PREVIOUS YEAR'S SOLUTION

CSIR-NET JUNE 2024

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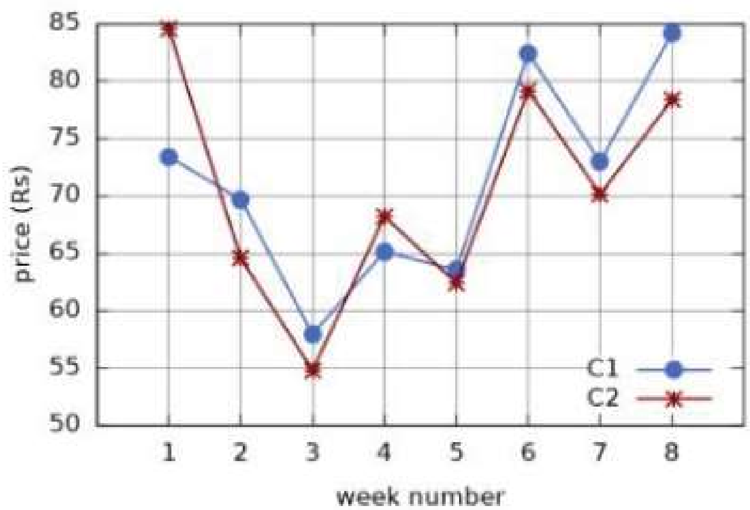
CSIR-NET Physics Course

CSIR NET JUNE 2024

Part A

Question ID: 705015:

The two graphs show the change in price of two commodities C1 and C2 over 8 weeks.



Which of the statements is correct?

1. C1 has higher fluctuation than C2
2. Average price of C1 is lower than that of C2
3. The largest change in a week is shown by C2
4. C1 shows a tendency of reduction

Ans. : (3)

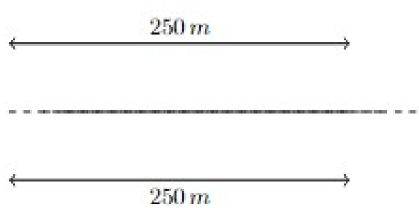
Question ID 705017:

On a one-way road, broken lines consisting of 2.5 m length segments separated by 2.5 m gaps are painted along the length of the road to demarcate 3 lanes, and continuous lines are painted along both the borders. What is the total length of the painted lines (in m) over a 250 m stretch of the road?

1. 500 2. 625 3. 750 4. 1000

Ans. : (3)

Solution:



Total $= 250 + 250 + 250$ (broken lines)

$= 750 \text{ m} = 0 + (3)$.

Question ID 705009:

In a class of 70 students, 20% of girls have spectacles and 40% of boys have spectacles. If the total number of students having spectacle is 23 , the number of boys in the class is

1. 45 2. 14 3. 18 4. 25

Ans. : (1)

Solution: Let the number of boys be x.

Given:

- Total number of students $= 70$
- Number of girls $= 70 - x$
- Total number of students wearing specs $= 23$

According to the question:

$\frac{30}{100}(70-x)+\frac{40}{100}x=23, x=20$

Question ID 705018:

A patient requires administration of 500 ml of an intravenous fluid in 1 hour. What is the approximate drip rate (number of drops per minute) at which the fluid should be administered, if the volume of a drop is 0.05 ml?

1. 76 2. 152 3. 167 4. 332

Ans. : (3)

Solution: Given:

- Total dose $= 500$ mL
- Time required $= 1$ hour
- Volume of a single drop $= 0.05$ mL

Solution:

1. Calculate the number of drops to be administered:

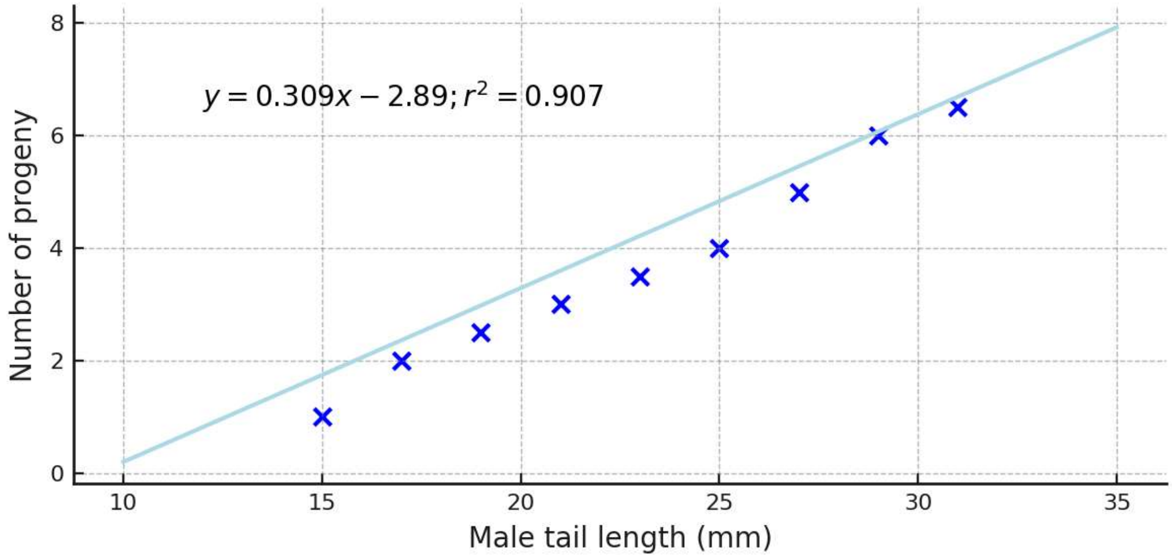
$$\text{Number of drops } = \frac{\text{Total dose}}{\text{Volume of a single drop}} = \frac{500}{0.05} = 10^4$$

2. Calculate the drip rate (number of drops per minute):

$$\text{Drip rate } = \frac{\text{Number of drops}}{\text{Time in minutes}} = \frac{10^4}{60} = 167.$$

Question ID 705016:

The graph shows observations and a regression line of the number of progeny on the tail length of male birds.



Which of the following can be inferred from the graph?

1. Producing less progeny decreases the tail length of the males.
2. Males cannot have a tail length lesser than 10 mm.
3. Males with longer tails tend to father more progeny.
4. For a male with a 25 mm tail, the expected number of progeny is 4.

Ans. : (3)

Solution: From the graph, it is clearly teen spt (3) is correct.

Question ID 705001

A large number of birds, half of which belong to specie A and the other half to specie B, rest on a tree where they are distributed randomly across the branches. In a random sample of 5 birds from the tree, what is the probability that at least one is from specie A ?

1. 0.03125 2. 0.15625 3. 0.84375 4. 0.96875

Ans. : (4)

Solution: Half of the birds belong to species A, and the other half belong to species B. Thus:

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$

If 5 birds are chosen at random, the probability that at least one is from species A is: $P($ At least one from species A$)=1-P($ None of the birds is from species A$)$. Probability that a bird is not from species A is:

$$P(\text{ Not species A})=\frac{1}{2}$$

Probability that all 5 birds are not from species A :

$$P(\text{ None of the 5 is from species A})=\left(\frac{1}{2}\right)^{5}=\frac{1}{32}.$$

Therefore, the probability that at least one bird is from species A is:

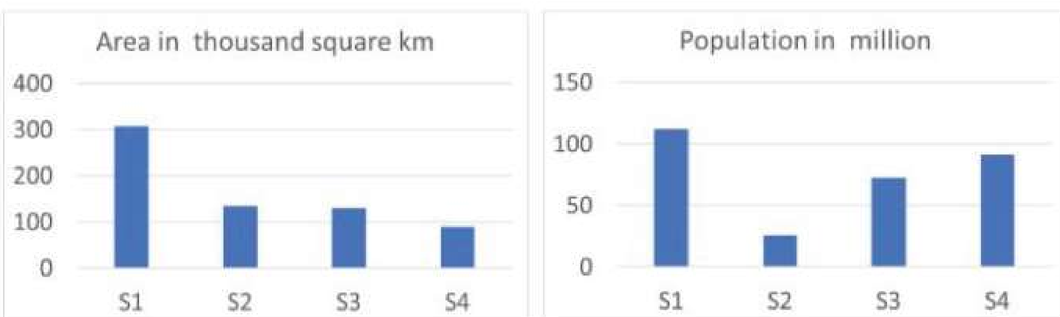
$$1-P(\text{ None of the birds is from species A})=1-\frac{1}{32}$$

Simplify:

$$1-\frac{1}{32}=\frac{31}{32}=0.96875$$

Question ID 705007

Areas and populations of four states S1, S2, S3 and S4 are shown.



Their arrangement in decreasing order of population density would be

1. S4, S3, S1, S2 2. S1, S2, S3, S4 3. S4, S1, S3, S2 4. S2, S1, S3, S4

Ans. : (1)

Solution: From the figure, the calculations for S_1, S_2, S_3, and S_4 are as follows:

$$S_1=\frac{110}{300}=0.367, S_2=\frac{25}{110}=0.227, S_3=\frac{70}{110}=0.63, S_4=\frac{90}{90}=1$$

Decreasing Order:

$$S_4>S_3>S_1>S_2$$

Question ID 705010

A referendum on a proposal involved 7000 participants. Among the participants 3600 were women and the rest were men. 2900 participants, of whom 1300 were women, voted against while 3000 participants voted in favour. 400 women abstained. The ratio of the number of men that did not vote to the total number of participants is

1. $11:70$ 2. $17:35$ 3. $1:10$ 4. $8:70$

Ans. : (3)

Solution: Data Table:

Category	For	Against	Abstained	Total
Men	1100	1600	700	3400
Women	1900	1300	400	3600
Total	3000	2900	1100	7000

Analysis:

Total participants: 7000

Total women participants: 3600

Total men participants: $7000-3600=3400$

Total participants who voted against: 2900

Out of these, 1300 are women who voted against. Therefore, men who voted against:

$2900-1300=1600$

Women who abstained from voting: 400

Women participants who voted in favor: $3600-(1300+400)=1900$

Men participants who voted in favor: $3400-1900=1500$

Total people who abstained from voting: $1100=700(\text{Men})+400(\text{ Women })$

Ratio of the number of men who did not vote to the total number of participants:

$$\text{Ratio } =\frac{\text{Number of men who did not vote}}{\text{Total number of participants}}=\frac{700}{7000}=\frac{1}{10}.$$

Question ID 705005

How many three-digit numbers exist whose first and last digits add up to 9?

1. 90 2. 81 3. 80 4. 72

Ans. : (1)

Solution: The data shows a grouping of numbers into intervals with their corresponding frequencies. The total count is 90. The data is structured as follows:

Range	Frequency (No. of numbers)
$109-199$	10
$207-297$	10
$306-396$	10
$405-495$	10
$504-594$	10
$603-693$	10
$702-792$	10
$801-891$	10
$900-990$	10
Total	90

Question ID 705013

If 32XY6 is divisible by 9, X and Y being even decimal digits, then X =

1. 2 2. 4 3. 6 4. 8

Ans. : (4)

Solution: Given the number $32XY6$, where X and Y are even numbers. The number $32XY6$ is divisible by 9.

Steps to Solve:

For a number to be divisible by 9, the sum of its digits must also be divisible by 9. 2. Possible values for X and Y (even numbers) are: 2, 4,6,8.

Condition for divisibility:

$3+2+X+Y+6$ must be divisible by 9

My choice: Let $X=8$ and $Y=8$.

Then: $3+2+8+8+6=27$

which is divisible by 9.

Question ID 705011

The population of a town is increasing at a uniform rate. If its population was 90,000 and 96,000 in 2022 and 2023 respectively, what would be its population in 2024?

1. 102,000 2. 102,400 3. 102,720 4. 102,960

Ans. : (2)

Solution: Given:

- Population in 2022 $= 90{,}000$
- Population in 2023 $= 96{,}000$

The question states that the population is increasing at a uniform rate. Therefore, the rate of increase can be calculated as:

$$\text{Rate of increase } = \frac{96{,}000 - 90{,}000}{90{,}000} = \frac{6{,}000}{90{,}000} = 0.067$$

Using this rate, we calculate the population in 2024:

Population in 2024 $= 96{,}000 \times (1 + 0.067) = 96{,}000 \times 1.067 = 102{,}400$

Explanation: All other options do not match the uniform growth rate.

Question ID 705014

Canals A and B join to form canal C, all having semi-circular cross-sections of radii which are in the ratio $3:4:5$, respectively. Assume smooth merger of A and B, and ignore the possibility of flooding. If the speed s of water is the same and uniform in both A and B then the speed of water flowing in C is

1. s 2. $7s/5$ 3. $2s$ 4. $5s/7$

Ans. : (1)

Solution: Given:

- Radius of $A = 3x$
- Radius of $B = 4x$
- Radius of $C = 5x$
- Speed of water in band $B = S$

Solution: According to the question:

$\frac{1}{2}\pi(3x)^2 S + \frac{1}{2}\pi(4x)^2 S + \frac{1}{2}\pi(5x)^2 S = \frac{1}{2}\pi(5x)^2 V$, $50\text{ S} = 25\text{ V}$, $V = \frac{50}{25} = 2S$

Question ID 705008

Among 1000 squirrel babies, 200 have three stripes on their back, 500 have two stripes on their back and the rest have four stripes on their back. While 90% of the three-striped babies survive to adulthood, only 80% of the two-striped and 70% of the four-striped babies survive to adulthood. The fraction of four-striped squirrels among the adults is nearest to

1. 0.21 2. 0.3 3. 0.266 4. 0.228

Ans. : (3)

Solution: Problem:

The total number of squirrel babies is 1000, categorized as follows:

- 3 -striped squirrel babies $= 200$
- 2-striped squirrel babies $= 500$
- 4 -striped squirrel babies $= 1000-(200+500)=300$

Solution: Number of 3-striped squirrel babies that survive to adulthood: $\frac{90}{100}\times 200=180$

Number of 2-striped squirrel babies that survive to adulthood: $\frac{80}{100}\times 500=400$

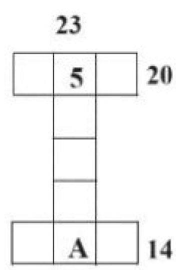
Number of 4 -striped squirrel babies that survive to adulthood: $\frac{70}{100}\times 300=210$

Total number of adult squirrels: $180+400+210=790$

Fraction of total adult 4-striped squirrels to total adult squirrels: $\frac{210}{790}\approx 0.266$

Question ID 705012

The squares in the following grid are filled with numbers 1 to 9, without repetition, such that the numbers in the squares forming the top and bottom rows add to 20 and 14 respectively and those forming the column to 23. What is the value of **A** ?



1. 4
2. 6
3. 7
4. 8

Ans. : (3)

Solution: Diagram Representation:

Below is the tabular arrangement of numbers:

6	5	9(23)
8	2	1(20)
3	7	4(14)

Explanation:

The sums of the numbers in each row and column are as follows:

- First row: $6 + 5 + 9 = 23$
- Second row: $8 + 2 + 1 = 20$
- Third row: $3 + 7 + 4 = 14$

The given data satisfies the condition, and the correct option is:

Question ID 705020

An egg tray has 30 cavities to hold eggs in 5 rows and 6 columns. Each cavity is surrounded by 4 raised corners shared by adjacent cavities. How many raised corners does the egg tray have?

1. 30 2. 35 3. 36 4. 42

Ans. : (4)

Solution: An egg tray has 5 rows and 6 columns of cavities. Each cavity is surrounded by 4 corners.

Solution: 1. The total number of intersections (or corners) can be calculated by counting the grid lines:

- There are 7 vertical lines (6 columns +1 extra line at the end).
- There are 6 horizontal lines (5 rows +1 extra line at the end).

2 The total number of intersections (corners) is:

$\text{Total intersections} = (7 \text{ vertical lines}) \times (6 \text{ horizontal lines}) = 42$

Thus, the total number of intersections equals the total number of shared corners in the grid.

