\right) = E_0\left(N+1\right)\frac{N}{2}$$

The difference in ground state

$$E_2 + E_1 = \frac{N^2}{4}E_0$$

Q21. The matrix $M = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ satisfies the equation

$M^3 + \alpha M^2 + \beta M + 3 = 0 \, if \, (\alpha, \beta)$ are

(a) $(-2,2)$ (b) $(-3,3)$ (c) $(-6,6)$ (d) $(-4,4)$

**Topic-Mathematical Physics**

**Sub topic: Matrix**

Ans.: (c)

Solution: $\lambda = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ $\quad |M - \lambda I| = \begin{vmatrix} 3-\lambda & -1 & 2 \\ -1 & 2-\lambda & 0 \\ 2 & 0 & 1-\lambda \end{vmatrix}$

$$(3-\lambda)\left[(2-\lambda)(1+\lambda)-0\right]+2\left(-2(2-\lambda)\right)+1\left(-1(1-\lambda)\right)=0$$

$$(3-\lambda)(2-\lambda)(1-\lambda)-4(2-\lambda)-1(1-\lambda)=0$$

$$(2-\lambda)\left(3+\lambda^2-3\lambda-\lambda+4\right)-1(1-\lambda)=0$$

$$(2-\lambda)\left(\lambda^2-4\lambda+1\right)-1+\lambda=0$$

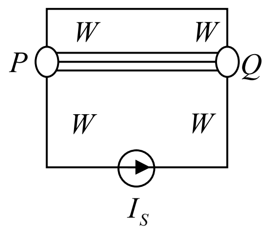
$$2\lambda^2-8\lambda-2-\lambda^3+4\lambda^2+\lambda-1+\lambda=0$$

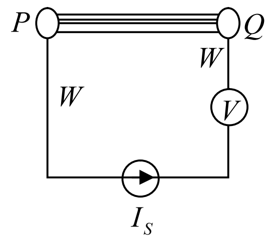
$$-\lambda^3-6\lambda^2+6\lambda-3=0$$

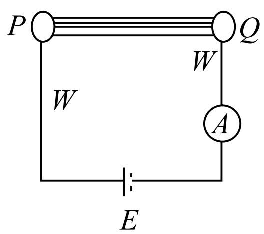
$$-\lambda^3-6\lambda^2+6\lambda+3=0$$

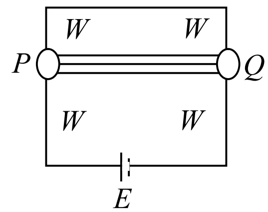
$$\alpha=-6,\ \beta=6$$

Q22. A circuit needs to be designed to measure the resistance $R$ of a cylinder $PQ$ to the best possible accuracy, using an ammeter $A$, a voltmeter $V$, a battery $E$ and a current source $I_s$ (all assumed to be ideal). The value of $R$ is known to be approximately $10\Omega$, and the resistance $W$ of each of the connecting wires is close to $10\Omega$. If the current from the current source and voltage from the battery are known exactly, which of the following circuits provides the most accurate measurement of $R$ ?

(a) 


(b) 

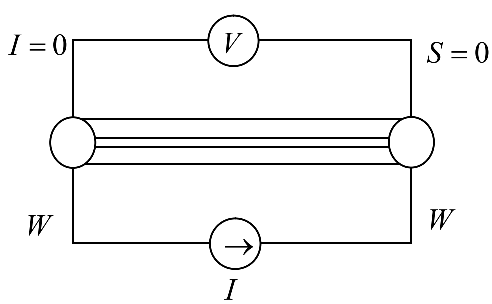

(c) 


(d) 


(a) B (b) A (c) C (d) A

Ans.: (b)

Solution: Since the constant current source is given. You can use the current source to floe the current to flow the current in the cylinder. Now we can use the four probe technique. Two leads will be used for the measurement of current and another two will be used for measurement of voltage. This is the ideal volt meter, the resistance will be very high. So the no current will be flow here. Now you are measuring voltage across the length, so the problem with contact. So, this technique will be used for medium or smaller resistance. This is the correct circuit to measure the accurate resistance of this cylinder. So, the correct option will be (b).



Q23. The electric potential on the boundary of a spherical cavity of radius $R$ as a function of the polar angle $\theta$ is $V_0 \cos^2 \frac{\theta}{2}$. The charge density inside the cavity is zero everywhere. The potential at a distance $\frac{R}{2}$ from the canter of the sphere is

(a) $\frac{V_0}{2}(1+\frac{\cos(\theta)}{2})$ (b) $\frac{V_0}{2}\cos(\theta)$ (c) $\frac{V_0}{2}(1+\frac{Sin(\theta)}{2})$ (d) $\frac{V_0}{2}\sin(\theta)$

**Topic-EMT**

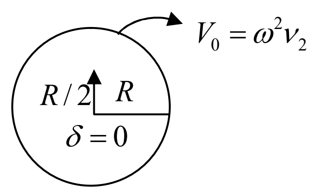
**Sub topic: Electrostatic**

Ans. : (a)

Solution: $V(r,\theta)=\sum\left(A_l r^{\ell}+\frac{B_l}{r^{\ell+1}}\right)P_l(\cos\theta)$

$V(r,\theta)=\sum A_l r^l P_\ell(\cos\theta), \quad at\, r=R$

$V_0 \text{co}^2\frac{\theta}{2}=\sum_{l=0}^{\infty}A_l R^l P_l(\cos\theta)$




$\frac{V_0}{2}[1+\cos\theta]=\sum_{l=0}^{\infty}A_l R^l P_l(\cos\theta)$

$\frac{V_0}{2}+\frac{V_0}{2}\cos\theta=\sum_{l=0}^{\infty}A_l R^l P_l(\cos\theta)$

$\frac{V_0}{2}P_0(\cos\theta)+\frac{V_0}{2}P_1(\cos\theta)=A_0 R^0 P_0(\cos\theta)+A_1 R^1 P_1(\cos\theta)$

$A_0=\frac{V_0}{2}, A_1 R=\frac{V_0}{2}, A_1=\frac{V_0}{2R}$

$V(r,\theta)=\frac{V_0}{2}\left(\frac{R}{2}\right)^0+\frac{V_0}{2R}\left(\frac{R}{2}\right)\cos\theta$

Q24. A jar J1 contains equal number of balls of red, blue and green colours, while another jar J2 contains balls of only red and blue colours, which are also equal in number. The probability of choosing J1 is twice as large as choosing J2. If a ball picked at random from one of the jars turns out to be red, the probability that it came from $J1$ is

