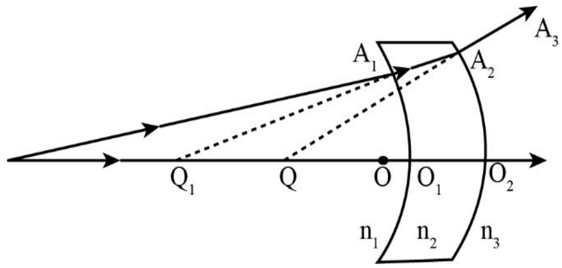
Refraction At Spherical Surfaces

Lens formula with optical centre as origin:



$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{1}{F} \dots \dots (17)$$

where, u = object distance, v = image distance, $\frac{1}{F} = \frac{n_2 - n_1}{r_1} - \frac{n_2 - n_3}{r_2}$......(18)

According to the definition of focal lengths, $u = f'$ (1^{st} focal length) when $v = \infty$;

$v = f'$ (2^{nd} focal length) when $u = \infty$;

Putting these conditions in equation (17), we get

$$-\frac{n_1}{f'} = \frac{n_3}{f} = \frac{1}{F} \dots .. (19)$$

$$\Rightarrow \quad \frac{f'}{f} = -\frac{n_1}{n_3} = -\frac{\text{r.i. of incident medium}}{\text{r.i. of emergent medium}}$$

Combining (17) and (19) we get, $\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_3}{f} \dots . (20)$

Case-I: When $n_1 = n_3 \Rightarrow \frac{1}{f} = -\frac{1}{f'} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \dots (21)$

And $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \dots .. (22)$

Case-II: When the surrounding medium is air, $n_1 = n_3 - 1$ and if $n_2 = n$ then $\frac{1}{f} = \frac{1}{f'} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Sum of the powers of the refracting surfaces.

The power of the lens is $P = \frac{1}{F} = \frac{n_2 - n_1}{r1} + \frac{n_3 - n_2}{r}$ Sum of the powers of the refracting surfaces.

Conjugate focí relation with the two foci as origin Newton s equation:

The conjugate foci relation for a thin lens with optical centre (O) as origin is given by

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

where we assume that the media on both sides of the lens are same.

$$\frac{f}{v}-\frac{f}{u}=1 \Rightarrow \frac{f}{v}+\frac{f'}{u}=1 \Rightarrow uv'-vf'+ff'=ff'$$

$$(u-f')(v-f)=ff'$$

Here U and V represent the object distance and its image distance measured from the first and second foci respectively.

$$\text{UV } = ff'$$

This equation is known as Newton's equation.

Equivalent Lens and Equivalent Focal Length:

A single lens is said to be equivalent to a system of lenses when that single lens produces an image of a given object at the same place and of the same size as that formed by the system of lenses. The focal length of such a single lens is called equivalent focal length of the system of lenses. The equivalent focal length F of two thin lenses having second focal lengths f_1 and f_2 and kept separated by a distance t in air is given by ,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{t}{f_1 f_2}$$

Some Important Problems with Lenses:

Minimum distance between an object and its real image produced by a convex lens:

Suppose we consider a convex lens in air forming real image on its right-hand side for an object placed or the left. Here object distance u is $-ve$, image distance v and focal length f are $+ve$.

So, by putting numerical values of u, v and f in the formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$________ (1)

For the distance $D=u+v$ to be minimum, $\frac{d}{du}(u+v)=0 \Rightarrow \frac{dv}{du}=-1$ Now from Equation(1), on differentiation, $-\frac{1}{v^2}-\frac{1}{u^2}\frac{du}{dv}=0$

Therefore, for D to be minimum, $-\frac{1}{v^2}-\frac{1}{u^2}(-1)=0 \Rightarrow u=v$ Thus, from (1) $u=v=2f$

Thus $D_{\text{min}}=u+v=2f+2f=4f$

Condition for the formation of two real images of a given object, on the same screen, for two different positions of a convex lens and expression for focal length in terms of lens displacement:

Here the distance between the object and the screen i.e., $D = u + v$ is constant. Therefore, putting $v = D - u$ in Equation (1) we get, $\frac{1}{D-u} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow\ u^2 - Du + Df = 0$....(2)

For real distinct roots of this quadratic equation we must have $D^2 > 4Df \Rightarrow D > 4f$ So, for getting two real images on the screen the distance between the object and the screen must be greater than four times the focal length of the convex lens. Solving Equation (2) we get for the two positions of the lens

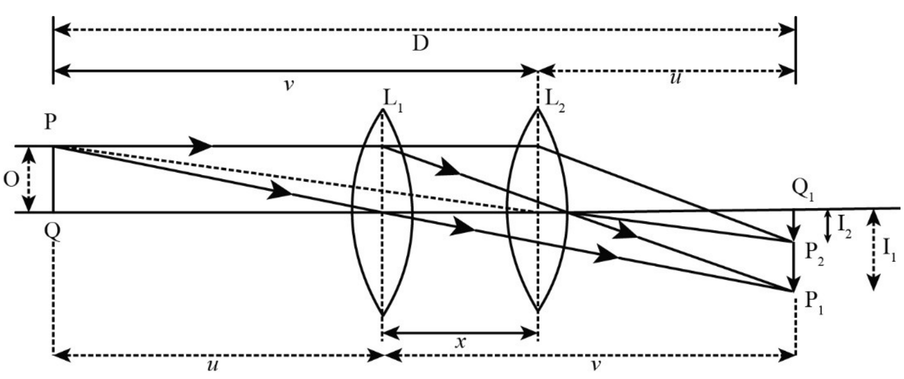
$u_1 = \frac{D+\sqrt{D^2-4D}}{2}$ and $u_2 = \frac{D-\sqrt{D^2-4Df}}{2}$....(3)

Therefore, displacement of the lens between the two positions is

$x = u_1 - u_2 = \sqrt{D^2 - 4Df} \Rightarrow f = \frac{D^2-x^2}{4D}$...(4)

Size of the object is the geometric mean of the sizes of the images produced by a convex lens at two positions:

Let u, v and I_1 be respectivelythe-object distance imáge distance and length of the image for the first position L_1 of the lens; while the corressponding quantities for the second position L_2 of the lens would be v, u and I_2 ($\because$ the positions of object and image are interchangeable) [See Figure below]. If O be the length of the object then magnifications for the two positions are given by



$m_1 = \frac{I_1}{O} = \frac{v}{u}$ and $m_2 = \frac{I_2}{O} = \frac{u}{v}$

$\frac{I_1 I_2}{O^2} = 1 \Rightarrow O = \sqrt{I_1 I_2}$