Question ID 705004

In how many distinct ways can 128 identical marbles be arranged in a complete rectangular grid (disregarding the orientation of the grid)?

1. 7 2. 6 3. 5 4. 4

Ans. : (4)

Solution: The number of distinct ways in which 128 identical marbles can be arranged in a rectangular grid is determined by finding the factor pairs of 128.

The factor pairs of 128 are:

$$1 \times 128,\ 2 \times 64,\ 4 \times 32,\ 8 \times 16$$

Thus, there are 4 distinct ways to arrange the marbles in a rectangular grid.

Question ID 705003

A rectangular tray of $30 \text{ cm} \times 60 \text{ cm}$ size is used for baking circular biscuits. The diameter of each biscuit is 3 cm before baking, which increases by 10% on baking. What is the maximum number of biscuits that can be baked in the tray such that the base of each biscuit is in contact with the tray?

1. 171 2. 162 3. 180 4. 200

Ans. : (2)

Solution: Problem:

- Size of the tray: $30 \text{ cm} \times 60 \text{ cm}$
- Diameter of the biscuit: 3 cm
- After baking, the size of the biscuit increases by 10%.

Solution: Calculate the increased diameter of the biscuit:

New diameter $= 3 \text{ cm} + 10\% \text{ of } 3 \text{ cm} = 3 + 0.3 = 3.3 \text{ cm}$

Determine the maximum number of biscuits along the vertical direction:

Number of biscuits along vertical direction $= \dfrac{30}{3.3} \approx 9.$

Determine the maximum number of biscuits along the horizontal direction:

Number of biscuits along horizontal direction $= \dfrac{60}{3.3} \approx 18.$

Total number of biscuits: Total biscuits $= 9 \times 18 = 162$

Question ID 705002

Suppose that the increase in a population can be modelled as

$$\left(\frac{dN}{dt}\right) = rN\frac{(K-N)}{K}$$

where N is the size of the population, K is the carrying capacity, r is the per capita growth rate and t is time. Which of the following statements is correct?

1. When $N \approx 0$, the change in population N is nearly exponential.
2. When $N = K$, the population goes extinct as dN/dt goes to zero.
3. When $N \approx 0$, the population growth dN/dt is maximum.
4. When $N \approx K/4$, the population growth dN/dt is maximum.

Ans. : (1)

Solution: $\frac{dN}{dt} = \gamma N\left(\frac{K-N}{K}\right), \quad N \simeq 0.$

$\frac{dN}{dt} = \gamma N(1) \left(\because \frac{N}{K} \simeq 0\right) \Rightarrow \frac{dN}{N} = \int \gamma dt \Rightarrow \ln\, N = \gamma t \Rightarrow \frac{IN}{r} = e^{\gamma t}\Big]$

Question ID 705006

Among A, B, C, D, E and F, D is taller than B but shorter than F. E is taller than B, but shorter than C. B is not the shortest of all. Then A is

1. The shortest of all.
2. The tallest of all.
3. Taller than E, but shorter than C.
4. Taller than C, but shorter than F.

Ans. : (1)

Solution: Among A, B, C, D, E, F, the following conditions are given:

$$D > B, F > D, E > B, C > E$$

From the given conditions, we can rank them as:

$$F > D > C > E > B > A$$

Conclusion: A is the shortest of all.

Question ID 705019

A record player stylus moves along a spiral groove cut on an annular portion of a disc with inner radius 4 cm and outer radius 10 cm. If the record turns 100 times when playing, the stylus travels approximately

1. 2.2 m
2. 4.4 m
3. 22 m
4. 44 m

Ans. : (4)

Solution: Given:

- Inner radius $= 4$ cm $= 0.04$ m
- Outer radius $= 10$ cm $= 0.10$ m
- Number of times the record spins $= 100$

The approximate distance the stylus travels can be calculated as:

$$\text{Distance } = 2\pi\left(\frac{\text{Inner radius } + \text{ Outer radius}}{2}\right) \times \text{ Number of spins}$$

Substitute the values:

$$\text{Distance } = 2 \times 3.14 \times \left(\frac{0.04 + 0.10}{2}\right) \times 100$$

Simplify:

$$\text{Distance } = 2 \times 3.14 \times 0.07 \times 100$$

$$\text{Distance } = 43.96 \text{ m.}$$

Part B

Question ID 705027

If $\vec{L}$ is the orbital angular momentum operator and $\vec{\sigma}$ are the Pauli matrices, which of the following operators commutes with $\vec{\sigma}\cdot\vec{L}$?

1. $\vec{L}-\frac{\hbar}{2}\vec{\sigma}$ 2. $\vec{L}+\frac{\hbar}{2}\vec{\sigma}$ 3. $\vec{L}+\hbar\vec{\sigma}$ 4. $\vec{L}-\hbar\vec{\sigma}$

Topic: Quantum Mechanics

Subtopic: Angular Momentum

Ans. : (2)

Solution: $\sigma.L=\frac{2}{\hbar}\vec{S}.\vec{L}$ $\quad \vec{L}+\frac{\hbar}{2}\sigma=\vec{L}+\vec{S}$ $\quad \left[\vec{S}.\vec{L},L_x\right]=i\hbar\left[L_yS_x-S_yL_z\right]$

So, $\left[\vec{S}.\vec{L},L_x\right]=i\hbar\left(\vec{L}\times\vec{S}\right)_x$

$\left[\vec{S}.\vec{L},\vec{L}\right]=i\hbar\left(\vec{L}\times\vec{S}\right)$ $\quad \left[\vec{S}.\vec{L},\vec{S}\right]=i\hbar\left(\vec{S}\times\vec{L}\right)$, $\left[\vec{S}.\vec{L},\vec{J}\right]=0$

Question ID 705032

The matrix A is given by

$$A=\begin{bmatrix}1&2&-3\\0&3&2\\0&0&-2\end{bmatrix}$$

The eigenvalues of $3A^3+5A^2-6A+2I$, where I is the identity matrix, are

1. $4,9,27$ 2. $1,9,44$ 3. $1,110,8$ 4. $4,110,10$

Topic: Mathematical Physics

Subtopic: Matrices Eigen Values

Ans. : (4)

Solution: Let

$$A=\begin{bmatrix}1&2&-3\\0&3&0\\0&0&-2\end{bmatrix}$$

The characteristic equation is obtained as:

$$\det\left(A-\lambda I\right)=\begin{vmatrix}1-\lambda&2&-3\\0&3-\lambda&0\\0&0&-2-\lambda\end{vmatrix}=0$$

Expanding the determinant:

$(1-\lambda)(3-\lambda)(-2-\lambda)-2(0)-3(0)=0.$

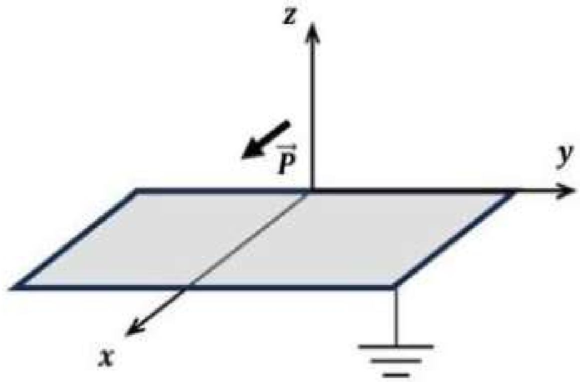
Simplifying: $(1-\lambda)((3-\lambda)(-2-\lambda))=0$, The eigenvalues are: $\lambda=1,\lambda=3,\lambda=-2.$

Hence eigen value for $3A^3 + 5A^2 - 6A + 2I$ is 4,110,10

(For ex. for 1 it is 3+5-6+2=4)

Question ID 705036

A point electric dipole $\vec{P} = p_x\hat{\imath}$ is placed at a vertical distance d above a grounded infinite conducting xy plane as shown in the figure.



At a point $\vec{r}$ $(r \gg d, z > 0)$ far away from the dipole, the electrostatic potential $V(r)$ varies approximately as

1. $\frac{1}{r^2}$ 2. $\frac{1}{r^6}$ 3. $\frac{1}{r^3}$ 4. $\frac{1}{r^4}$

Topic: Electromagnetic Theory

Subtopic: Dipole Moment

Ans. : (3)

Solution: For $r \gg d$, the system effectively forms a quadrupole due to the dipole and its image. The potential at a point P decays faster than a dipole, with:

$$V(r) \propto \frac{1}{r^3}$$

This behavior is characteristic of a quadrupole field.

Question ID 705031

Vorticity of a vector field $\vec{B}$ is defined as $\vec{V} = \vec{\nabla} \times \vec{B}$. Given $\vec{B} = kxyz\hat{r}$, where k is a constant, which one of the following is correct?

1. Vorticity is a null vector for all finite x, y, z

2. Vorticity is parallel to the vector field everywhere

3. The angle between vorticity and vector field depends on x, y, z

4. Vorticity is perpendicular to the vector field everywhere

Topic: Electromagnetic Theory

Subtopic: Magnetostat

Ans. : (4)

Solution: $\nabla \times B = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ kxyz & 0 & 0 \end{vmatrix}$

Here:

$= \hat{r}(0-0) - \hat{\theta}(0-0) + \hat{\phi}(0-0), = A_1\hat{\theta} + A_2\hat{\varphi}$

Hence Vorticity is perpendicular to the vector field ($\hat{r}$) everywhere

Question ID 705028

A quantum mechanical system is in the angular momentum state $|l = 4, l_z = 4\rangle$. The uncertainty in L_x is

1. $\hbar\sqrt{2}$ 2. $2\hbar$ 3. 0 4. $\hbar$

Topic: Quantum Mechanics

Subtopic: Angular Momentum

Ans. : (1)

Solution: The operator L_x in terms of the ladder operators L_+ and L_- is given by: $L_x = \frac{L_+ + L_-}{2}$

$\langle L_x \rangle = 0$ on $|4,4\rangle$ and $L_x^2 = \frac{L_+^2 + L_-^2 + 2\left(L^2 - L_z^2\right)}{4} = \frac{0 + 0 + 2(l.(l+1) - m^2)\hbar^2}{4}$

$\langle L_x^2 \rangle = \frac{2(4.5-16)\hbar^2}{4} = 2\hbar^2 \quad \Delta L_x = \sqrt{\langle L_x^2 \rangle - \langle L_x \rangle^2} = \sqrt{2}\hbar$

Question ID 705033

An integral is given by $\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \exp\left[-(x^2 + y^2 + 2axy)\right]$, where a is a real parameter. The full range of values of a for which the integral is finite, is

1. $-\infty < a < \infty$ 2. $-2 < a < 2$ 3. $-1 < a < 1$ 4. $-1 \le a \le 1$

Topic: Mathematical Physics

Subtopic: Jacobian transformation

Ans. : (3)

Solution: $x^2 + y^2 + 2xay > 0$

Transform, $x = u + v \qquad y = u - v$

$x^2 + y^2 + 2xay = (u-v)^2 + (u-v)^2 + 2a(u+v)(u-v) > 0$

$= 2(u^2 + v^2) + 2\text{a}(u^2 - v^2) > 0 = (2 + 2a)u^2 + (2 - 2a)v^2 > 0$

$2 + 2a > 0;\ 2 - 2a > 0 - 1 < a < 1$

Question ID 705026

If A and B are hermitian operators and C is an antihermitian operator, then

1. $[[A,B],C]$ is hermitian and $[[A,C],B]$ is antihermitian

2. $[[A,B],C]$ and $[[A,C],B]$ are both antihermitian

3. $[[A,B],C]$ and $[[A,C],B]$ are both hermitian

4. $[[A,B],C]$ is antihermitian and $[[A,C],B]$ is hermitian

Topic: Quantum Mechanics

Subtopic: Commutator

Ans. : (2)

Solution: $\left[[A,B],C\right]^{\dagger}=\left[C^{\dagger},[A,B]^{\dagger}\right]=\left[C^{\dagger},\left[B^{\dagger},A^{\dagger}\right]\right]=\left[-C,[B,A]\right]=-\left[[A,B],C\right]$

$$\left[[A,C],B\right]^{\dagger}=\left[B^{\dagger},[A,C]^{\dagger}\right]=\left[B^{\dagger},\left[C^{\dagger},A^{\dagger}\right]\right]=\left[B,[-C,A]\right]=-\left[[A,C],B\right]$$

Question ID 705039

A single particle can exist in two states with energies 0 and E respectively. At high temperatures $(k_BT \gg E)$ the specific heat of the system (C_V) will be approximately

1. proportional to $\frac{1}{T}$

2. proportional to $\frac{1}{T^2}$

3. proportional to $e^{\frac{E}{k_BT}}$

4. Constant

Topic: Thermodynamics and Statistical Mechanics

Subtopic: Canonical Ensemble

Ans. : (2)

Solution: The partition function is given as: $Z = e^{-\beta E(0)} + e^{-\beta E} = 1 + e^{-\beta E}$

Average Energy $< E >= \frac{E(0)e^{-\beta E(0)}+Ee^{-\beta E}}{1+e^{-\beta}} = \frac{Ee^{-\beta}}{1+e^{-\beta}}$

The heat capacity is defined as: $C_V = \left(\frac{\partial E}{\partial T}\right)$

Substituting: $C_V = E\left[\frac{\frac{Ee^{-\beta E}}{k_BT^2}(1+e^{-\beta E})-e^{-\beta E}\frac{Ee^{-\beta E}}{k_BT^2}}{(1+e^{-\beta E})^2}\right]$

For small values of βE, where $\beta = \frac{1}{k_BT}$: $C_V = \frac{E^2}{4k_BT^2}$, $C_V \propto \frac{1}{T^2}$

Question ID 705038

Quantum particles of unit mass, in a potential

$$V(x)=\begin{cases}\frac{1}{2}\omega^2x^2 & x>0\\ \infty & x\le 0\end{cases}$$

are in equilibrium at a temperature T. Let n_2 and n_3 denote the numbers of the particles in the second and third excited states respectively. The ratio n_2/n_3 is given by

1. $\exp\left(\frac{2\hbar\omega}{k_BT}\right)$ 2. $\exp\left(\frac{\hbar\omega}{k_BT}\right)$ 3. $\exp\left(\frac{3\hbar\omega}{k_BT}\right)$ 4. $\exp\left(\frac{4\hbar\omega}{k_BT}\right)$

Topic: Thermodynamics and Statistical Mechanics

Subtopic: Canonical Ensemble

Ans. : (1)

Solution: Half Harmonic Oscillator

The potential function is given as:

$$V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & \text{for } x>0\\ \infty & \text{for } x\le 0\end{cases}$$

In this system, the allowed quantum numbers are $n=1,3,5,7,\ldots$ (only odd quantum numbers are permitted due to boundary conditions).

The energy levels for the half-harmonic oscillator are given by: $E_n=\left(n+\frac{1}{2}\right)\hbar\omega,\ n=1,3,5,\ldots$

For the second energy level $(n=3)$: $E_2=\left(5+\frac{1}{2}\right)\hbar\omega=\frac{11}{2}\hbar\omega$

For the third energy level $(n=5)$: $E_3=\left(7+\frac{1}{2}\right)\hbar\omega=\frac{15}{2}\hbar\omega$

Population Ratio

The ratio of populations in the states n_2 and n_3 is given by the Boltzmann distribution:

$$\frac{n_2}{n_3}=\frac{\frac{e^{-E_2/k_BT}}{Z}}{\frac{e^{-E_3/k_BT}}{Z}}$$

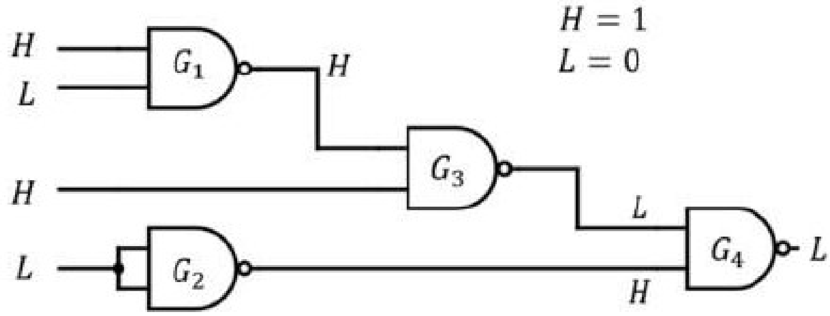
Substituting the energy values: $\frac{n_2}{n_3}=\frac{e^{-1\ \hbar\omega/2k_BT}}{e^{-15\hbar\omega/2k_BT}}$

Simplify the exponent: $\frac{n_2}{n_3}=e^{-\frac{2\hbar\omega}{k_B\omega^T}}$

Thus, the population ratio between the second and third levels depends on the Boltzmann factor.