(a) $2/3$ (b) $3/5$ (c) $2/5$ (d) $4/7$

**Topic-Mathematical Physics**

**Sub topic: Probability**

Ans. (d)

Q25. Two energy levels, 0 (non-degenerate) and $\varepsilon$ (Doubly degenerate), are available to $N$ non-interacting distinguishable particles. If U is the total energy of the system, for large values of $N$ the entropy of the system is $k_B\left[N\ln N-(N-\frac{U}{\varepsilon})\ln(N-\frac{U}{\varepsilon})+X\right]$. In this expression $X$ is

(a) $-\frac{U}{\varepsilon}\ln(\frac{U}{2\varepsilon})$ (b) $-\frac{U}{\varepsilon}\ln(\frac{2U}{\varepsilon})$ (c) $-\frac{2U}{\varepsilon}\ln(\frac{2U}{\varepsilon})$ (d) $-\frac{U}{\varepsilon}\ln(\frac{U}{\varepsilon})$

Topic-Thermodynamics and statistical mechanics

Sub topic: Entropy

Ans.: (a)

Solution: $N=n_1+n_2+n_3$

$$U=n_1.0+n_2\times\varepsilon+n_3\times\varepsilon\Rightarrow\varepsilon=\frac{U}{n_2+n_3}......(2)$$

From equation (1)

$$N=n_1+n_2+n_3\Rightarrow n_1=N-\frac{U}{\varepsilon}$$

$$\Omega={}^{N}C_{n_1}\times{}^{N-n_1}C_{n_2}\times{}^{N-n_1-n_2}C_{n_3}$$

$$S=k_B\left[N\ln N-N-n_1\ln n_1+n_1-n_2\ln n_2+n_2-n_3\ln n_3+n_3\right]$$

$$S=k_B\left[N\ln N-\left(N-\frac{U}{\varepsilon}\right)\ln\left(N-\frac{U}{\varepsilon}\right)-2\frac{U}{2\varepsilon}\ln\frac{U}{2\varepsilon}\right]$$

If we compare with original equation then we will get $X=-\frac{U}{\varepsilon}\ln(\frac{U}{2\varepsilon})$

**Part-C**

Q1. A jar J1 contains equal number of balls of red, blue and green colours, while another jar J2 contains balls of only red and blue colours, which are also equal in number. The probability of choosing J1 is twice as large as choosing J2. If a ball picked at random from one of the jars turns out to be red, the probability that it came from J1 is

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{4}{7}$

Ans.: (d)

**Topic-Mathematical Physics**

**Sub topic: Probability**

Q2. Two random walkers A and B walk on a one-dimensional lattice. The length of each step taken by A is one, while the same for B is two, however, both move towards right or left with equal probability. If they start at the same point, the probability that they meet after 4 steps, is

(a) $\frac{9}{64}$ (b) $\frac{5}{32}$ (c) $\frac{11}{64}$ (d) $\frac{3}{16}$

Ans.: (c)

**Topic-Mathematical Physics**

**Sub topic: Probability**

Q3. Let the separation of the frequencies of the first Stokes and the first anti-Stokes lines in the pure rotational Raman Spectrum of the $H_2$ molecule be $\Delta\nu(H_2)$ while the corresponding quantity for $D_2$ is $\Delta\nu(D_2)$. The ratio $\frac{\Delta\nu(H_2)}{\Delta\nu(D_2)}$ is

(a) 0.6 (b) 1.2 (c) 1 (d) 2

**Topic-Atomic, Molecular and Laser**

**Sub topic: Raman effect**

Ans.: (d)

Solution: The separation between first Stokes and first Anti-Stokes line for $H_2$ is $\Delta\nu(H_2)=12B_1$.

The separation between first Stokes and first Anti-Stokes line for $D_2$ is $\Delta\nu(D_2)=12B_2$.

$$\frac{\Delta\nu(H_2)}{\Delta\nu(D_2)}=\frac{12B_1}{12B_2}=\frac{I_2}{I_1}=\frac{\mu_1}{\mu_2};\ \mu_2=\frac{M_D\times M_D}{2M_D}=\frac{M_D}{2}=\frac{2M_H}{2}=M_H;\ \mu_2=\frac{M_H\times M_H}{2M_H}=\frac{M_H}{2}$$

$$\Rightarrow\frac{\Delta\nu(H_2)}{\Delta\nu(D_2)}=2$$

Q4. A random variable Y obeys a normal distribution $P(y)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(Y-\mu)^2/2\sigma^2}$

The mean value of $e^Y$ is

(a) $e^{\mu+\frac{\sigma^2}{2}}$ (b) $e^{\mu-\sigma^2}$ (c) $e^{\mu+\sigma^2}$ (d) $e^{\mu-\frac{\sigma^2}{2}}$

**Topic-Mathematical physics**

**Sub topic: Probability**

Ans. (a)

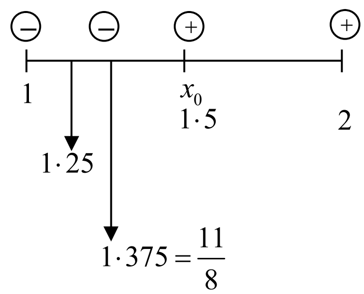
Solution: $\left\langle e^y\right\rangle=\int\limits_{-\infty}^{\infty}e^y\frac{1}{\sigma\sqrt{2\pi}}-\frac{1}{e^{2r^2}}(y-m)^2$

$$-\frac{m^2}{e^{2\sigma^2}}\frac{1}{\sigma\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}e^{-\frac{1}{2\sigma^2}y^2}+\left(\frac{1}{\sigma^2}\mu+1\right)y$$

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-m^2/2\sigma^2}\sqrt{\frac{\hbar}{\sqrt{2\sigma^2}}}e^{\left(\frac{\mu}{\sigma^2}+1\right)^2}$$

$$\int\limits_{-\infty}^{\infty}e^{-ax^2+bx}dx=\sqrt{\frac{\pi}{a}}e$$

$$e^{-\mu^2/2\sigma^2},e^{\frac{(\mu+1)^2}{\sigma^2}\cdot\frac{\sigma^2}{2}},\ e^{-\frac{\mu^2}{2\sigma}}\cdot e^{\left(\frac{\mu^2}{\sigma^4}+1+2\frac{\mu}{\sigma^2}\right)}\frac{\sigma^2}{2},\ e^r\cdot e^x+\frac{\sigma^2}{2}\cdot e^{-\frac{\mu^2}{2\sigma^2}+\frac{\mu^2}{2\sigma^2}}$$