Question ID 705044

The logic levels H and L at different locations in a digital circuit are found to be as shown in the figure.



Based on these observations, which of the logic gates is not behaving as an ideal NAND gate?

1. G_2 2. G_3 3. G_4 4. G_1

Topic: Electronics

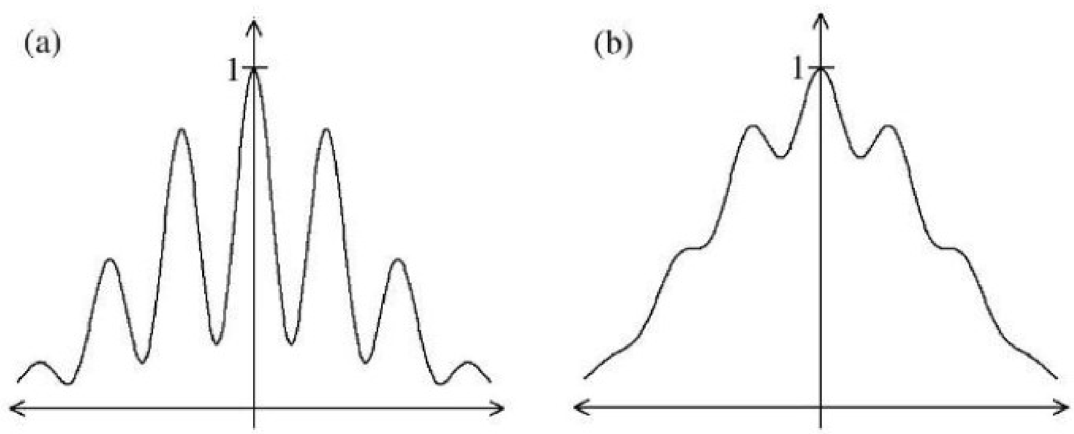
Subtopic: Logic Gate

Ans. : (3)

Solution:

Question ID 705037

A finite sized light source is used in a double slit experiment. The observed intensity pattern changes from figure (a) to figure (b), as shown below.



The observed change can occur due to

1. narrowing of the slits.
2. a reduction in the distance between the slits.
3. a decrease in the coherence length of the light source.
4. a reduction in the size of the light source.

Topic: Electromagnetic Theory

Subtopic: Optics

Ans. : (3)

Solution: From the figure, we see the pattern is completely lost, which shows that the interference fringes are no longer visible. This is a result of the decrease in the coherence length of the light source.

Question ID 705025

The Hamiltonian for a one-dimensional simple harmonic oscillator is given by $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. The harmonic oscillator is in the state $|\psi\rangle = \frac{1}{\sqrt{1+\lambda^2}}\left(|1\rangle + \lambda e^{i\vartheta}|2\rangle\right)$, where $|1\rangle$ and $|2\rangle$ are the normalised first and second excited states of the oscillator and λ, ϑ are positive real constants. If the expectation value $\langle\psi|x|\psi\rangle = \beta\sqrt{\frac{\hbar}{m\omega}}$, the value of β is

1. $\frac{1}{\sqrt{2}(1+\lambda^2)}$
2. $\frac{\sqrt{2}\lambda\cos\vartheta}{1+\lambda^2}$
3. $\frac{2\lambda\cos\vartheta}{1+\lambda^2}$
4. $\frac{\lambda^2\cos\vartheta}{1+\lambda^2}$

Topic: Quantum Mechanics

Subtopic: Harmonic-Oscillator

Ans. : (3)

Solution: The Hamiltonian is given as: $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$

The state $|\psi\rangle$ is: $|\psi\rangle = \frac{1}{\sqrt{1+\lambda^2}}\left(|1\rangle + \lambda e^{i\theta}|2\rangle\right)$

The operator $\hat{x}$ is: $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^\dagger)$

Now applying $\hat{x}$ on $|\psi\rangle$: $\hat{x}|\psi\rangle = \sqrt{\frac{\hbar}{2m\omega}}\frac{1}{\sqrt{1+\lambda^2}}\left[\hat{a}\left(|1\rangle + \lambda e^{i\theta}|2\rangle\right) + \hat{a}^\dagger\left(|1\rangle + \lambda e^{i\theta}|2\rangle\right)\right]$

Expanding this: $\hat{x}|\psi\rangle = \sqrt{\frac{\hbar}{2m\omega}}\frac{1}{\sqrt{1+\lambda^2}}\left[\sqrt{1}|0\rangle + \sqrt{2}\lambda e^{i\theta}|1\rangle + \sqrt{2}|1\rangle + \lambda e^{i\theta}\sqrt{3}|3\rangle\right]$

The expectation value $\langle\psi|\hat{x}|\psi\rangle$ is: $\langle\psi|\hat{x}|\psi\rangle = \sqrt{\frac{\hbar}{2m\omega}}\frac{1}{1+\lambda^2}\left[\langle 1| + \lambda e^{i\theta}\langle 2|\right]\cdot\left[\sqrt{2}|1\rangle + \sqrt{2}\lambda e^{i\theta}|2\rangle\right]$

Simplifying: $\langle\psi|\hat{x}|\psi\rangle = \sqrt{\frac{\hbar}{2m\omega}}\frac{1}{1+\lambda^2}\left[\lambda e^{i\theta}\sqrt{2}\lambda e^{-i\theta} + \sqrt{2}\lambda e^{i\theta}e^{-i}\ \right]$

This becomes: $\langle\psi|\hat{x}|\psi\rangle = \sqrt{\frac{\hbar}{2m\omega}\frac{2\lambda\cos\theta}{1+\lambda^2}}$

Finally: $\beta = \frac{2\lambda\cos\theta}{1+\lambda^2}$

Question ID 705022

A uniform plane square sheet of mass m is centered at the origin of an inertial frame. The sheet is rotating about an axis passing through the origin. At an instant when all its vertices lie on x and y axes, the angular momentum is $\vec{L} = I_0\omega_0(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$, where I_0 is the moment of inertia about the x axis. At this instant, the angular velocity of the sheet is

1. $(2\hat{\imath} + \hat{\jmath} + 2\hat{k})\omega_0$ 2. $(2\hat{\imath} + \hat{\jmath} + \hat{k})\omega_0$ 3. $(2\hat{\imath} + \hat{\jmath})\omega_0$ 4. $(\hat{\imath} + \hat{\jmath})\omega_0$

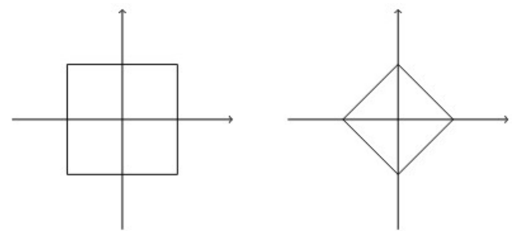
Topic: Classical Mechanics

Subtopic: Moment of Inertia Tensor

Ans. : (2)

Solution: Moment of Inertia Calculations

Mass Element



$$I_{XX} = I_{yy} = \frac{Ma^2}{12}, I_{zz} = \frac{Ma^2}{6}$$

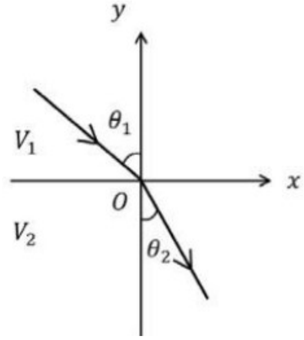
$$I = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix} = \begin{pmatrix} \frac{ma^2}{12} & 0 & 0 \\ 0 & \frac{ma^2}{12} & 0 \\ 0 & 0 & \frac{ma^2}{6} \end{pmatrix} = \begin{pmatrix} I_0 & 0 & 0 \\ 0 & I_0 & 0 \\ 0 & 0 & 2I_0 \end{pmatrix}$$

assume $\vec{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$ and $\vec{L} = I_0\omega_0(2\hat{\imath} + \hat{\imath} + 2\hat{k})$,

$$I_0\omega_0\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = I_0\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \Rightarrow \omega_x = 2\omega_0, \omega_y = \omega_0, \omega_z = \omega_0, \ \omega = 2\omega_0\hat{\imath} + \omega_0\hat{\imath} + \omega_0\hat{k}.$$

Question ID 705034

The region $y > 0$ has a constant electrostatic potential V_1 and $y < 0$ has a constant electrostatic potential $V_2 \neq V_1$. A charged particle with momentum $\overrightarrow{p_1}$ is incident at an angle θ_1 on the interface of the two regions (see figure below).



If the particle has momentum $\overrightarrow{p_2}$ in the region $y < 0$, then the angle θ_2 is given by

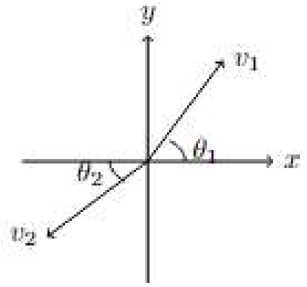
1. $\cos^{-1}\left(\frac{p_2}{p_1}\cos\ \theta_1\right)$
2. $\cos^{-1}\left(\frac{p_1}{p_2}\cos\ \theta_1\right)$
3. $\sin^{-1}\left(\frac{p_2}{p_1}\sin\ \theta_1\right)$
4. $\sin^{-1}\left(\frac{p_1}{p_2}\sin\ \theta_1\right)$

Topic: Electromagnetic Theory

Subtopic: Boundary Value Problem

Ans. : (4)

Solution: Diagram The system is represented as follows:



Applying Conservation of Momentum in the x-Direction From the conservation of momentum in the x-direction: $p_1 \sin\ \theta_1 - p_2 \sin\ \theta_2$

Rearranging to solve for sin θ_2 : $\sin\ \theta_2 = \frac{p_1 \sin\ \theta_1}{p_2}$

Taking the inverse sine to solve for θ_2 :

$\theta_2 - \arcsin\left(\frac{p_1 \sin\ \theta_1}{p_2}\right)$, $\theta_2 - \arcsin\left(\frac{p_1 \sin\ \theta_1}{p_2}\right)$

Question ID 705041

Two non-interacting classical particles having masses m_1 and m_2 are moving in a one-dimensional box of length L. For total energy not exceeding a given value E, the phase space "volume" is given by

1. $\pi L^2 E\left(\frac{m_1 m_2}{m_1+m_2}\right)$ 2. $\pi L^2 E\sqrt{m_1 m_2}$ 3. $2\pi L^2 E\left(\frac{m_1 m_2}{m_1+m_2}\right)$ 4. $2\pi L^2 E\sqrt{m_1 m_2}$

Topic: Thermodynamics and Statistical Mechanics

Subtopic: Microcanonical Ensemble

Ans. : (4)

Solution: Phase Space Volume: The phase space volume is given by: $\int\int \; dp_1 dp_2 dx_1 dx_2$

Consider two classical particles present in a 1D box of length L, and assume they are not interacting.

The spatial integral over x_1 and x_2 gives: $\int_0^L \; dx_1 \int_0^L \; dx_2 = L^2$

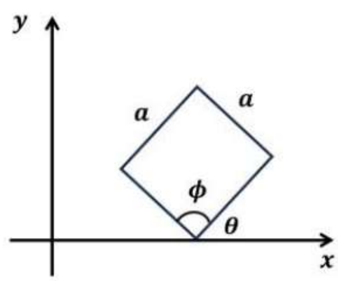
For Momentum: The momentum condition is: $\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = E$

Rewriting in terms of p_1 and p_2 : $\frac{p_1^2}{\sqrt{2m_1 E}} + \frac{p_2^2}{\sqrt{2m_2 E}} = 1$

Phase Space Volume (using area of ellipse) $= 2\pi L^2 E\sqrt{m_1 m_2}$

Question ID 705024

A square plate of dimension $a \times a$ makes an angle $\theta = \pi/4$ with the x axis in its rest frame (S) as shown in the figure.



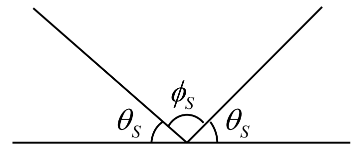
It is moving with a speed $v = \sqrt{\frac{2}{3}}c$ along the x axis with respect to an observer S' (where c is the speed of light in vacuum). The value of the interior angle ϕ indicated in the figure (which is obviously $\pi/2$ in the frame S), as measured in S' is

1. $\frac{\pi}{3}$ 2. $\frac{2\pi}{3}$ 3. $\frac{\pi}{6}$ 4. $\frac{4\pi}{3}$

Topic: Classical Mechanics

Subtopic: STR

Ans. : (1)



Solution: $l_x = a\cos\theta\sqrt{1-\frac{v^2}{c^2}} = a\cos\frac{\pi}{4}\sqrt{1-\frac{2}{3}} = a.\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}$

$l_y = a\sin\theta = \frac{1}{\sqrt{2}}$ $\quad \tan\theta_s = \frac{l_y}{l_x} = \frac{1}{\frac{1}{\sqrt{3}}} \Rightarrow \sqrt{3} \Rightarrow \theta_s = \frac{\pi}{6}$

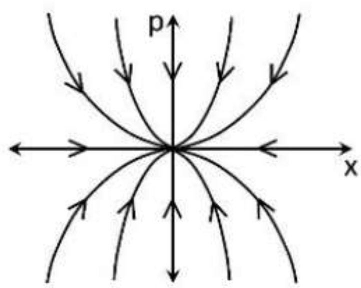
From geometry $2\theta_s + \phi_s = \pi \Rightarrow \phi_s = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

Question ID 705021

The evolution of the dynamical variables $x(t)$ and $p(t)$ is given by

$\dot{x} = ax, \dot{p} = -p$

where a is a constant. The trajectory in (x, p) space for $-1 < a < 0$ is best described by

1.