Q5. Two distinguishable non-interacting particles, each of mass $m$ are in a one-dimensional infinite square well in the interval $[0,a]$. If $x_1$ and $x_2$ are position operators of the two particles, the expectation value $\langle x_1x_2\rangle$ in the state in which one particle is in the ground state and the other one is in the first excited state, is

(a) $\frac{a^2}{2}$ (b) $\frac{\pi^2a^2}{2}$ (c) $\frac{a^2}{4}$ (d) $\frac{\pi^2a^2}{4}$

**Topic-Quantum Mechanics**

**Sub topic: Particle in a box**

Ans.: (c)

Solution: $\psi(x_1,x_2)=\left(\sqrt{\frac{2}{a}}\sin\frac{\pi x_1}{a}\right)\sqrt{\frac{2}{a}}\sin\frac{2\pi x_2}{a}$ for $0\le x_1\le a, 0\le x_2\le a$

$$\langle x_1.x_2\rangle=\left(\frac{2}{a}\int\limits_0^a x_1\sin^2\frac{\pi x_1}{a}dx_1\right)\left(\frac{2}{a}\int\limits_0^a x_2\sin^2\frac{2\pi x_2}{a}dx_2\right)=\langle x_1\rangle\langle x_2\rangle=\frac{a}{2}\times\frac{a}{2}=\frac{a^2}{4}$$

Q6. In a one-dimensional system of $N$ spins the allowed values of each spin are $\sigma_i = \{1, 2, ......q\}$ where $q \geq 2$ is an integer. The energy of the system is $-J\sum \delta_{\sigma i, \sigma i+1}$

Where $j > 0$ is a constant. If periodic boundary conditions are imposed, the number of ground states of the

(a) q (b) Nq (c) $q^N$ (d) 1

**Topic-Solid state Physics**

**Sub topic: Band theory**

Ans. : (a)

Q7. An infinitely long solenoid of radius $r_0$ centred at origin which produces a time-dependent magnetic field $\frac{\alpha}{\pi r_0^2}\cos(\omega t)$ (where $\alpha$ and $\omega$ are constants) is placed along the z-axis. A circular loop of radius R, which carries unit line charge density is placed, initially at rest, on the xy-plane with its centre on the z-axis. If $R > r_0$, the magnitude of the angular momentum of the loop is

(a) $\alpha R(1-\cos\omega t)$ (b) $\alpha R\sin(\omega t)$ (c) $\frac{\alpha R}{2}(1-\cos 2\omega t)$ (d) $\frac{\alpha R}{2}\sin(2\omega t)$

**Topic-EMT**

**Sub topic-Electrodynamics**

Ans. : (a)

Q8. Two electrons in thermal equilibrium at temperature $T = \frac{k_B}{\beta}$ can occupy two sites. The energy of the configuration in which they occupy the different sites is $JS_1.S_2$ (where J>0is a constant and S denotes the spin of an electron), while it is U if they are at the same site. If U=10J, the probability for the system to be in the first excited state is

(a) $e^{-3\beta J/4}/\left(3e^{\beta J/4} + e^{-3\beta J/4} + 2e^{-10\beta}\right)$

(b) $3e^{-\beta J/4}/\left(3e^{-\beta J/4} + e^{3\beta J/4} + 2e^{-10\beta}\right)$

(c) $e^{-\beta J/4}/\left(2e^{-\beta J/4} + 3e^{3\beta J/4} + 2e^{-10\beta}\right)$

(d)$3e^{-3\beta J/4}/\left(2e^{\beta J/4} + 3e^{-3\beta J/4} + 2e^{-10\beta J}\right)$

**Topic-Atomic, Molecular and Laser Physics or Statistical mechanics**

**Sub-topic: Spin-spin interaction**

Ans. (b)

Solution: Net spin for two electron system can be written as follows

$\vec{S}=\vec{S}_1+\vec{S}_2=1,0 \qquad \because \vec{S}_1 \& \vec{S}_2=1/2$

The interaction Hamiltonian when they are occupying different sites is given as

$$H=JS_1.S_2$$

$$\vec{S}^2=\left(\vec{S}_1+\vec{S}_2\right)^2 \Rightarrow S_1.S_2=\frac{S^2-S_1^2-S_2^2}{2} \Rightarrow \langle S_1.S_2\rangle=\frac{S(S+1)-S_1(S_1+1)-S_2(S_2+1)}{2}$$

$$H=JS_1.S_2 \Rightarrow E=\langle H\rangle=\frac{J}{2}S(S-1)-S_1(S_1+1)-S_2(S_2+1);$$

$$E_{S=1}=\frac{J}{2}\left[1(1+1)-1/2(1/2+1)-1/2(1/2+1)\right]=J/4$$

$$E_{S=0}=\frac{J}{2}\left[0-1/2(1/2+1)-1/2(1/2+1)\right]=-3J/4$$

The first excited state energy is $J/4$

Since, the electrons are spin-1/2 particle so they will follow Pauli exclusion principles.

If we have two sites A and B. Then the degeneracy to be staying in the same site is 2.

Now the partition function is defined as

$$Z=\sum_{i=1}^{N} g_i e^{-\beta E_i}=2e^{-10\beta J}+e^{3/4\beta J}+3e^{-1/4\beta J}$$

The probability to be staying in the first excited state is

$$3e^{-\beta J/4}/\left(3e^{-\beta\ /4}+e^{3\beta\ /4}+2e^{-10\beta J}\right)$$

Q9. For the transformation $x \to X=\frac{\alpha p}{x}, p \to P=\beta x^2$ between conjugate pairs of a coordinate and its momentum, to be canonical, the constants $\alpha$ and $\beta$ must satisfy

(a) $1+\frac{1}{2}\alpha\beta=0$ (b) $1-\frac{1}{2}\alpha\beta=0$ (c) $1+2\alpha\beta=0$ (d) $1-2\alpha\beta=0$

Ans.: (c)

Solution: $X=\dfrac{\alpha p}{x}, P=\beta x^2$

$$[X,P]=1 \Rightarrow \frac{\partial X}{\partial x}.\frac{\partial P}{\partial p}-\frac{\partial X}{\partial p}.\frac{\partial P}{\partial x}=1 \Rightarrow -\frac{\alpha p}{x^2}.0-\frac{\alpha}{x}.2\beta x=1 \Rightarrow 1+2\alpha\beta=0$$