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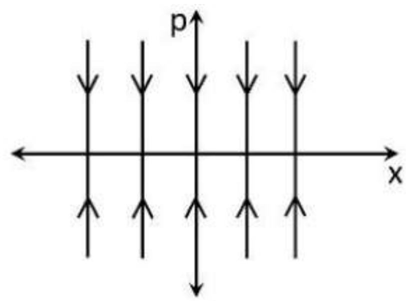

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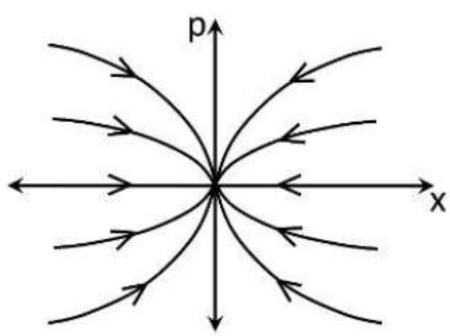

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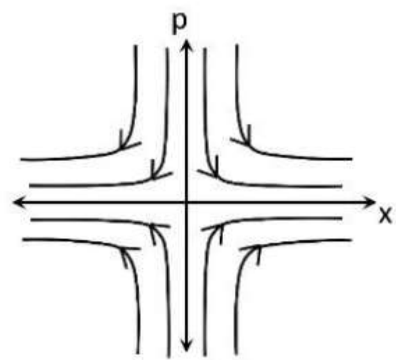


Topic: Classical Mechanics

Subtopic: Phase Curve

Ans. : 1

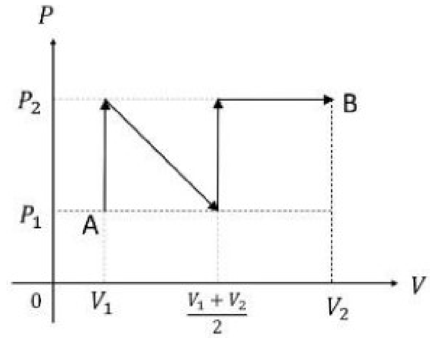
Solution: $\dot{x} = \alpha x \Rightarrow \frac{dx}{dt} = \alpha x, \dot{p} = -p \Rightarrow \frac{dp}{dt} = -p$

From above these equation $\frac{dx}{dp} = -\frac{\alpha x}{p} \Rightarrow \frac{dx}{x} = -\alpha\frac{dp}{p} \Rightarrow \ln x = \ln p^{-\alpha} \Rightarrow x = p^{-\alpha}$

Where $-1 < \alpha < 0$ so option 1 is correct

Question ID 705040

The following $P-V$ diagram shows a process, where an ideal gas is taken quasi-statically from A to B along the path as shown in the figure.



The work done W in this process is

1. $\frac{1}{4}(V_2 - V_1)(3P_2 + P_1)$
2. $\frac{1}{4}(V_2 - V_1)(3P_2 - P_1)$
3. $\frac{1}{2}(V_2 - V_1)(P_1 + P_2)$
4. $\frac{1}{2}(V_2 + V_1)(P_2 - P_1)$

Topic: Thermodynamics and Statistical Mechanics

Subtopic: First Law of Thermodynamics

Ans. : (1)

Solution: Region I: $W_{\mathrm{I}} = \frac{1}{2}\left(V_1 + \frac{V_a - V_1}{2} - V_1\right)(P_2 - P_1)$

Simplify: $W_{\mathrm{I}} = \frac{1}{2}(P_2 - P_1)\left(V_1 + \frac{V_a - V_1}{2} - V_1\right)$, $W_{\mathrm{I}} = \frac{1}{2}(P_2 - P_1)\frac{V_a - V_1}{2}$

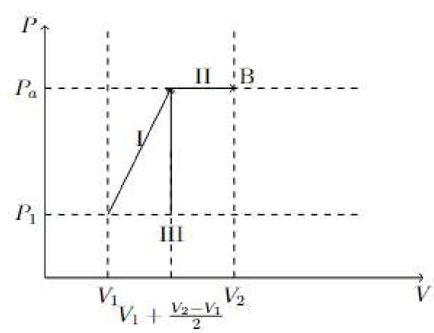
Region II: $W_{\mathrm{II}} = (P_2 - P_1)\left(\frac{V_a - V_1 + V_a}{2}\right)$

Simplify: $W_{\mathrm{II}} = (P_2 - P_1)\left(\frac{2V_a - V_1 - V_a}{2}\right)$

$$W_{\mathrm{II}} = (P_2 - P_1)\frac{V_a - V_1}{2}$$

Region III: $W_{\mathrm{III}} = (P_1 - 0)(V_a - V_1)$

Total Area: $I + II + III$



$$\text{Area } = \frac{1}{2}(P_2 - P_1)\left(\frac{V_2 - V_1}{2}\right) + (P_2 - P_1)\left(\frac{V_2 - V_1}{2}\right) + (P_1 - 0)(V_2 - V_1)$$

$$\text{Area } = \frac{(3P_2 + P_1)}{4}(V_2 - V_1)$$

Question ID 705035

The electric field of an electromagnetic wave in free space is given by

$$\vec{E} = E_0 \sin(\omega t - k_z z)\hat{j}.$$

The magnetic field $\vec{B}$ vanishes for $t = \frac{k_z z}{\omega}$. The Poynting vector of the system is

1. $\frac{k_z}{2\mu_0 \omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$
2. $\frac{4k_z}{\mu_0 \omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$
3. $\frac{2k_z}{\mu_0 \omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$
4. $\frac{k_z}{\mu_0 \omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$

Topic: Electromagnetic Theory

Subtopic: Electromagnetic Waves

Ans. : (4)

Solution: Finding B :

From Maxwell's equations: $\nabla \times E = -\frac{\partial B}{\partial t}$; Taking ∇ and $\partial/\partial t$: $\nabla \to k_z$, $\frac{\partial}{\partial t} \to -\omega$

Substitute into the equation: $k_z \times E = \omega B$; Rearrange to solve for B : $B = \frac{k_z \times E}{\omega}$

$B = \frac{1}{\omega}(k_z \times E_0 \sin(\omega t - k_z z)\hat{j})$; Since $k_z \times \hat{j} = -\hat{\imath}$: $\vec{B} = \frac{1}{\omega} k_z E_0 \sin(\omega t - k_z z)(-\hat{\imath})$

Thus: $\vec{B} = -\frac{k_z E_0}{\sin} \sin(\omega t - k_z z)\hat{\imath}$

The Poynting vector is: $S = \frac{1}{\mu_0} E \times B,\ \vec{S} = \frac{1}{\mu_0} \frac{k_z E_0^2}{\omega} \sin^2(\omega t - k_z z)\hat{k}$

Question ID 705023

A body of mass m is acted upon by a central force $\vec{f}(\vec{r}) = -k\vec{r}$, where k is a positive constant. If the magnitude of the angular momentum is l, then the total energy for a circular orbit is

1. $2\sqrt{\frac{kl^2}{m}}$
2. $\frac{1}{2}\sqrt{\frac{kl^2}{m}}$
3. $\frac{3}{2}\sqrt{\frac{kl^2}{m}}$
4. $\sqrt{\frac{kl^2}{m}}$

Topic: Classical Mechanics

Subtopic: Central Force Problem

Ans. : (4)

Solution: The potential is $\frac{1}{2}kr^2$ effective potential is given by $V_{eff} = \frac{l^2}{2mr^2} + \frac{1}{2}kr^2$ for circular orbit radius is given by $\frac{\partial V_{eff}}{\partial r} = -\frac{l^2}{mr^3} + kr = 0 \Rightarrow r_0 = \left(\frac{l^2}{mk}\right)^{1/4}$

Total energy is $E = \frac{l^2}{2mr_0^2} + \frac{1}{2}kr_0^2 \Rightarrow l\sqrt{\frac{k}{m}}$

Question ID 705043

A battery with an open circuit voltage of 10 V is connected to a load resistor of 485Ω and the voltage measured across the battery terminals using an ideal voltmeter is 9.7 V . The internal resistance of the battery is closest to

1. 30Ω 2. 15Ω 3. 20Ω 4. 40Ω

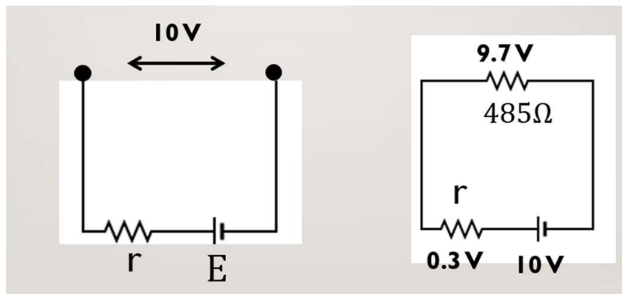
Topic: Electronics

Subtopic: Network Analysis

Ans. : (2)

Solution: $\mathrm{I} = \frac{9.7}{485} = 20\ \mathrm{mA}$

$$\mathrm{r} = \frac{0.3}{20\times10^{-3}} = \frac{300}{20} = 15\Omega$$



Question ID 705029

A hydrogen atom is in the state $|\psi\rangle = \sqrt{\frac{8}{21}}|\psi_{200}\rangle + \sqrt{\frac{3}{7}}|\psi_{210}\rangle + \sqrt{\frac{4}{21}}|\psi_{311}\rangle$, where $|\psi_{\text{nlm}}\rangle$ are normalised eigenstates. If $\hat{L}^2$ is measured in this state, the probability of obtaining the value $2\hbar^2$ is

1. $\frac{13}{21}$ 2. $\frac{4}{21}$ 3. $\frac{17}{21}$ 4. $\frac{3}{7}$

Topic: Quantum Mechanics

Subtopic: Hydrogen Atom

Ans. : (1)

Solution: Calculation

The given state is: $|\psi\rangle = \frac{\sqrt{8}}{\sqrt{21}}|4,00\rangle + \frac{\sqrt{3}}{\sqrt{7}}|4,10\rangle + \frac{\sqrt{4}}{\sqrt{21}}|4,11\rangle$

The square of the state is: $|\psi|^2 = \frac{\sqrt{8}}{\sqrt{21}}|4,00\rangle^2 + \frac{\sqrt{3}}{\sqrt{7}}|4,10\rangle^2 + \frac{\sqrt{4}}{\sqrt{21}}|4,11\rangle^2$

$$p\left(2\hbar^2\right) = \frac{\frac{3}{7}+\frac{4}{21}}{\frac{8}{21}+\frac{3}{7}+\frac{4}{21}} = \frac{13}{21}$$

Question ID 705042

A set of 100 data points yields an average $\bar{x} = 9$ and a standard deviation $\sigma_x = 4$. The error in the estimated mean is closest to

1. 3.0 2. 0.4 3. 4.0 4. 0.3

Topic: Mathematical Physics

Subtopic: Probability

Ans. : (2)

Solution: $Error\ in\ mean = \frac{Standard\ deviation}{\sqrt{N}} = \frac{4}{10} = 0.4$

Question ID 705030

Probability density function of a variable x is given by $P(x) = \frac{1}{2}[\delta(x-a) + \delta(x+a)]$. The variance of x is

1. a^2 2. 0 3. $2a^2$ 4. $\frac{a^2}{2}$

Topic: Mathematical Physics

Subtopic: Probability

Ans. : (1)

Solution: The probability density function is: $P(x) = \frac{1}{2}[\delta(x-a) + \delta(x+a)]$

The variance is given by: $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$

Calculating $\langle x \rangle : \langle x \rangle = \int_{-\infty}^{+\infty} xP(x)dx$

Substitute $P(x) : \langle x \rangle = \int_{-\infty}^{+\infty} x\frac{1}{2}[\delta(x-a) + \delta(x+a)]dx$

Evaluate the delta function: $\langle x \rangle = \frac{1}{2}(-a) + \frac{1}{2}(a), \langle x \rangle = 0$

Calculating $\langle x^2 \rangle : \langle x^2 \rangle = \int_{-\infty}^{+\infty} x^2 P(x)dx$

Evaluate the delta function: $\langle x^2 \rangle = \frac{1}{2}(a^2) + \frac{1}{2}(a^2), \langle x^2 \rangle = a^2$

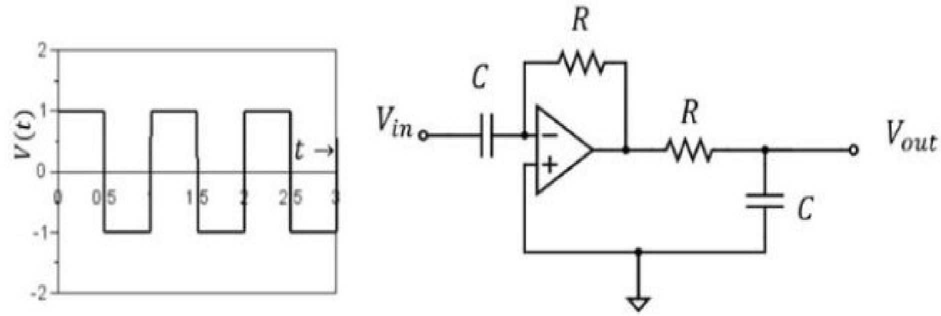
Variance: $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$

Final Result: $(\Delta x)^2 = a^2$

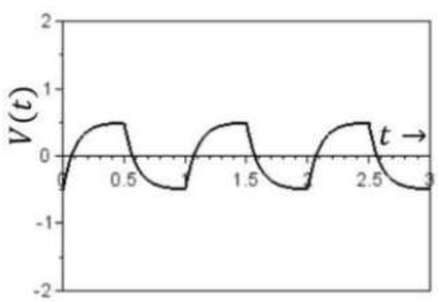
Question ID 705045

A train of square wave pulses is given to the input of an ideal op-amp circuit shown below.

Given that the time period of the input pulses $T \ll RC$ and the opamp does not get into saturation, which of the following best represents the output waveform?



1.



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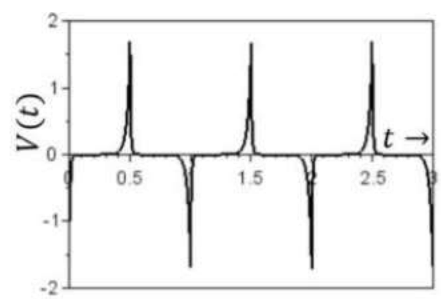

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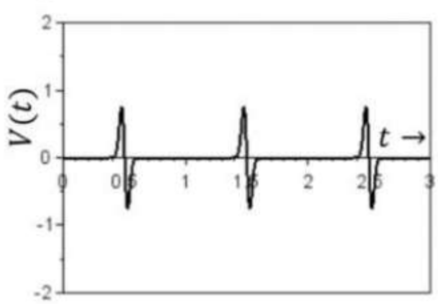
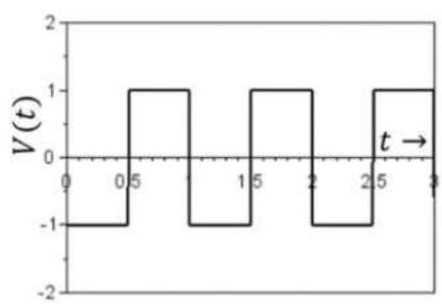


4.

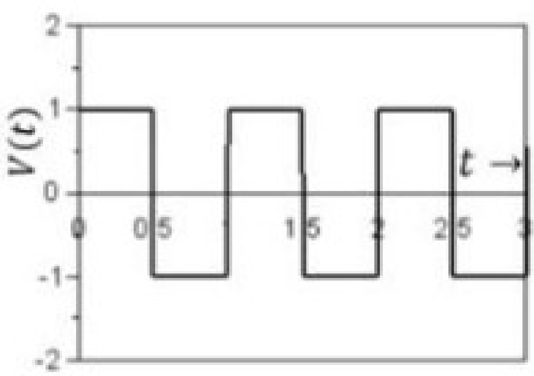


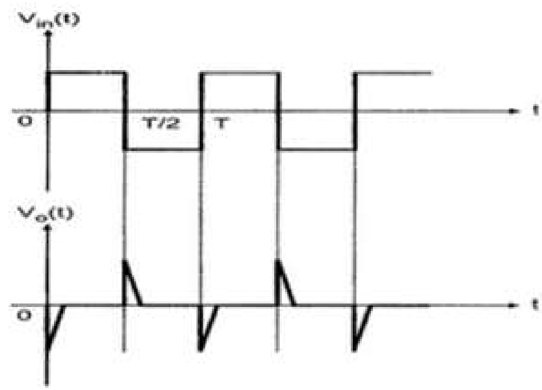
Topic: Electronics

Subtopic: OP-Amp

Ans. : (4)

Solution: Given circuit is combination of Op Amp differentiator circuit with RC integrator circuit. For given input wave $T \ll RC$. So, first circuit will act as ideal differentiator and output will differentiation of square wave which is zero. The spike in differentiation output is because of sudden change of plus from 0 to 1 in amplitude of input. The differentiation output has phase change because of inverting Op Amp property. The next RC circuit is ideal integrator. Hence output will be square wave again.





Part C

Question ID 705070

The bond dissociation energy of a molecule is defined as the energy required to dissociate it. For H_2 and H_2^+molecules, the bond dissociation energies are 4.478 eV and 2.651 eV respectively. If the equilibrium bond lengths of both H_2 and $\mathrm{H}_2^+$are identical, the value of the ionization potential of hydrogen molecule will be closest to

1. $15.427\mathrm{eV}$ 2. $11.773\mathrm{eV}$ 3. $20.729\mathrm{eV}$ 4. 6.471 eV

Topic: Atomic Physics

Subtopic: Hydrogen Atom

Ans. : (1)

Solution: Calculation of Energy in the Process

We need to calculate the energy involved in the following process: $\mathrm{H}_2 \rightarrow \mathrm{H}_2^+ + e^-$

This process can be divided into three steps as follows:

$\mathrm{H}_2 \rightarrow \mathrm{H} + \mathrm{H}, \mathrm{H} \rightarrow \mathrm{H}^+ + e^-, \mathrm{H}^+ + \mathrm{H} \rightarrow \mathrm{H}_2^+.$

Energy Required for Each Step.

The energy required to dissociate H_2 into two hydrogen atoms is: $E_1 = 4.478\mathrm{eV}$

The energy required to ionize a hydrogen atom is: $E_2 = 13.6\mathrm{eV}$

The energy released when $\mathrm{H}^+$and H combine to form H_2^+ is: $E_3 = -2.651\mathrm{eV}$

Total Energy Change The total energy change, ΔE, is calculated as: $\Delta E = E_1 + E_2 + E_3$.

Substituting the values: $\Delta E = 4.478 + 13.6 - 2.651$.

Simplifying: $\Delta E = 15.427\mathrm{eV}$

Final Answer The total energy required for the process is: $\Delta E = 15.427\mathrm{eV}$

Question ID 705059

A particle of unit mass and unit charge is moving in a magnetic field, which varies as $\vec{B}(\vec{r}) = b_0\vec{r}/r^3$ (b_0 is a constant) over a region far away from the origin. If $\vec{L}$ is the instantaneous angular momentum of the particle within that region, then $d\vec{L}/dt$ is

1. $2b_0\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ 2. $-b_0\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ 3. $b_0\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ 4. 4. 0

Topic: Electromagnetic Theory

Subtopic: Charge Particle in Magnetic field

Ans. : (3)

Solution: The magnetic field is given as: $\vec{B} = b_0\frac{\hat{r}}{r^2}$

Force $\vec{F} = q[\vec{v} \times \vec{B}]$, q $= 1$

$\vec{F} = \vec{v} \times \vec{B}, \vec{\tau} = \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$

Torque $= \vec{r} \times \vec{v} \times \vec{B}$

$$= \vec{v}(\vec{r} \cdot \vec{B}) - \vec{B}(\vec{r} \cdot \vec{v}) \ = \vec{v}(\vec{r} \cdot \vec{B}) - \vec{B}(\vec{r} \cdot \vec{v}) \ = \vec{v}\frac{b_0}{r} - \frac{b_0\hat{r}}{r^2} r v_r$$

$$= \frac{b_0}{r}[\vec{v} - v_r\hat{r}] = \frac{b_0}{r}\left[v_\theta\hat{\theta} + v_\varphi\hat{\varphi}\right]$$

Where, $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}, + r\sin\ \theta\dot{\varphi}\hat{\varphi}$

$$\vec{\tau} = \frac{b_0}{r}[r\dot{\theta}\hat{\theta} + r\sin\ \theta\dot{\varphi}\hat{\varphi}] = b_0[\dot{\theta}\hat{\theta} + \sin\ \theta\dot{\varphi}\hat{\varphi}] \ = b_0\frac{d}{dt}\hat{r} = b_0\frac{d\vec{r}/\mathrm{r}}{dt}$$

Option 3 is the correct result.

Question ID 705057

In a non-magnetic material with no free charges and no free currents, the permittivity ϵ is a function of position. If $\vec{E}$ represents the electric field and μ_0, ϵ_0 are free space permeability and permittivity respectively, which one of the following expressions is correct?

1. $\nabla^2\vec{E} - \mu_0\frac{\partial^2(\epsilon\vec{E})}{\partial t^2} - \frac{1}{\epsilon_0}\vec{\nabla}(\vec{E} \cdot \vec{\nabla}\epsilon) = 0$
2. $\nabla^2\vec{E} - \mu_0\frac{\partial^2(\epsilon\vec{E})}{\partial t^2} + \frac{1}{\epsilon_0}\vec{\nabla}(\vec{E} \cdot \vec{\nabla}\epsilon) = 0$
3. $\nabla^2\vec{E} - \mu_0\frac{\partial^2(\epsilon\vec{E})}{\partial t^2} + \vec{\nabla}\left(\frac{1}{\epsilon}\vec{E} \cdot \vec{\nabla}\epsilon\right) = 0$
4. $\nabla^2\vec{E} - \mu_0\frac{\partial^2(\epsilon\vec{E})}{\partial t^2} - \vec{\nabla}\left(\frac{1}{\epsilon}\vec{E} \cdot \vec{\nabla}\epsilon\right) = 0$

Topic: Electromagnetic Theory

Subtopic: Electromagnetic Wave

Ans. : (3)

Solution: Electromagnetic Wave Equation Derivation

Given: $\rho_f = 0,\ J_f = 0.$

From Maxwell's equations: $\nabla \times B = \mu_0\left(J_f + J_d\right)$,

where: $J_d = \epsilon\frac{\partial E}{\partial t}$.

Substitute: $\nabla \times B = \mu_0\epsilon\frac{\partial E}{\partial t}$.

From Faraday's law: $\nabla \times E = -\frac{\partial B}{\partial t}$

Take the curl of both sides: $\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times B)$

Substitute for $\nabla \times B$: $\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}\left(\mu_0\epsilon\frac{\partial E}{\partial t}\right)$.

Using the vector identity: $\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E$, We have: $\nabla(\nabla \cdot E) - \nabla^2 E = -\mu_0 \epsilon \frac{\partial^2 E}{\partial t^2}$

Simplifying with $\nabla \cdot E$

From Gauss's law: $\nabla \cdot D = \rho_f \implies \nabla \cdot (\epsilon E) = 0.$

Thus: $\epsilon \nabla \cdot E + E \cdot \nabla \epsilon = 0$

Solve for $\nabla \cdot E : \nabla \cdot E = -\frac{1}{\epsilon}(E \cdot \nabla \epsilon)$

Substitute Equation (ii) into the Wave Equation Substituting (ii) into:

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu_0 \epsilon \frac{\partial^2 E}{\partial t^2}$$

we get: $\nabla\left(-\frac{1}{\epsilon}(E \cdot \nabla \epsilon)\right) - \nabla^2 E = -\mu_0 \epsilon \frac{\partial^2 E}{\partial t^2}.$

$$\nabla^2 E - \mu_0 \epsilon \frac{\partial^2 E}{\partial t^2} + \nabla\left(\frac{1}{\epsilon}(E \cdot \nabla \epsilon)\right) = 0.$$

Question ID 705046

A particle of mass m is moving in a potential $V(r) = -\frac{k}{r}$, where k is a positive constant. If $\vec{L}$ and $\vec{p}$ denote the angular momentum and linear momentum respectively, the value of α for which $\vec{A} = \vec{L} \times \vec{p} + \alpha m k \hat{r}$ is a constant of motion, is

1. -2 2. -1 3. 2 4. 1

Topic: Classical Mechanics

Subtopic: Central Force Problem

Ans. : (4)

Solution: $\dot{P} = f(r)\frac{\vec{r}}{|r|}, \quad \vec{A} = \vec{L} \times \vec{P} + \alpha m k \hat{r}$

$$\dot{P} \times \vec{L} = m\frac{f(r)}{r}\left(\vec{r} \times \left(\vec{r} \times \dot{\vec{r}}\right)\right) = m\frac{f(r)}{r}\left[\vec{r}\left(\vec{r} \cdot \vec{r} - r^2 \dot{\vec{r}}\right)\right]$$

$$\vec{r} \cdot \vec{r} = \frac{1}{2}\frac{d}{dt}(\vec{r} \cdot \vec{r}) = 2\dot{r}$$

$$\frac{d}{dt}\left(\vec{P} \times \vec{L}\right) = -mf(r)r^2\left[\frac{\vec{r}}{r} - \frac{\vec{r}\dot{r}}{r^2}\right] = mf(r)r^2\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$$

$$A = \vec{L} \times \vec{p} + \alpha m k \hat{r}, \quad \frac{dA}{dt} = 0$$

$$\frac{d}{dt}\left(\vec{L} \times \vec{P}\right) + d\frac{d}{dt}(mk\hat{r}) = 0 - \frac{d}{dt}(mk\hat{r}) + d\frac{d}{dt}(mk\hat{r}) = 0, \ -1 + \alpha = 0, \qquad \alpha = +1$$

Question ID 705047

A linear molecule is modelled as two atoms of equal mass m placed at coordinates x_1 and x_2, connected by a spring of spring constant k. The molecule is moving in one dimension under an additional external potential $V(x_1, x_2) = \frac{1}{2}m\omega_0^2(x_1^2 + x_2^2)$. If one frequency of molecular vibration is ω_0, the other frequency is

1. $\sqrt{\omega_0^2 - \frac{k}{m}}$ 2. $\sqrt{\omega_0^2 + \frac{k}{m}}$ 3. $\sqrt{\omega_0^2 + \frac{2k}{m}}$ 4. $\sqrt{\omega_0^2 - \frac{2k}{m}}$

Topic: Classical Mechanics

Subtopic: Small Oscillation

Ans. : (3)

Solution: The kinetic energy is given by $T = \frac{m(\dot{x}_1^2 + \dot{x}_2^2)}{2}$ the potential energy is given by

$$V(x_1, x_2) = \frac{k(x_2 - x_1)^2}{2} + \frac{1}{2}m\omega_0^2(x_1^2 + x_2^2)$$

$T = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$ and potential energy matrix $V = \begin{bmatrix} k + m\omega_0^2 & -k \\ -k & k + m\omega_0^2 \end{bmatrix}$

For solving normal frequency $|V - \omega^2 T| = 0 \Rightarrow \begin{bmatrix} k + m\omega_0^2 - \omega^2 m & -k \\ -k & k + m\omega_0^2 - \omega^2 m \end{bmatrix} = 0$ solving this

equation $\omega = \omega_0$ $\omega = \sqrt{\omega_0^2 + \frac{2k}{m}}$

Question ID 705073

In a scattering experiment a beam of e^- with an energy of 420 MeV scatters off an atomic nucleus. If the first minimum of the differential cross section is observed at a scattering angle of 45°, the radius of the nucleus (in fermi) is closest to

1. 0.4 2. 8.0 3. 2.5 4. 0.8

Topic: Nuclear Physics

Subtopic: Properties of Nucleus

Ans. : (3)

Solution: Calculation of Nuclear Radius Using Diffraction

Given: - Energy of the incident particle: $E = 420\text{MeV}$, - Scattering angle: $\theta = 45°$.

Step 1: Calculation of Wavelength

The wavelength is calculated using the relation: $\lambda = \frac{hc}{pc} = \frac{hc}{E}$.

Substituting the values: $\lambda=\frac{1240\text{MeV}-\text{fm}}{420\text{MeV}}$

Simplifying: $\lambda=2.95\text{fm}$.

Step 2: Condition for the First Minimum in Diffraction

The condition for the first minimum in the diffraction pattern is: $\sin\,\theta=\frac{1.22\lambda}{d}$

where d is the diameter of the nucleus.

Rearranging for d : $d=\frac{1.22\lambda}{\sin\,\theta}$.

Step 3: Calculation of Radius r

The radius r is half the diameter: $r=\frac{d}{2}=\frac{1.22\lambda}{2\sin\,\theta}$.

Substituting the values: $r=\frac{1.22\times2.95\times10^{-15}}{2\times\frac{1}{\sqrt{2}}}$

Simplifying: $r=2.5\times10^{-15}\text{ m}$

Final Answer: The radius of the nucleus is: $r=2.5\times10^{-1}\ \ \text{m}$

Question ID 705064

A piezoresistive pressure sensor utilizes change in electrical resistance (ΔR) with change in pressure (ΔP) as $\Delta R=-R_0\log_{10}\left(\frac{\Delta P}{P_0}\right)$, where $R_0=500\Omega$ and $P_0=1000\text{mbar}$. A current of $2\mu\text{ A}$ is passed through the sensor and the resultant voltage drop is measured using an analog-to-digital (ADC) converter having a range 0 to 1 V . If a pressure change of 1 mbar is to be measured, amongst the given options, the minimum number of bits needed for the ADC is

1. 12 2. 14 3. 8 4. 10

Topic: Electronics

Subtopic: Digital

Ans. : (4)

Solution: $3\text{mV}\geq\frac{1000mV}{2^n-1}$

$2^n-1\geq\frac{1000mV}{3mV}$, $2^n-1\geq333.33$, $2^n\geq332.33$

$n\geq\log_2\ 332.33$, $\text{n}\geq8.37$

Amongst the given options, the minimum number of bits needed for the ADC is 10

Question ID 705068

Consider a body-centered tetragonal lattice with lattice constants $a = b = a_0$ and $c = \frac{a_0}{2}$. The number of nearest neighbours, the nearest neighbour distance, the number of next nearest neighbours and the next nearest neighbour distance, respectively, are

1. $6, \frac{1}{2}a_0, 8, \frac{\sqrt{3}}{2}a_0$ 2. $8, \frac{\sqrt{3}}{2}a_0, 6, a_0$ 3. $2, \frac{1}{2}a_0, 8, \frac{3}{4}a_0$ 4. $8, a_0, 6, \frac{4}{3}a_0$

Topic: Condensed Matter Physics

Subtopic: Crystallography

Ans. : (3)

Solution: Consider the cubic structure shown in the figure below:

$$a - a_0,\ b - a_0,\ c - \frac{a_0}{2}$$

Nearest Neighbours The number of nearest neighbours:

No. of nearest neighbours -2

The nearest-neighbour distance is given by:

Nearest neighbour distance $-\frac{a_0}{2}$

Next-Nearest Neighbours The next-nearest neighbour distance is calculated as:

$$\sqrt{\left(\frac{a_0}{2}\right)^2 + \left(\frac{a_0}{4}\right)^2}$$

Simplify: $= \sqrt{\frac{a_0^2}{4} + \frac{a_0^2}{16}} = \sqrt{\frac{4a_0^2 + a_0^2}{16}} = \sqrt{\frac{5a_0^2}{16}} = \sqrt{\frac{5a_0^2}{16}} = \frac{\sqrt{5}a_0}{4}$

The number of next-nearest neighbours:

No. of next-nearest neighbours -8

Result: Nearest-neighbour distance: $\frac{a_0}{2}$, Next-nearest neighbor distance: $\frac{\sqrt{5}a_0}{4}$

Question ID 705062

A random walker takes a step of unit length towards right or left at any discrete time step. Starting from $x = 0$ at time $t = 0$, it goes right to reach $x = 1$ at $t = 1$. Hereafter if it repeats the direction taken in the previous step with probability p, the probability that it is again at $x = 1$ at $t = 3$ is

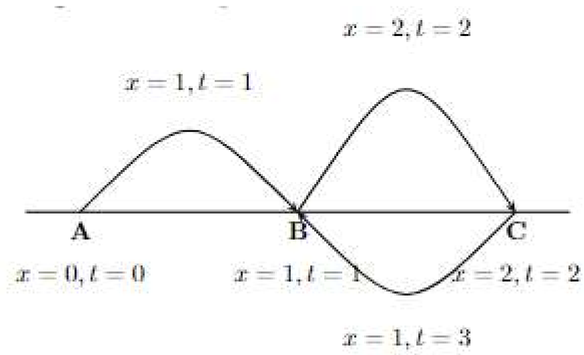
1. $1 - p$ 2. $(1 - p)^2$ 3. $2p(1 - p)$ 4. $4p^2(1 - p)$

Topic: Thermodynamics and Statistical Mechanics

Subtopic: Random Walk Problem

Ans. : (1)

Solution:

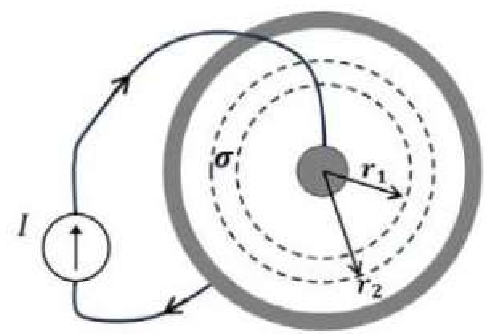


Explanation:

- If the event starts at $A \rightarrow B$ at $t = 1$, then:
- $B \rightarrow C$ at $t = 2$ occurs with probability p.
- $C \rightarrow B$ at $t = 3$ occurs with probability $1 - p$.