**Topic-Classical mechanics**

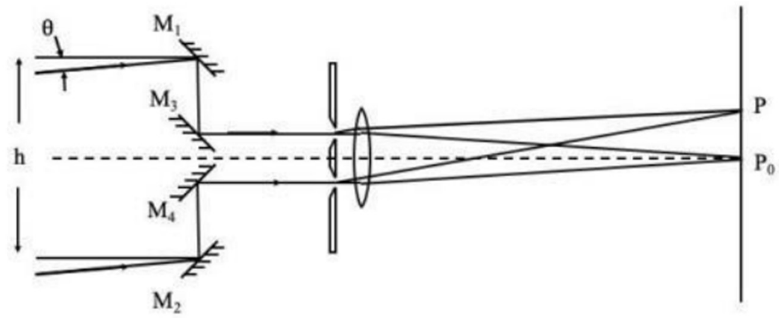
**Sub-topic-canonical transformation**

Q10. The bisection method is used to find a zero $x_0$ of the polynomial $f(x)=x^3-x^2-1$. Since $f(1)=-1$, while $f(2)=3$ the values $a=1$ and $b=2$ are chosen as the boundaries of the interval in which the $x_0$ lies. If the bisection method is iterated three times, the resulting value of $x_0$ is

(a) $\frac{15}{8}$ (b) $\frac{13}{8}$ (c) $\frac{11}{8}$ (d) $\frac{9}{8}$

Ans.: (c)

Q11. The angular width $\theta$ of a distant star can be measured by the Michelson radiofrequency stellar interferometer (as shown in the figure below).




The distance $h$ between the reflectors $M_1$ and $M_2$ (assumed to be much larger than the aperture of the lens), is increased till the interference fringes (at $P_0, P$ on the plane as shown) vanish for the first time. This happens for $h = 3$ m for a star which emits radiowaves of wavelength 2.7 cm. The measured value of $\theta$ (in degrees) is closest to

(a) 1.0.63 (b) 2.0.32 (c) 3.0.52 (d) 4.0.26

**Topic-EMT**

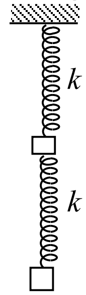
**Sub topic-EM Waves**

Ans.: (a)

Solution: $h\sin(\theta)=\lambda$

$$h\tan(\theta)=\lambda \Rightarrow \theta=\tan^{-}\left(\frac{2.7}{3}\right)=0.7^{0}$$

Q12. A system of two identical masses connected by identical springs, as shown in the figure, oscillates along the vertical direction.




The ratio of the frequencies of the normal modes is

(a) $\sqrt{3-\sqrt{5}}:\sqrt{3+\sqrt{5}}$ (b) $3-\sqrt{5}:3+\sqrt{5}$

(c) $\sqrt{5-\sqrt{3}}:\sqrt{5+\sqrt{3}}$ (d) $5-\sqrt{3}:5+\sqrt{3}$

**Topic-Classical mechanics**

**Sub topic-Small Oscillation**

Ans.: (a)

Solution: Kinetic energy is given by $T=\frac{1}{2}m\dot{y}_1^2+\frac{1}{2}m\dot{y}_2^2 \Rightarrow T=\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$

The potential energy is $\frac{1}{2}ky_1^2+\frac{1}{2}k\left(y_2-y_1\right)^2-mgy_1-mgy_2 \Rightarrow V=\begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}$

The secular equation is given as

$$\left[V-\omega^2T\right]=0\Rightarrow\begin{bmatrix} 2k-\omega^2m & -k \\ -k & k-\omega^2m \end{bmatrix}=0\Rightarrow\left(2k-\omega^2m\right)\left(k-\omega^2m\right)-k^2=0\Rightarrow$$

$$2k^2-3k\omega^2m+\omega^4m^2-k^2=0\Rightarrow k^2-3k\omega^2m+\omega^4m^2=0\Rightarrow\omega^2=\frac{3km\pm\sqrt{9k^2m^2-4k^2m^2}}{m^2}$$

$$\omega^2=\frac{k}{m}\left(3\pm\sqrt{5}\right)\Rightarrow\omega=\sqrt{\frac{k}{m}}\left(\left(3\pm\sqrt{5}\right)\right)^{1/2}$$

Ratio is $\left(3+\sqrt{5}\right)^{1/2}:\left(3-\sqrt{5}\right)^{1/2}$

Q13. The red line of wavelength $644\ \mathrm{nm}$ in the emission spectrum of Cd corresponds to a transition from the $^1D_2$ level to the $^1P_1$ level. In the presence of a weak magnetic field, this spectral line will split into (ignore hyperfine structure)

(a) 9 lines (b) 6 lines (c) 3 lines (d) 2 lines

**Topic-Atomic, Molecular and Laser physics**

**Sub topics-Zeeman effect**

Ans.: (c)

Solution: The given transition is singlet to singlet which is normal Zeeman effect. In normal zeeman effect we always get three spectral lines.

Q14. A neutral particle $X^0$ is produced in $\pi^- + p \rightarrow X^0 + n$ by $s$-wave scattering. The branching ratios of the decay of $X^0$ to $2\gamma, 3\pi$ and $2\pi$ are $0.38, 0.30$ and less than $10^{-3}$, respectively. The quantum numbers $J^{CP}$ of $X^0$ are

(a) $0^{-+}$ (b) $0^{+-}$ (c) $1^{-+}$ (d) $4 \cdot 1^{+-}$

**Topic-Nuclear and particle physics**

**Sub-topic: Particle Physics**

Ans.: (b)

Solution: $X^0 \rightarrow 2\pi \; or \; X^0 \rightarrow 3\pi$ clearly shows that the spin $J = 0$

$$X^0 \rightarrow \gamma + \gamma$$

$$J = \vec{1} + \vec{1} = 2,1,0$$

$$X^0 \rightarrow \pi + \pi$$

$$0 = \quad 0 + 0$$

$$X^0 \rightarrow \pi + \pi$$

$$0 = \quad 0 + 0$$

Since, the maximum number of X decaying to $2\gamma$.