Question ID 705060

A two-dimensional sheet with a uniform sheet conductivity of σ has a central metallic point contact and a circular metal contact at the boundary as shown in the figure.



If a constant current I is injected through the central contact and collected at the boundary, then the voltage difference between two points on the sheet at radius r_1 and r_2 is proportional to

1. $\frac{I}{\sigma}\left[\tan^{-1}\left(\frac{r_2}{r_1}\right) - \frac{\pi}{4}\right]$ 2. $\frac{I}{\sigma}\left[\ln\left(\frac{r_2}{r_1}\right)\right]$ 3. $\frac{I}{\sigma}\left(\frac{r_2-r_1}{r_2+r_1}\right)$ 4. $\frac{I}{\sigma}\left(\frac{r_2-r_1}{r_2+r_1}\right)^3$

Topic: Electromagnetic Theory

Subtopic: Electrostatics

Ans. : (2)

Solution: Current Density and Electric Field: The current density is given by: $J = \frac{I}{A}$

For a cylindrical surface of radius r : $A = 2\pi r\delta$

where δ is the thickness.

Substituting A into J : $J = \frac{I}{2\pi r\delta}$

The electric field is related to the current density by: $E = \frac{J}{\sigma} = \frac{I}{2\pi r\delta\sigma}$

Potential Difference: The potential difference V between two points r_1 and r_2 is given by:

$V_2 - V_1 = -\int_{r_1}^{r_2} E dr$

Final Result: $V_1 - V_2 = \frac{I}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$

Question ID 705053

The following four matrices form a representation of a group

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Which of the following represents the multiplication table for the same group?

1.

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	A	I
C	C	B	I	A

2.

	I	A	B	C
I	I	A	B	C
A	A	B	C	I
B	B	C	I	A
C	C	I	A	B

3.

	I	A	B	C
I	I	A	B	C
A	A	C	I	B
B	B	I	C	A
C	C	B	A	I

4.

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	I	A
C	C	B	A	I

Topic: Mathematical Physics

Subtopic: Group Theory

Ans. : (4)

Solution: $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

$$IA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = A, \; IB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = B$$

$$IC = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = C$$

$$AA = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I, \; B,B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Question ID 705067

The Debye temperature of a two-dimensional insulator is 150 K. The ratio of the heat required to raise its temperature from 1 K to 2 K and from 2 K to 3 K is

1. $7:19$ 2. $3:13$ 3. $1:1$ 4. $3:5$

Topic: Condensed Matter Physics

Subtopic: Specific-Heat

Ans. : (1)

Solution: Given: $\Theta_D = 150 \text{ K}$, $c_v \propto T^{d/s}$, $d = 2$ (for phonons) , $s = 1$, $c_v \propto T^2$.

From the Debye model: $c_v = \frac{KT^2}{\Theta_D^3}$

The heat required is: $Q = \int \ c_v dT = \int \ \frac{KT^2}{\Theta_D^3} dT = \frac{K}{\Theta_D^3} \int \ T^2 dT$

Evaluate the integral: $Q = \frac{K}{\Theta_D^3} \cdot \frac{T^3}{3}$.

For, $T_1 = 2 \text{ K}$ to $T_2 = 3 \text{ K} : Q_{1\to2} = \frac{K}{3\Theta_D^3}(2^3 - 1^3) = \frac{7K}{3\Theta_D^3}$.

For, $T_2 = 3 \text{ K}$ to $T_3 = 3 \text{ K} : Q_{2\to3} = \frac{K}{3\Theta_D^3}(3^3 - 2^3) = \frac{27K-8}{3\Theta_D^3} = \frac{19K}{3\Theta_D^3} = \frac{19k}{3Q_D^3}, \frac{Q_{1\to2}}{Q_{2\to3}} = \frac{7}{19}$

Question ID 705075

The Δ^{++}can be produced by colliding a pion beam onto a H_2 target, in a reaction $\pi^+ + p \to \Delta^{++} \to \pi^+ + p$. In the rest frame of Δ^{++}, the energy and momentum of the pion in the final state (in MeV) are closest to (assume $\mathrm{c} = 1$, and $m_\pi \approx 140\mathrm{MeV}, m_p \approx 1\mathrm{GeV}, m_{\Delta^{++}} \approx 1.2\mathrm{GeV}$)

1. 210,156 2. 230,182 3. 175,105 4. 190,130

Topic: Nuclear and Particle Physics

Subtopic: Particle Physics

Ans. : (4)

Solution: The total energy of the pion $(\pi^+)$is given by: $E_{\pi^+} = \frac{m_{\Delta^+}^2 + m_{\pi^+}^2 - m_p^2}{2m_{\Delta^+}}$

where: $m_{\Delta^+} = 1.2 \times 10^3 \mathrm{MeV},\ m_{\pi^+} = 140\mathrm{MeV},\ m_p = 1 \times 10^3 \mathrm{MeV}$

Substitute values into the equation: $E_{\pi^+} = \frac{\left(1.2\times10^3\right)^2 + (140)^2 - \left(1\times10^3\right)^2}{2\cdot(1.2\times10^3)}$

First, compute the terms: $(1.2 \times 10^3)^2 = 1440000,\ (140)^2 = 19600,\ (1 \times 10^3)^2 = 1000000$

Substitute back: $E_{\pi^+} = \frac{1440000+19600-100000}{2\cdot1200}$

Simplify: $E_{\pi^+} = \frac{460000}{2400} = 191.5\mathrm{MeV}$

The energy-momentum relation is: $E_{\pi^+}^2 = p^2c^2 + m_{\pi^+}^2c^4$

Rearrange for p^2c^2 : $p^2c^2 = E_{\pi^+}^2 - m_{\pi^+}^2c^4$

Substitute values: $E_{\pi^+} = 191.5\text{MeV}$, $m_{\pi^+} = 140\text{MeV}$

Compute: $E_{\pi^+}^2 = (191.5)^2 = 36672.25$, $m_{\pi^+}^2 = (140)^2 = 19600.$

Substitute back: $p^2c^2 = 36672.25 - 19600 = 17072.25$

Take the square root: $pc = \sqrt{17072.25} \approx 130.69\text{MeV}$

The momentum of the pion (π^+) is: $pc \approx 130.7\text{MeV}$

Question ID 705066

An astronomer observes 500 objects and classifies them as either of type A or type B. She finds 148 objects to be of type B. Assuming a binomial distribution, the best estimate of the fraction of type A objects and its associated standard deviation respectively are

1. 0.704,0.002 2. 0.70,0.02 3. 0.704,0.031 4. 0.72,0.03

Topic: Mathematical Physics

Subtopic: Probability Distribution

Ans. : (2)

Solution: Given:

Total number of objects - 500

Number of Type B objects - 148

Calculation of Type A Objects: Number of Type A objects $= 500 - 148 = 352$

The fraction of Type A objects is: $\frac{\text{Number of Type A objects}}{\text{Total number of objects}} = \frac{352}{500} - 0.704$

Binomial Distribution: For a binomial distribution, the standard deviation (σ) is given by:

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$

Substitute $p = 0.704, 1 - p = 1 - 0.704 = 0.296$, and $n = 500$: $\sigma = \sqrt{\frac{0.704 \times 0.296}{500}}$

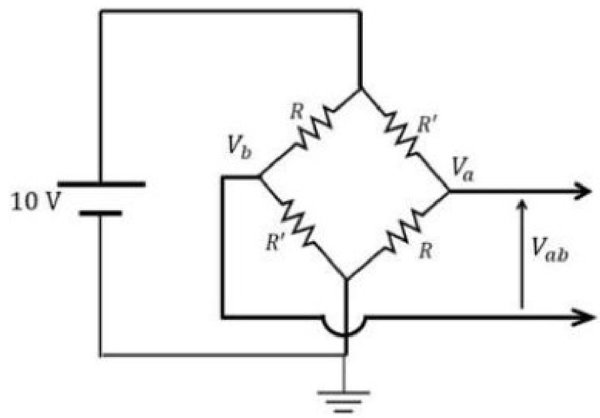
Simplify:

$$\sigma = \sqrt{\frac{0.208384}{500}}, \ \sigma = \sqrt{0.000416768}, \sigma \approx 0.02041 \approx 0.02$$

Final Result: $\sigma \approx 0.02$

Question ID 705065

In the circuit shown in the figure, the resistances R and R' change due to strain. While R increases, R' decreases by the same amount ΔR due to the applied strain. The unstrained values of R and R' are 100Ω each. If same strain is applied to all the resistors, and the output voltage (V_{ab}) changes to 0.3 V, then ΔR is closest to



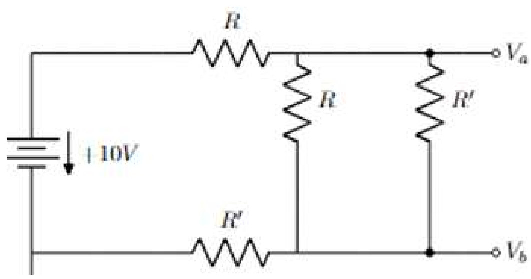
1. 3Ω 2. 1.5Ω 3. 4.5Ω 4. 6Ω

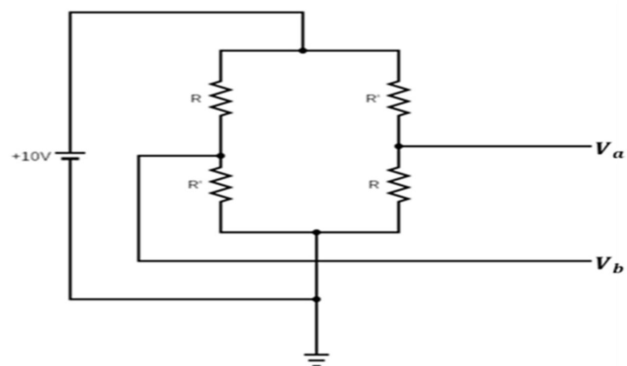
Topic: Electronics

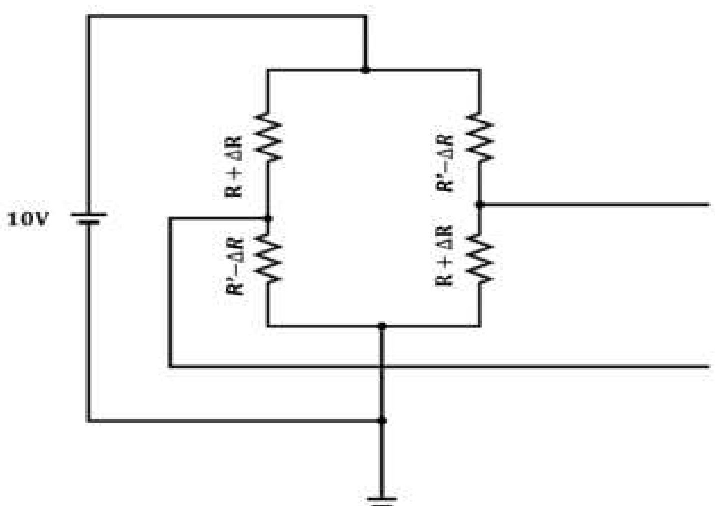
Subtopic: Network Analysis

Ans. : (1)

Solution:







Case Unstrained Resistance

$$V_a = \frac{10}{R+R'} \times R = \frac{10}{200} \times 100 = 5\text{ V},\ V_b = \frac{10}{R+R'} \times R' = \frac{10}{200} \times 100 = 5\text{ V}$$

$V_{ab} = 0, V_a = \frac{10}{(R+\Delta R)+(R'-\Delta R)} \times (R+\Delta R) = \frac{10}{R+R'} \times (R+\Delta R)$

$$V_b = \frac{10}{(R+\Delta R)+(R'-\Delta R)} \times (R'-\Delta R) = \frac{10}{R+R'} \times (R'-\Delta R)$$

$V_{ab} = 0.3 \text{ V}, \frac{10}{R+R'} \times (R+\Delta R) - \frac{10}{R+R'} \times (R'-\Delta R) = 0.3 \text{ V}$

$\frac{10}{200} \times (2\Delta R) = 0.3 \text{ V}, \Delta R = 0.3 \times 10 = 3\Omega$

$\frac{10}{R+R'} \times (R+\Delta R) - \frac{10}{R+R'} \times (R'-\Delta R) = 0.3 \text{ V}, \frac{10}{200} \times (2\Delta R) = 0.3 \text{ V}, \Delta R = 3\ \Omega$

Question ID 705056

The integral $I = \int_0^1 \frac{2x}{1+x^2} dx$ is estimated using Simpson's $1/3^{\text{rd}}$ rule with a grid value of $h = 0.5$.