The charge conjugation for photon is -1

$$X^0 \rightarrow \gamma + \gamma$$

$$C = (-1) \times (-1) = 1$$

Also, the main equation is strong where the parity is conserved. so the parity is conserved. The parity of fermion is +1 and for boson it is -1.

$$\pi^- + p \rightarrow X^0 + n$$

$$(-1) \times 1 = (-1) \times 1$$

Thus, $J^{CP} = 0^{+-}$

Q15. A lattice A consists of all points in three-dimensional space with coordinates $(n_x, n_y, n_z)$ where $n_x, n_y$ and $n_z$ are integers with $n_x + n_y + n_z$ being odd integers. In another lattice B, $n_x + n_y + n_z$ are even integers. The lattices $A$ and $B$ are

(a) Both $BCC$ (b) Both $FCC$

(c) $BCC$ and $FCC$ respectively (d) $FCC$ and $BCC$ respectively

**Topic-Solid State Physics**

**Sub-topic-Crustal structure**

Ans.: (b)

Q16. The charge density and current of an infinitely long perfectly conducting wire of radius $a$, which lies along the $z$-axis, as measured by a static observer are zero and a constant $I$, respectively. The charge density measured by an observer, who moves at a speed $v = \beta c$ parallel to the wire along the direction of the current, is

(a) $-\frac{I\beta}{\pi a^2 c\sqrt{1-\beta^2}}$ (b) $-\frac{I\beta\sqrt{1-\beta^2}}{\pi a^2 c}$ (c) $\frac{I\beta}{\pi a^2 c\sqrt{1-\beta^2}}$ (d) $\frac{I\beta\sqrt{1-\beta^2}}{\pi a^2 c}$

**Topic-Electromagnetic theory**

**Sub-topic-Relativistic electrodynamics**

Ans.: (a)

Solution: $\rho = \dfrac{\rho' + \dfrac{(-v)J_z'}{c^2}}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{0 - \dfrac{v}{c^2}.\dfrac{I}{\pi a^2}}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{-\beta I}{\pi a^2 c\sqrt{1-\beta^2}}$

Q17. The electric and magnetic fields at a point due to two independent sources are $\mathrm{E}_1 = E(\alpha\hat{\imath} + \beta\hat{\jmath}), \mathrm{B}_1 = B\hat{k}$ and $\mathrm{E}_2 = E\hat{\imath}, \mathrm{B}_2 = -2B\hat{k}$, where $\alpha, \beta, E$ and $B$ are constants. If the Poynting vector is along $\hat{\imath} + \hat{\jmath}$, then

(a) $\alpha + \beta + 1 = 0$ (b) $\alpha + \beta - 1 = 0$ (c) $\alpha + \beta + 2 = 0$ (d) $\alpha + \beta - 2 = 0$

**Topic-Electromagnetic theory**

**Sub topic-Poynting vector**

Ans.: (d)

Solution: $S_1 = E_1 \times B_1 = EB(-\alpha j + \beta i)$

$S_2 = E_2 \times B_2 = -2EBj \Rightarrow S = S_1 + S_2 = EBi + EB(2-\alpha)j$

$\Rightarrow \vec{A} = \hat{i} + \hat{j}$

$$\Rightarrow S \times A = 0 \Rightarrow S \times A = 0 \begin{vmatrix} i & j & k \\ EB & EB(2-\alpha) & 0 \\ 1 & 1 & 0 \end{vmatrix}$$

$\Rightarrow EB(2-\alpha) - EB\beta = 0 \Rightarrow EB(2-\alpha-\beta) = 0 \Rightarrow \alpha + \beta = 2$

Q18. The electron cloud (of the outermost electrons) of an ensemble of atoms of atomic number Z is described by a continuous charge density $\rho(r)$ that adjusts itself so that the electrons at the Fermi level have zero energy. If $V(r)$ is the local electrostatic potential, then $\rho(r)$ is

(a) $\frac{e}{3\pi^2\hbar^3}\left[2m_e eV(r)\right]^{3/2}$

(b) $\frac{Ze}{3\pi^2\hbar^3}\left[2m_e eV(r)\right]^{3/2}$

(c) $\frac{Ze}{3\pi^2\hbar^3}\left[Zm_e eV(r)\right]^{3/2}$

(d) $\frac{e}{3\pi^2\hbar^3}\left[m_e eV(r)\right]^{3/2}$

Ans.: (a)

Solution: From the concept of Fermi gas model

$$eV(r)=\frac{\hbar^2}{2m_e}\left(3\pi^2\rho(r)\right)^{2/3}\Rightarrow\rho(r)=\frac{e}{3\pi^2\hbar^3}\left(2m_e eV(r)\right)^{3/2}$$

Q19. The matrix $R_{\hat{n}}(\theta)$ represents a rotation by an angle $\theta$ about the axis $\hat{n}$. The value of $\theta$ and $\hat{n}$ corresponding to the matrix $\begin{pmatrix}-1 & 0 & 0\\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3}\\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3}\end{pmatrix}$ respectively, are

(a) $\pi/2$ and $\left(0,-\sqrt{\frac{2}{3}},\frac{1}{\sqrt{3}}\right)$

(b) $\pi/2$ and $\left(0,\frac{1}{\sqrt{3}},\sqrt{\frac{2}{3}}\right)$

(c) $\pi$ and $\left(0,-\sqrt{\frac{2}{3}},\frac{1}{\sqrt{3}}\right)$

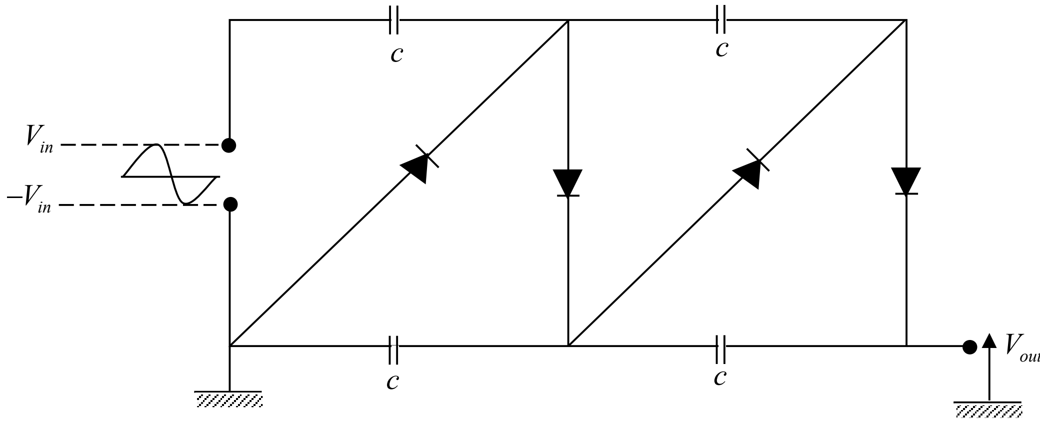
(d) $\pi$ and $\left(0,\frac{1}{\sqrt{3}},\sqrt{\frac{2}{3}}\right)$

**Topic-Mathematical Physics**

**Sub topic-Matrix**

**Ans.:**

Q20. In the circuit shown below, four silicon diodes and four capacitors are connected to a sinusoidal voltage source of amplitude $V_{\text{in}} > 0.7$ V and frequency 1kHz. If the knee voltage for each of the diodes is 0.7 V and the resistances of the capacitors are negligible, the DC output voltage $V_{\text{out}}$ after 2 seconds of starting the voltage source is closest to



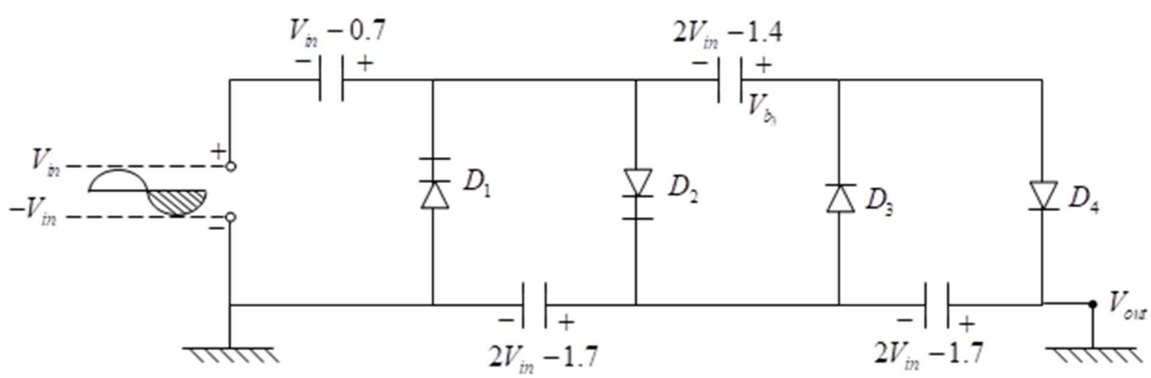

(a) $4V_{in} - 0.7V$ (b) $4V_{in} - 2.8V$ (c) $V_{in} - 0.7V$ (d) $V_{in} - 2.8V$

Topic-Electronics

Sub-Diode

Ans.: (b)

Solution:




If we apply KVL that will provide

$$-V_{in} + V_{D1} + V_{C1} = 0 \Rightarrow V_{C1} = V_{in} - V_{D1} = V_{in} - 0.7V$$

In the positive half cycle of input $D_1$ and $D_3$ becomes reverse bias. On the other hand $D_2$ and $D_4$ are forward bias.

$$-V_{in} + V_{D2} - V_{C1} + V_{C2} = 0 \Rightarrow V_{C2} = 2V_{in} - 1.4V$$

In the positive half cycle of input $D_1$ and $D_3$ becomes forward bias. On the other hand $D_2$ and $D_4$ are reverse bias.

$V_{D1}-V_{C3}-V_{D3}+V_{C2}=0 \Rightarrow V_{C3}=2V_{in}+1.4V$

In the next half cycle, $D_1$ and $D_3$ becomes reverse bias. On the other hand $D_2$ and $D_4$ are forward bias. If we apply KVL

$V_{C4}=2V_{in}-1.4V$

$V_{out}=4V_{in}-2.8V$

Q21. A layer of ice has formed on a very deep lake. The temperature of water, as well as that of ice at the ice-water interface, are $0^o C$ whereas the temperature of the air above is $-10^o C$. The thickness $L(t)$ of the ice increases with time t. Assuming that all physical properties of air and ice are independent of temperature, $L(t) \sim L_0 t^{\alpha}$ for large t. The value of $\alpha$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $4.1$

**Topic-Mathematical Physics**

**Sub-Laplace transformation**

Ans. : (c)

Solution: The Laplace Transform Pair

$$\sin x \Leftrightarrow \frac{1}{s^2+1}$$

$$x\sin x \Leftrightarrow -\frac{d}{ds}\frac{1}{s^2+1}$$

$$x\sin x \Leftrightarrow -\frac{d}{ds}\left(s^2-1\right)^{-1}$$

$$\left(s^2+1\right)^{-2}\cdot 2s$$

$s=1$, So, the value of integral is $\left(1+1\right)^{-2}\cdot 2\cdot 1=\frac{2}{4}=\frac{1}{2}$

Q22. The Hall coefficient $R_H$ of a sample can be determined from the measured Hall voltage $V_H = \frac{1}{d} R_H BI + RI$ where $d$ is the thickness of the sample, $B$ is the applied magnetic field, $I$ is the current passing through the sample and $R$ is an unwanted offset resistance. A lock-in detection technique is used by keeping $I$ constant with the applied magnetic field being modulated as $B = B_0 \sin \Omega t$, where $B_0$ is the amplitude of the magnetic field and $\Omega$ is frequency of the reference signal. The measured $V_H$ is

(a) $B_0 \frac{R_H I}{d}$ (b) $\frac{B_0}{\sqrt{2}} \frac{R_H I}{d}$ (c) $\frac{I}{\sqrt{2}}\left(\frac{B_0 R_H I}{d} + R\right)$ (d) $I\left(\frac{B_0 R_H}{d} + R\right)$

Topic-Solid State Physics

Sub topic-Hall coefficient

Ans.: (b)

Solution: $V_H = \frac{1}{d} R_H BI + RI$

$$B = B_0 \sin(\omega t)$$

$$(V_H)_{AC} = \frac{1}{d} R_H B(t) I = \frac{1}{d} R_H I B_0 \sin(\omega t)$$

$$\left((V_H)_{AC}\right)_{rms} = \frac{1}{\sqrt{2}d} R_H I B_0$$

$$(V_H)_{RMS} = \frac{\beta_0}{\sqrt{2}} \frac{R_H I}{d}$$

Q23. A train of impulses of frequency 500 Hz, in which the temporal width of each spike is negligible compared to its period, is used to sample a sinusoidal input signal of frequency 100 Hz. The sampled output is

(a) Discrete with the spacing between the peaks being the same as the time period of the sampling signal

(b) a sinusoidal wave with the same time period as the sampling signal

(c) discrete with the spacing between the peaks being the same as the time period of the input signal

(d) a sinusoidal wave with the same time period as the input signal

Topic-Electronics

Ans.: (a)

Q24. The value of the integral $\int_{-\infty}^{\infty} dx 2^{-\frac{|x|}{\pi}} \delta(\sin x)$ where $\delta(x)$ is the Dirac delta function, is