The difference ($I_{\text{estimated}} - I_{\text{exact}}$) is closest to

1. 0.007 2. 0.001 3. 0.0007 4. -0.005

Topic: Mathematical Physics

Subtopic: Numerical Analysis Simpsons

Ans. : (1)

Solution: The integral to evaluate is: $I = \int_0^1 \frac{2x}{1+x^2} dx$

Using Simpson's $1/3$ rd Rule:

$$I = \frac{h}{3}[y_0 + 4(y_1 + y_3 + y_5 + \cdots) + 2(y_2 + y_4 + y_6 + \cdots + y_{n-1}) + y_n]$$

Here, $h = 0.5$, and the function is: $y = \frac{2x}{1+x^2}$

Compute y_0, y_1, and y_2 : $y_0 = \frac{2(0)}{1+(0)^2} = 0$

$$y_1 = \frac{2(0.5)}{1+(0.5)^2} = \frac{1}{1+0.25} = \frac{1}{1.25} = \frac{4}{5}, y_2 = \frac{2(1)}{1+(1)^2} = \frac{2}{2} = 1$$

Substitute into Simpson's $1/3$ rd Rule: $I \approx \frac{0.5}{3}[y_0 + 4y_1 + y_2]$

Substitute $y_0 = 0, y_1 = \frac{4}{5}, y_2 = 1 : I \approx \frac{0.5}{3}\left[0 + 4\left(\frac{4}{5}\right) + 1\right]$

Final Result: $I = 0.7$

$I_{\text{exact}} = 0.6931$

$I_{\text{estimated}} - I_{\text{exact}} = 0.007$

Question ID 705050

Using a normalized trial wavefunction $\psi(x) = \sqrt{\alpha}e^{-\alpha|x|}$ (α is a positive real constant) for a particle of mass m in the potential $V(x) = -\lambda\delta(x), (\lambda > 0)$, the estimated ground state energy is

1. $-\frac{m\lambda^2}{\hbar^2}$ 2. $\frac{m\lambda^2}{\hbar^2}$ 3. $\frac{m\lambda^2}{2\hbar^2}$ 4. $-\frac{m\lambda^2}{2\hbar^2}$

Topic: Quantum Mechanics

Subtopic: Variational Principle

Ans. : (4)

The given wave function is exact wave function for given potential so the exact eigen value will be optimize solution so eigen value is $-\frac{m\lambda^2}{2\hbar^2}$

Question ID 705055

The general solution for the second order differential equation

$$\frac{d^2y}{dx^2} = y = x\sin\, x$$

will be

1. $C_1e^x + C_2e^{-x} - \frac{1}{2}(x\sin\, x + \cos\, x)$ 2. $C_1e^x + C_2e^{-x} - \frac{1}{2}(\sin\, x - x\cos\, x)$

3. $C_1e^x + C_2e^{-x} + \frac{1}{2}x(\sin\, x - \cos\, x)$ 4. $C_1e^x + C_2e^{-x} + \frac{1}{2}x(\sin\, x + \cos\, x)$

(where C_1 and C_2 are arbitrary constants)

Topic: Mathematical Physics

Subtopic: Differential Equation

Ans. : (1)

Solution: Solution to the Differential Equation

The given differential equation is:

$$\frac{d^2y}{dx^2} = y = x\sin\, x$$

Using the operator D, where $D = \frac{d}{dx}$, the equation becomes:

$$(D^2 - 1)y = x\sin\, x$$

Solution of the Homogeneous Equation:

The auxiliary equation is: $D^2 - 1 = 0$

The roots of this equation are: $m = \pm 1$

Thus, the complementary function (CF) is: $y_c = C_1 e^{x} + C_2 e^{-x}$

Particular Integral (PI):

To find the particular integral (PI), we use the right-hand side $x\sin\ x$: $\mathrm{PI} = \frac{1}{D^2-1} x\sin\ x$

Write $\sin\ x$ as the imaginary part of e^{ix} : $\mathrm{PI} = \mathrm{Im}\left(\frac{1}{D^2-1} x e^{ix}\right)$

Substitute $D \rightarrow D + i$ for e^{ix} : $\mathrm{PI} = \mathrm{Im}\left(e^{ix} \cdot \frac{1}{(D+i)^2-1} x\right)$

Simplify the denominator: $(D+i)^2 - 1 = D^2 + 2iD - 1$

Thus: $\mathrm{PI} = \mathrm{Im}\left(e^{ix} \cdot \frac{1}{D^2+2iD-} x\right)$

General Solution: The general solution of the differential equation is: $y(x) = y_c + \mathrm{PI}$

$$y(x) = C_1 e^{x} + C_2 e^{-x} + \mathrm{Im}\left(e^{ix} \cdot \frac{1}{D^2 + 2iD - 2} x\right)$$

We aim to solve the particular integral (PI): $\mathrm{PI} = e^{ix} \cdot \frac{1}{D^2+2iD-2} x$

Rewrite the denominator: $\mathrm{PI} = -\frac{e^{ix}}{2} \cdot \frac{1}{1-\frac{(D^2+2iD)}{2}} x$

Express as a series expansion: $\mathrm{PI} = -\frac{e^{ix}}{2}\left[1 + \frac{D^2+2iD}{2} + \cdots\right] x$

Since higher-order terms vanish: $\mathrm{PI} = -\frac{e^{ix}}{2}\left[x + \frac{D^2+2iD}{2} x\right]$

Expand D^2 and D : $D(x) = 1,\ D^2(x) = 0$

Substitute into the expression: $\mathrm{PI} = -\frac{e^{ix}}{2}\left[x + \frac{2i}{2}\right]$

Using $e^{ix} = \cos\ x + i\sin\ x$: $\mathrm{PI} = -\frac{1}{2}[(x+i)(\cos\ x + i\sin\ x)]$

Simplify: $\mathrm{PI} = -\frac{1}{2}[x\cos\ x + ix\sin\ x + \cos\ x + i\sin\ x]$

Separate real and imaginary parts: $\mathrm{Re}\ (\mathrm{PI}) = -\frac{1}{2}(x\cos\ x + \cos\ x) = -\frac{1}{2}\cos\ x(x+1)$

$$\mathrm{Im}\ (\mathrm{PI}) = -\frac{1}{2}(x\sin\ x + \sin\ x) = -\frac{1}{2}\sin\ x(x+1)$$

Thus: $\mathrm{PI} = -\frac{1}{2}(x\sin\ x + \cos\ x)$

General Solution

The general solution is: $y(x) = y_c + \mathrm{PI}$

Substitute $y_c = C_1 e^{x} + C_2 e^{-x}$ and PI: $y(x) = C_1 e^{x} + C_2 e^{-x} - \frac{1}{2}(x\sin\ x + \cos\ x)$

Final Solution: $y(x) = C_1 e^{x} + C_2 e^{-x} - \frac{1}{2}(x\sin\ x + \cos\ x)$

Question ID 705074

π^-has spin 0 and negative intrinsic parity. In a reaction a deuteron in its ground state ($J = 1$, parity is +1) captures a π^-in p-wave to produce a pair of neutrons (intrinsic parity is +1). The neutrons will be produced in a state with

1. $l = 1, S = 0$ 2. $l = 0, S = 1$ 3. $l = 1, S = 1$ 4. $l = 0, S = 0$

Topic: Nuclear and Particle Physics

Subtopic: Particle Physics

Ans. : (4)

Solution: The reaction is: $\pi^- + d \rightarrow n + n$

Given that the process involves a p-wave: $l - 1$

The parity of the initial state is: $\pi^- + d \rightarrow (-1)(+1)$

The parity of the final state is: $n + n \rightarrow (-1)(+1)$

For the p-wave $(l - 1)$, the parity is multiplied by $(-1)^l : (-1)(+1) \times (-1)^1$

Thus, the total parity conservation gives: $(-1)^1 \times 1 - +1$

Therefore, $l - 0$ satisfies the conservation of parity. Conservation of Angular Momentum In the initial state: $\pi^- + d \rightarrow$ spin $- 0 + 1$

In the final state: $n + n \Rightarrow \text{spin} - \frac{1}{2} + \frac{1}{2}$

Key conditions: $\Delta S - 0,\ l - 0,\ S = 0$

Hence, the conservation of angular momentum is satisfied: $l - 0$

Question ID 705051

The Hamiltonian of a particle of mass m is given by $H = \frac{p^2}{2m} + V(x)$, with

$$\mathrm{V}(x) = \begin{cases} -\alpha x & \text{for } x \leq 0 \\ \beta x & \text{for } x > 0 \end{cases}$$

where α, β are positive constants. The n^{th} energy eigenvalue E_n obtained using WKB approximation is

$$E_n^{3/2} = \frac{3}{2}\left(\frac{\hbar^2}{2m}\right)^{1/2} \pi \left(n - \frac{1}{2}\right) f(\alpha, \beta)\ (n = 1,2, \dots)$$

The function $f(\alpha, \beta)$ is

1. $\sqrt{\frac{\alpha^2\beta^2}{2(\alpha^2+\beta^2)}}$ 2. $\frac{\alpha\beta}{\alpha+\beta}$ 3. $\frac{\alpha+\beta}{4}$ $\frac{1}{2}\sqrt{\frac{\alpha^2+\beta^2}{2}}$

Topic: Quantum Mechanics

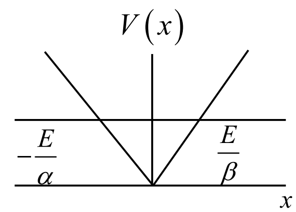
Subtopic: WKB Approximation

Ans. : (2)

Solution: Hamiltonian and Potential

The Hamiltonian is given by: $H = \frac{p^2}{2m} + V(x)$, where $V(x) = \begin{cases} -\alpha x, x < 0 \\ \beta x, x > 0 \end{cases}$

Using wkb approximation $\int_{-\alpha/E}^{\beta/E} p\,dx = \left(n + \frac{1}{2}\right)\pi\hbar$



$$\int_{-\alpha/E}^{0} \sqrt{2m(E + \alpha x)}dx + \int_{0}^{\beta/E} \sqrt{2m(E + \beta x)}dx = \left(n + \frac{1}{2}\right)\pi\hbar$$

$$\sqrt{2mE}\left(\int_{-\alpha/E}^{0} \sqrt{\left(1 + \frac{\alpha}{E}x\right)}dx + \int_{0}^{\beta/E} \sqrt{\left(E - \frac{\beta}{E}x\right)}dx\right) = \left(n + \frac{1}{2}\right)\pi\hbar$$

Put $1 + \frac{\alpha}{E}x = t_1$ and $1 - \frac{\beta}{E}x = t_2$

$$\sqrt{2mE} \times \frac{E}{\alpha}\int_0^1 \sqrt{t_1}dt_1 + \sqrt{2mE} \times \frac{E}{\beta}\int_0^1 \sqrt{t_2}dt_2 = \left(n + \frac{1}{2}\right)\pi\hbar$$

$$E^{3/2} = \frac{\alpha\beta}{\alpha + \beta}\frac{3}{2}\left(\frac{\hbar^2}{2m}\right)^{1/2}\left(n + \frac{1}{2}\right)\pi\hbar \text{ so } f(\alpha, \beta) = \frac{\alpha\beta}{\alpha + \beta}$$

Question ID 705054

An integral transform $\tilde{f}(x)$ of a function $f(x)$ can be regarded as a result of applying an operator F to the function such that

$$(Ff)(x) \equiv \tilde{f}(x) = \int_{-\infty}^{\infty} dy e^{-ixy} f(y)$$

If I is the identity operator, then the operator F^4 is given by

1. $(2\pi)^4 I$ 2. $(2\pi)I$ 3. I 4. $(2\pi)^2 I$

Topic: Mathematical Physics

Subtopic: Fourier Transformation

Ans. : (4)

Solution: Starting with the definition of $F(f(x))$: $F(f(x)) = \tilde{f}(x) = \int_{-\infty}^{+\infty} e^{-ixy} f(y)dy$

For $F^2(f(x))$, we have: $F^2(f(x)) = F\left(\tilde{f}(x)\right) = \int_{-\infty}^{+\infty} e^{-ixz}\tilde{f}(z)dz$

Substituting the expression for $\tilde{f}(z)$: $= \int_{-\infty}^{+\infty} e^{-ixz}\left(\int_{-\infty}^{+\infty} e^{-izy} f(y)dy\right)dz$

Changing the order of integration: $= \int_{-\infty}^{+\infty} f(y)\left(\int_{-\infty}^{+\infty} e^{-i(x+z)y}dy\right)dz$

The inner integral evaluates as: $\int_{-\infty}^{+\infty} e^{-i(x+z)y}dy = 2\pi\delta(x+z)$

Substituting this result back: $F^2(f(x)) = \int_{-\infty}^{+\infty} f(y)2\pi\delta(x+z)dz$

Using the sifting property of the Dirac delta function: $F^2(f(x)) = 2\pi f(-x)$

Now, the parity $P(f(x))$ is defined as: $P(f(x)) = f(-x)$

Thus, $F^2 = 2\pi P$

For F^4, squaring F^2 : $F^4 = (2\pi)^2 P^2$

Finally, $F^4 = (2\pi)^2 I$

where I represents the identity operator.

Question ID 705072

An atom of mass m, initially at rest, resonantly absorbs a photon. It makes a transition from the ground state to an excited state and also gets a momentum kick. If the difference between the energies of the ground state and the excited state is $\hbar\Delta$, the angular frequency of the absorbed photon is closest to

1. $\Delta\left(1+\frac{3}{2}\frac{\hbar\Delta}{mc^2}\right)$ 2. $\Delta\left(1+\frac{1}{2}\frac{\hbar\Delta}{mc^2}\right)$ 3. $\Delta\left(1+\frac{\hbar\Delta}{mc^2}\right)$ 4. $\Delta\left(1+2\frac{\hbar\Delta}{mc^2}\right)$

Topic: Atomic Physics

Subtopic: Atomic Transition

Ans. : (2)

Solution: According to the conservation of energy,

Energy of photon = Energy difference of two states (excited and ground state) + K.E. of the atom. $\hbar\omega = \hbar\Delta + \frac{p^2}{2m}$

From the conservation of momentum: $p = \frac{\hbar\omega}{c}$

Substituting Eq. (2) into Eq. (1), $\hbar\omega = \hbar\Delta + \frac{\left(\frac{\hbar\omega}{c}\right)^2}{2m}$

But the kinetic energy of the atom is very small, so the momentum is also very small.

Hence, we approximate: $p^2 \approx 0$

Thus, $\hbar\omega \approx \hbar\Delta$

Expanding the original equation: $\hbar\omega = \hbar\Delta + \frac{\hbar^2\omega^2}{2mc^2}$

Dividing throughout by $\hbar$: $\omega = \Delta + \frac{\Delta^2\hbar}{2mc^2}$

Finally, $\omega = \Delta\left(1+\frac{\hbar\Delta}{2mc^2}\right)$

Question ID 705071

Helium atom is excited to a state with the configuration ($2s2p$) with an energy 58.3 eV . After some time, this atom spontaneously ejects a single electron. The value of the orbital angular momentum quantum number (l) of the ejected electron in the final state of the system is (Ionization potential of $\mathrm{He}(1s)^2$ is 24.6 eV)

1. 1 2. 0 3. 2 4. 3

Topic: Atomic Physics

Subtopic: Hydrogen Atom

Ans. : (1)

Solution: The configuration of the excited state is: $2s2p$

When an electron is ejected from the excited state, it is most likely to come from the p-orbital rather than the s-orbital.

Thus, the angular momentum of the ejected electron is: $l=1$

Question ID 705061

Rotational energy of a molecule in the angular momentum state j is given by $E_j=\frac{\hbar^2}{2I}j(j+1)$, where I is the moment of inertia of the molecule. The probability that the molecule will be in its ground state at temperature T (such that $k_BT\gg\frac{\hbar^2}{2I}$), is

1. $\frac{3}{2}\frac{\hbar^2}{1k_BT}$ 2. $\frac{2}{3}\frac{\hbar^2}{Ik_BT}$ 3. $\frac{1}{2}\frac{\hbar^2}{Ik_BT}$ 4. $\frac{\hbar^2}{Ik_BT}$

Topic: Thermodynamics and Statistical Mechanics

Subtopic: Canonical Ensemble

Ans. : (3)

Solution: The energy levels for a molecule are given by: $E_j=\frac{h^2}{8I}j(j+1)$

The probability that the molecule is in the ground state $(j=0)$ is: $P=\frac{(2j+1)e^{-\beta E_j}}{Z}$,

The rotational partition function is: $Z=\int_0^{\infty}(2j+1)e^{-\beta\frac{h^2}{8I}j(j+1)}dj$.

Let: $\beta\frac{h^2}{8I}j(j+1)=t\Longrightarrow\frac{d}{dj}\left(\beta\frac{h^2}{8I}j(j+1)\right)=\frac{2j+1}{\beta\frac{h^2}{8I}}dt$.

Thus: $(2j+1)dj=\frac{\beta\frac{h^2}{8I}}{1}dt$.

Substitute into the integral for Z : $Z=\int_0^{\infty}e^{-t}\cdot\frac{8I}{\beta h^2}dt$

Thus: $Z = \frac{8I}{\beta h^2}$.

For the ground state $(j = 0) : E_0 = 0$

The probability becomes: $P = \frac{(2\cdot 0+1)e^{-\beta E_0}}{Z} = \frac{1}{Z}$

Thus, the probability for the ground state is: $P = \frac{1}{2}\frac{\hbar^2}{Ik_BT}$

Question ID 705048

For a simple harmonic oscillator, the Lagrangian is given by

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$$

If $H(q,p)$ is the Hamiltonian of the system and $A(p,q) = \frac{1}{\sqrt{2}}(p + iq)$, the Poisson bracket $\{A, H\}$ is

1. i A
2. A^*
3. $-iA^*$
4. $-i$ A

Topic: Classical Mechanics

Subtopic: Poisson Bracket

Ans. : (1)

Solution: Lagrangian and Hamiltonian Formulation. The Lagrangian is given by: $L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$.