(a) 3 (b) 0 (c) 5 (d) 1

**Topic-Mathematical Physics**

**Sub topic-Integration**

Ans. : (a)

Solution: $\delta(\sin x) = \frac{\sum (x - n\pi)}{|\cos n\pi|}$

$$\int_{-\infty}^{\infty} 2^{-\frac{x}{\pi}} \delta(\sin x) dx = \sum_n \int_{-\infty}^{\infty} 2^{-\frac{x}{\pi}} \delta(x - n\pi) dx = \sum_{n=-\infty}^{n=\infty} 2^{-|n|} = 1 + (2^{-1} + 2^{-2} + 2^{-3}........) = 1 + 2\frac{2^{-1}}{1 - \frac{1}{2}} = 3$$

Q25. The energy (in keV) and spin-parity values $E(J^P)$ of the low-lying excited states of a nucleus of mass number $A = 152$ are $122(2^+), 366(4^+), 707(6^+)$ and $1125(8^+)$. It may be inferred that these energy levels correspond to a

(a) rotational spectrum of a deformed nucleus

(b) rotational spectrum of a spherically symmetric nucleus

(c) vibrational spectrum of a deformed nucleus

(d) vibrational spectrum of a spherically symmetric nucleus

**Topic-Nuclear and particle physics**

**Sub-topic: Collective model**

Ans. : (a)

Solution: As we know that the large size nucleus either shows rotational or vibration spectrum. The nucleus having mass no $A < 150$ shows vibrational spectra. On the other hand, the nucleus having mass no $150 < A < 200$ *or* $A > 230$ shows the rotational spectra. They have high electrical quadrupole moment implies they have deformed shape.

Q26. Electrons polarized along the x-direction are in a magnetic field

$$B_1\hat{i} + B_2(\cos \omega t\, \hat{j} + \sin \omega t\, \hat{k})$$

where $B_1 > B_2$ and $\omega$ are positive constants. The value of $\hbar\omega$ for which the polarization-flip process is a resonant one, is

(a) $2\mu_B |B_2|$ (b) $\mu_B |B_1|$ (c) $\mu_B |B_2|$ (d) $2\mu_B |B_1|$

**Topic-Electromagnetic theory**

**Sub-Polarization**

Ans.: (d)

Solution: Since the polarization along z direction, thus $S = S\hat{x}$

The interaction energy $h\omega = \vec{S}.\vec{B} = 2\mu_B |B_1|$

Q27. The dispersion relation of electrons in three dimensions is $\varepsilon(k) = \hbar v_F k$, where $v_F$ is the Fermi. If at low temperature $T << T_F$ the Fermi energy $\varepsilon_F$ depends on the number density n as $\varepsilon_F(n) \sim n^{\alpha}$, the value of $\alpha$ is

(a) 1/3 (b) 2/3 (c) 1 (d) 3/5

**Topic-Solid State Physics**

**Sub-Dispersion relation**

Ans.: (a)

Solution: $\varepsilon(k) = \hbar v_F k$

At Low temperature $T << T_F$

$$\varepsilon_F \, \alpha n^{\alpha}$$

$$N = \frac{2 \times \frac{4}{3}\pi k^3}{(2\pi)^3}$$

$$N = \frac{8\pi}{3}\frac{k_F{}^2 L^3}{8\pi^3} = \frac{k_F{}^2 v}{3\pi^2}$$

$$n = \frac{N}{V} = \frac{1}{3\lambda^2}\left(\frac{E_F}{\hbar v_F}\right)^3$$

$$\varepsilon_F = n^{1/2}$$

Q28. If the Bessel function of integer order n is defined as $J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!}\left(\frac{x}{2}\right)^{2k+n}$ then $\frac{d}{dx}[x^{-n}J_n(x)]$ is

(a) $-x^{n+1}J_{n+1}(x)$ (b) $-x^{n+1}J_{n-1}(x)$ (c) $-x^{n}J_{n-1}(x)$ (d) $-x^{n}J_{n+1}(x)$

**Topic-Mathematical Physics**

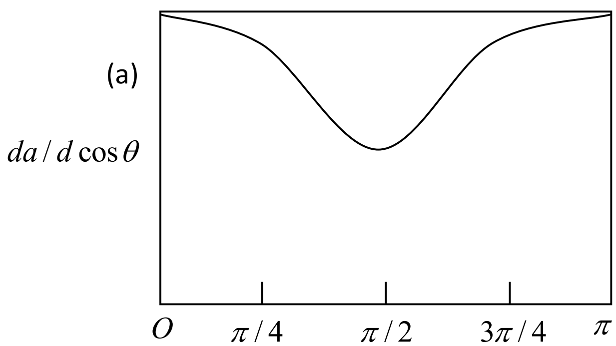
**Sub-Special function**

Ans.: (a)

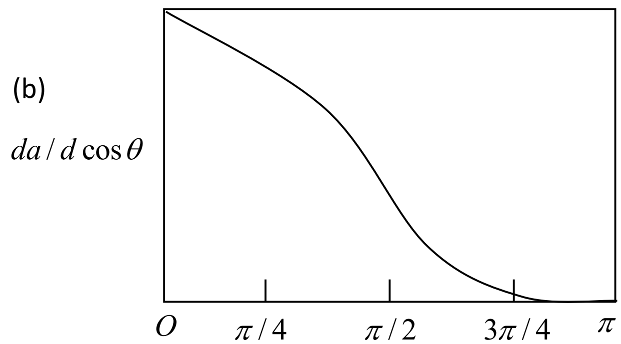
Solution: Put $n = 0$, $\frac{d}{dx}J_0(x) = -xJ_1(x)$

Q29. The phase shifts of the partial waves in an elastic scattering at energy E are $\delta_0 = 12^0, \delta_1 = 4^0 \; and \; \delta_{l\geq 2} = 0^0$ . The best qualitative depiction of $\theta$ dependence of the differential scattering cross-section $\frac{d\sigma}{d\cos(\theta)}$ is

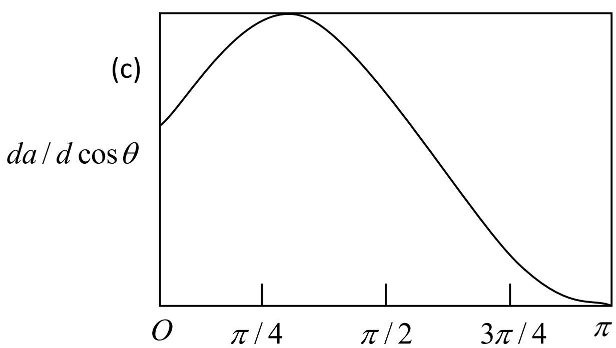
(a)



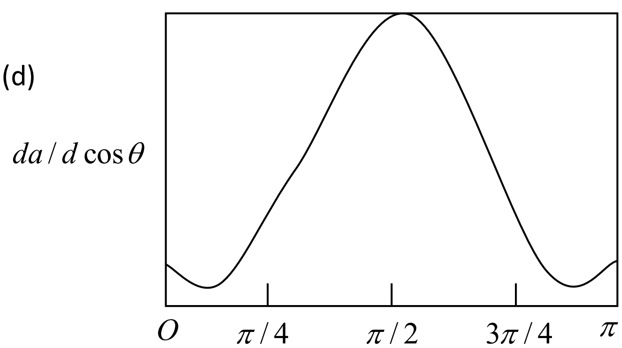

(b)




(c)




(d)




**Topic-Quantum mechanics**

**Sub-Scattering**

Ans.: (b)

Solution: The form factor is given by $f(\theta) = \frac{1}{k}\sum_l (2l+1)\exp i\delta_l \sin\delta_l \, p_l\left(\cos\theta\right)$

The differential scattering cross section is given by

$$D(\theta) = |f(\theta)|^2 = f^*(\theta) f(\theta) = \sum_{l'}\sum_{l}\left(2l'+1\right)\left(2l+1\right)\exp - i\delta_{l'} \exp i\delta_l \sin\delta_{l'} \sin\delta_l \, p_{l'}(\cos\theta) \, p_l\left(\cos\theta\right)$$

It is given only $l = 0$ and $l = 1$ is active

So $D(\theta) = \sin^2\delta_0 + 9\sin^2\delta_1\left(p_1\left(\cos\theta\right)\right)^2 + 3\exp - i\delta_0 \exp i\delta_1 \sin\delta_1 \sin\delta_0 p_1\left(\cos\theta\right) +$

$3\exp - i\delta_1 \exp i\delta_0 \sin\delta_1 \sin\delta_0 \, p_1\left(\cos\theta\right)$

$$D(\theta) = \sin^2\delta_0 + 9\sin^2\delta_1\left(p_1\left(\cos\theta\right)\right)^2 + 3\sin\delta_1 \sin\delta_0 p_1\left(\cos\theta\right)\left(\exp i\left(\delta_1 - \delta_0\right) + \exp - i\left(\delta_1 - \delta_0\right)\right)$$

$$D(\theta) = \sin^2\delta_0 + 9\sin^2\delta_1 \cos^2\theta + 6\sin\delta_1 \sin\delta_0 \cos\theta \cos\left(\delta_1 - \delta_0\right) =$$

$\sin^2\delta_0 + 9\sin^2\delta_1\cos^2\theta + 6\sin\delta_1\sin\delta_0\cos(\delta_1-\delta_0)\cos\theta =$

$D(\theta) = \sin^2\delta_0 + 9\sin^2\delta_1\cos^2\theta + 6\sin\delta_1\sin\delta_0\cos(\delta_1-\delta_0)\cos\theta =$

$\delta_0 = 12, \delta_1 = 4$ $\sin 12 = 0.2, \sin 4 = 0.07.\cos 8 = 0.99$

$D(\theta) = .04 + 9\times 0.0049\cos^2\theta + 6\times .2\times .07\times .99\cos\theta = 0.04 + 0.044\cos^2\theta + .08\cos\theta$

$D(\theta) = .04 + 9\times 0.0049\cos^2\theta + 3\times .14\cos\theta = 0.04 + 0.044\cos^2\theta + 0.08\cos\theta$

For $\theta = 0, D(\theta) = 0.16$, $\theta = \frac{\pi}{4}, D(\theta) = 0.11$, $\theta = \frac{\pi}{2}, D(\theta) = 0.04$, $\theta = \pi D(\theta) \simeq 0$

Q30. Two operators A and B satisfy the commutation relations $[H,A] = -\hbar\omega B$ and $[H,B] = \hbar\omega A$ where $\omega$ is a constant and $H$ is the Hamiltonian of the system. The expectation value $\langle A\rangle_\varphi t = \langle\varphi|A|\varphi\rangle$ in a state $\varphi$ such that at time $t=0$ $A_\varphi(0) = 0$ and $B_\varphi(0) = 0$ is

(a) $\sin(\omega t)$ (b) $\sinh(\omega t)$ (c) $\cos(\omega t)$ (d) $\cosh(\omega t)$

Topic-Quantum mechanics

Sub-topic- Ehrenfest theorem

Ans. (b)

Solution: $[H,A] = -\hbar\omega B, [H,B] = \hbar\omega A$

Using Ehrenfest theorem $\frac{d\langle A\rangle}{dt} = \frac{1}{i\hbar}\langle [A,H]\rangle + \left\langle \frac{\partial A}{\partial t}\right\rangle$

$$\frac{d\langle A\rangle}{dt} = \frac{1}{i\hbar}\hbar\omega\langle B\rangle \Rightarrow \frac{d\langle A\rangle}{dt} = \frac{1}{i}\omega\langle B\rangle \Rightarrow \frac{d}{dt}\frac{d\langle A\rangle}{dt} = \frac{\omega}{i}\frac{d\langle B\rangle}{dt}$$

$$\frac{d^2\langle A\rangle}{dt^2} = \frac{\omega}{i}\frac{d\langle B\rangle}{dt} = \frac{\omega}{i}\left\langle\frac{[B,H]}{i\hbar}\right\rangle = \frac{\omega}{i}\frac{(-\omega\langle A\rangle)}{i} \Rightarrow \frac{d^2\langle A\rangle}{dt^2} = \omega^2\langle A\rangle$$

$\langle A(t)\rangle = c_1\exp\omega t + c_2\exp(-\omega t)$ using boundary condition

$$\langle A(0)\rangle = 0 \Rightarrow c_2 = -c_1 = c_1(\exp(\omega t) - \exp(-\omega t))$$

$$\langle B(t)\rangle = \frac{i}{\omega}\frac{d\langle A\rangle}{dt} = \frac{i\omega}{\omega}c_1(\exp\omega t + \exp(-\omega t)) = ic_1(\exp\omega t + \exp(-\omega t))$$

$$\langle B(0)\rangle = i \Rightarrow 2ic_1 \Rightarrow c_1 = \frac{1}{2}$$

$$\langle A(t)\rangle = \frac{1}{2}(\exp\omega t - \exp(-\omega t)) = \sinh\omega t$$