Simplify further: $H = \frac{\dot{q}^2}{2} + \frac{q^2}{2}$.

Using $p = \dot{q}$, the Hamiltonian becomes: $H = \frac{p^2}{2} + \frac{q^2}{2}$.

Complex Variable Transformation Define: $A(p,q) = \frac{1}{\sqrt{2}}(p + iq)$

Compute $\{A, H\}$, the Poisson bracket: $\{A,H\} = \frac{\partial A}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial A}{\partial p}\frac{\partial H}{\partial q}$.

Substitute into the Poisson bracket: $\{A,H\} = \left(\frac{i}{\sqrt{2}}\cdot p\right) - \left(\frac{1}{\sqrt{2}}\cdot q\right)$.

Simplify: $\{A,H\} = \frac{ip}{\sqrt{2}} - \frac{q}{\sqrt{2}}$. $[A,H] = iA$

Question ID 705063

Five classical spins are placed at the vertices of a regular pentagon. The Hamiltonian of the system is $H = J\sum S_iS_j$, where $J > 0, S_i = \pm 1$ and the sum is over all possible nearest neighbour pairs. The degeneracy of the ground state is

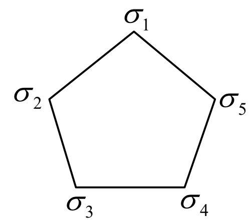
1. 8
2. 5
3. 4
4. 10

Topic: Thermodynamics and Statical Mechanics

Subtopic: Ising-Model

Ans. : (4)

Solution: $H = J\sum S_i S_j = S_1S_2 + S_2S_3 + S_3S_4 + S_4S_5 + S_5S_1$



S.NO	σ1	σ2	σ3	σ4	σ5	E
1	1	1	1	1	1	5
2	1	1	1	1	-1	1
3	1	1	1	-1	1	1
4	1	1	1	-1	-1	1
5	1	1	-1	1	1	1
6	1	1	-1	1	-1	-3
7	1	1	-1	-1	1	1
8	1	1	-1	-1	-1	1
9	1	-1	1	1	1	1
10	1	-1	1	1	-1	-3
11	1	-1	1	-1	1	-3
12	1	-1	1	-1	-1	-3
13	1	-1	-1	1	1	1
14	1	-1	-1	1	-1	-3
15	1	-1	-1	-1	1	1
16	1	-1	-1	-1	-1	1
17	-1	1	1	1	1	1
18	-1	1	1	1	-1	1
19	-1	1	1	-1	1	-3
20	-1	1	1	-1	-1	1
21	-1	1	-1	1	1	-3
22	-1	1	-1	1	-1	-3
23	-1	1	-1	-1	1	-3
24	-1	1	-1	-1	-1	1
25	-1	-1	1	1	1	1
26	-1	-1	1	1	-1	1
27	-1	-1	1	-1	1	-3
28	-1	-1	1	-1	-1	1
29	-1	-1	-1	1	1	1
30	-1	-1	-1	1	-1	1
31	-1	-1	-1	-1	1	1
32	-1	-1	-1	-1	-1	5

Question ID 705069

The band dispersion of electrons in a two-dimensional square lattice (lattice constant a) is given by,

$$E(k_x, k_y) = -2(t_x \cos\ k_x a + t_y \cos\ k_y a)$$

where $t_x, t_y > 0$. The effective mass tensor $m^* = \begin{pmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{pmatrix}$ of electrons at $\vec{k} = \left(\frac{\pi}{a}, \frac{\pi}{a}\right)$ is

1. $\begin{pmatrix} 0 & \frac{\hbar^2}{2a^2\sqrt{t_x t_y}} \\ \frac{\hbar^2}{2a^2\sqrt{t_x t_y}} & 0 \end{pmatrix}$

2. $\begin{pmatrix} \frac{\hbar^2}{2a^2 t_x} & 0 \\ 0 & \frac{\hbar^2}{2a^2 t_y} \end{pmatrix}$

3. $\begin{pmatrix} -\frac{\hbar^2}{2a^2 t_x} & 0 \\ 0 & -\frac{\hbar^2}{2a^2 t_y} \end{pmatrix}$

4. $\begin{pmatrix} 0 & -\frac{\hbar^2}{2a^2(t_x+t_y)} \\ -\frac{\hbar^2}{2a^2(t_x+t_y)} & 0 \end{pmatrix}$

Topic: Condensed Matter Physics

Subtopic: Band Theory

Ans. : (3)

Solution: The energy expression is: $E(k_x, k_y) = -2(t_x \cos\ k_x a + t_y \cos\ k_y a)$.

The effective mass tensor is defined as: $m^* = \begin{pmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{pmatrix}$,

where: $m^* = \begin{pmatrix} \frac{\partial^2 E}{\partial k_x^2} & \frac{\partial^2 E}{\partial k_2 \partial k_y} \\ \frac{\partial^2 E}{\partial k_y \partial k_x} & \frac{\partial^2 E}{\partial k_y^2} \end{pmatrix}$.

For the second derivative with respect to k_x : $\frac{\partial^2 E}{\partial k_x^2} = \frac{\partial}{\partial k_x}\left(\frac{\partial E}{\partial k_x}\right)$.

First derivative: $\frac{\partial E}{\partial k_x} = -2t_x \cos\ k_x a - 2t_y \cos\ k_y a$.

Taking the second derivative: $\frac{\partial^2 E}{\partial k_x^2} = \frac{\partial}{\partial k_x}[-2t_x \cos\ k_x a] = -2t_x \frac{\partial}{\partial k_x} \cos\ k_x a$

Using $\frac{\partial}{\partial k_x} \cos\ k_x a = -a \sin\ k_x a$, $\frac{\partial^2 E}{\partial k_x^2} = 2t_x a^2 \cos\ k_x a$.

At $k_x = \frac{\pi}{a}$, $\cos\left(\frac{\pi}{a} \cdot a\right) = \cos\ \pi = -1$

Substituting: $\frac{\partial^2 E}{\partial k_x^2} = 2t_x a^2(-1) = -2t_x a^2$.

For the second derivative with respect to k_y : $\frac{\partial^2 E}{\partial k_y^2} = \frac{\partial}{\partial k_y}\left(\frac{\partial E}{\partial k_y}\right)$.

First derivative: $\frac{\partial E}{\partial k_y} = -2t_x \cos\ k_x a - 2t_y \cos\ k_y a$.

Taking the second derivative: $\frac{\partial^2 E}{\partial k_y^2} = \frac{\partial}{\partial k_y}\left[-2t_y \cos\, k_y a\right] = -2t_y \frac{\partial}{\partial k_y}\cos\, k_y a$

Using $\frac{\partial}{\partial k_y}\cos\, k_y a = -a\sin\, k_y a : \frac{\partial^2 E}{\partial k_y^2} = 2t_y a^2 \cos\, k_y a$

At $k_y = \frac{\pi}{a}$: $\cos\left(\frac{\pi}{a}\cdot a\right) = \cos\, \pi = -1$

Substituting: $\frac{\partial^2 E}{\partial k_y^2} = 2t_y a^2(-1) = -2t_y a^2$

Finally, the effective mass tensor becomes: $m^* = \begin{pmatrix} \frac{\hbar^2}{-2t_x a^2} & 0 \\ 0 & \frac{\hbar^2}{-2t_y a^2} \end{pmatrix}$.

We begin by analyzing the second partial derivatives for $E(k_x, k_y)$.

Given: $E(k_x, k_y) = -2(t_x \cos\, k_x a + t_y \cos\, k_y a)$.

Mixed Partial Derivative: $\frac{\partial^2 E}{\partial k_x \partial k_y}$

$$\frac{\partial^2 E}{\partial k_x \partial k_y} = \frac{\partial}{\partial k_x}\left(\frac{\partial E}{\partial k_y}\right).$$

First, calculate: $\frac{\partial E}{\partial k_y} = -2t_y a\sin\, k_y a$

Differentiating with respect to k_x : $\frac{\partial^2 E}{\partial k_x \partial k_y} = \frac{\partial}{\partial k_x}\left[-2t_y a\sin\, k_y a\right] = 0$.

Thus: $\frac{\partial^2 E}{\partial k_x \partial k_y} = 0$.

Mixed Partial Derivative: $\frac{\partial^2 E}{\partial k_y \partial k_x}, \frac{\partial^2 E}{\partial k_y \partial k_x} = \frac{\partial}{\partial k_y}\left(\frac{\partial E}{\partial k_x}\right)$.

From: $\frac{\partial E}{\partial k_x} = -2t_x a\sin\, k_x a$,

We compute: $\frac{\partial^2 E}{\partial k_y \partial k_x} = \frac{\partial}{\partial k_y}\left[-2t_x a\sin\, k_x a\right] = 0$.

Thus: $\frac{\partial^2 E}{\partial k_y \partial k_x} = 0$.

The effective mass tensor m^* is defined as:

Substituting the calculated values: $\frac{\partial^2 E}{\partial k_x^2} = -2t_x a^2$, $\frac{\partial^2 E}{\partial k_y^2} = -2t_y a^2$,

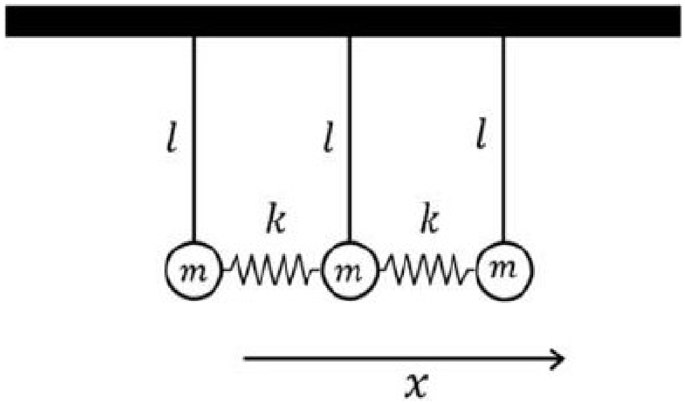
and: $\frac{\partial^2 E}{\partial k_x \partial k_y} = \frac{\partial^2 E}{\partial k_y \partial k_x} = 0$.

The tensor simplifies to: $m^* = \begin{pmatrix} -\frac{\hbar^2}{2t_x a^2} & 0 \\ 0 & -\frac{\hbar^2}{2t_y a^2} \end{pmatrix}$.

This concludes the derivation of the effective mass tensor.

Question ID 705049

Three identical simple pendula (of mass m and equilibrium string length l) are attached together by springs of spring constant k, as shown in the figure.



The frequencies of small oscillations are given by $\sqrt{\frac{g}{l}}, \sqrt{\frac{k}{m}+\frac{g}{l}}, \sqrt{\frac{3k}{m}+\frac{g}{l}}$. The normal modes (without normalisation) corresponding to these frequencies respectively are

1. $(1,1,1),(1,0,1),(1,-2,1)$
3. $(1,1,1),(1,0,-1),(1,2,1)$
3. $(1,1,1),(1,0,-1),(1,-2,1)$
4. $(1,2,1),(1,0,-1),(1,1,1)$

Topic: Classical Mechanics

Subtopic: Small Oscillation

Ans. : (3)

Solution: If we ignore the pendulum part then the system will reduce to linear triatomic molecular with angular frequency $\omega=0,\sqrt{k/m},\sqrt{3k/m}$ with normal modes $(1,1,1),(1,0,-1),(1,-2,1)$ so option c is correct.

Question ID 705058

A radio station antenna on the earth's surface radiates 50 kW power isotropically. Assume the electromagnetic waves to be sinusoidal and the ground to be a perfect absorber. Neglecting any transmission loss and effects of earth's curvature, the peak value of the magnetic field (in Tesla) detected at a distance of 100 km is closest to

1. 1.5×10^{-11}
2. 5.5×10^{-11}
3. 8.5×10^{-11}
4. 3.5×10^{-11}

Topic: Electromagnetic Theory

Subtopic: Electromagnetic Waves

Ans. : (2)

Solution: Given: $P = 50\ \mathrm{kW} = 50 \times 10^3\ \mathrm{W},\ R = 100\ \mathrm{km} = 100 \times 10^3\ \mathrm{m}$

The intensity I is calculated using: $I = \frac{P}{A},\ A = 4\pi R^2$

Calculation of I : $I = \frac{50\times10^3}{4\pi\times(1\times10^{10})},\ I = 3.98 \times 10^{-7}\ \mathrm{W/m^2}$

Electric Field E_0 : The relation between intensity and electric field is: $I = \frac{1}{2}\varepsilon_0 c E_0^2$

Rearranging for E_0 : $E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$

Substitute the values: $\varepsilon_0 = 8.854 \times \frac{10^{-12}\ \mathrm{F}}{\mathrm{m}},\ c = 3 \times 10^8\ \mathrm{m/s}$

$$E_0 = \sqrt{\frac{2 \times 3.98 \times 10^{-7}}{8.854 \times 10^{-1}\ \ \times 3 \times 10^8}}$$

Simplify: $E_0 \approx 0.0173\ \mathrm{V/m}$

$B_0 = 5.7 \times 10^{-1}\ \ \mathrm{T}$

Question ID 705052

A particle of energy E is scattered off a one-dimensional potential $\lambda\delta(x)$, where λ is a real positive constant, with a transmission amplitude t_+. In a different experiment, the same particle is scattered off another one-dimensional potential $-\lambda\delta(x)$, with a transmission amplitude t_-. In the limit $E \to 0$, the phase difference between $t_+$and $t_-$is

1. $\pi/2$ 2. π 3. 0 $3\pi/2$

Topic: Quantum Mechanics

Subtopic: Dirac Delta Function

Ans. : (2)

Solution: For potential $V(x) = \lambda\delta(x)$

$\psi_1 = A_1 \exp ikx + B_1 \exp - ikx$

$\psi_2 = C_1 \exp - ikx$

At $x = 0$ wave function is continuous $A_1 + B_1 = C_1$ where $t_+ = \frac{C_1}{A_1}$

The solution of Schrodinger equation $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \lambda\delta(x)\psi = E\psi$

Integrate both side we will get $-\frac{\hbar^2}{2m}\int_{-\varepsilon}^{\varepsilon}\frac{d^2\psi}{dx^2}dx + \int_{-\varepsilon}^{\varepsilon}\lambda\delta(x)\psi(x)dx = \int_{-\varepsilon}^{\varepsilon}E\psi(x)dx$

$-\frac{\hbar^2}{2m}\frac{d\psi}{dx}\bigg|_{-\varepsilon}^{\varepsilon}+\lambda\psi(0)=0\Rightarrow -\frac{\hbar^2}{2m}\left(ik\left(C_1-\left(A_1-B_1\right)\right)\right)+\lambda C_1=0$ put $B_1=A_1-C_1$

$$\Rightarrow -\frac{\hbar^2}{2m}\left(ik\left(C_1-\left(A_1+A_1-C_1\right)\right)\right)+\lambda C_1=0\Rightarrow C_1-\left(2A_1-C_1\right)=\frac{2m\lambda}{\hbar^2 ik}C_1$$

Dividing both side by A_1 $t_+-2+t_+=\frac{2m\lambda}{\hbar^2 ik}t_+\Rightarrow 2t_+-2=\frac{2m\lambda}{\hbar^2 ik}t_+\Rightarrow t_+=\frac{1}{1-\frac{\lambda m}{\hbar^2 ik}}$

Similarly for $-\lambda\delta(x)$ $t_-=\frac{1}{1+\frac{\lambda m}{\hbar^2 ik}}$, $\frac{t_+}{t_-}=\frac{1+\frac{\lambda m}{\hbar^2 ik}}{1-\frac{\lambda m}{\hbar^2 ik}}=\frac{\hbar^2 ik+\lambda m}{\hbar^2 ik-\lambda m}$ $\quad k\to 0$ $\frac{t_+}{t_-}=-1$

So, phase change is π.