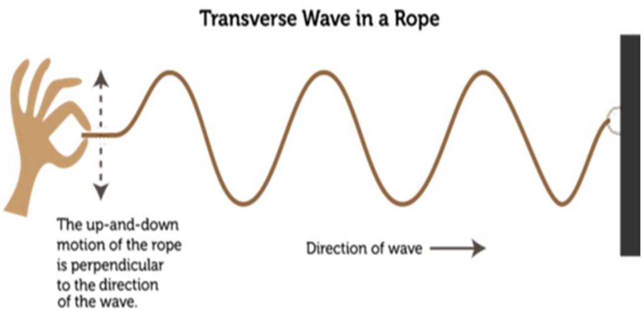
Wave Motion

Speed of transverse wave in string:

$$v=\sqrt{\frac{T}{\mu}};\qquad T=\text{Tension},\ \mu=\text{Mass per unit length}$$

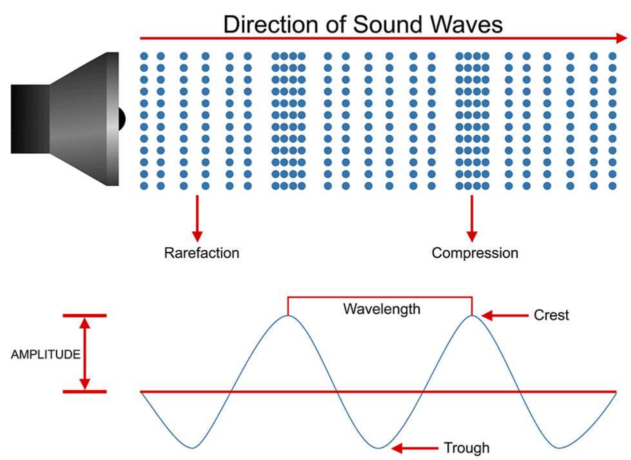


Speed of transverse wave in string:

$y = A\sin(\omega t - kx)$

The velocity of sound wave can be written as

$$v=\sqrt{\frac{E}{\rho}},\quad E=\text{Modulus of elasticity},\ \rho=\text{Density of medium}$$



For Solid

$E = Y =$ Young modulus, $Y =$ Stress/Strain

$$v=\sqrt{\frac{Y}{\rho}}$$

For liquid

$E = B =$ Bulk modulus, $B =$ Excess pressure/Volume strain

$$B=-v\frac{\Delta p}{\Delta v}$$

$$v=\sqrt{\frac{B}{\rho}}$$

Velocity of sound in glass/air

Newtons assume that velocity propagation is isothermal process.

$$pv = \text{constant}$$

$$vdp + pdv = 0$$

$$\Rightarrow p = -\frac{vdp}{dv} = B \Rightarrow v = \sqrt{\frac{p}{\rho}}$$

In air $p = 1.01 \times 10^5\ Pa,\ \rho = 1.293\ kg/m^3$

$v = 279.43\ m/s$

But in experiment, $v = 332\ m/s$

Later the correction is made by Laplace. He assumed that the speed of sound in air/glass is adiabatic process.

$$pv^{\gamma} = \text{constant}$$

$$v^{\gamma} dp + \gamma p v^{\gamma-1} dv = 0 \Rightarrow p = -\frac{vdp}{dv} = B$$

Thus, $v_{sound} = \sqrt{\frac{\gamma p}{\rho}} = 332\ m/s$

We know that

$$PV = nkT,\ n = \frac{m}{M} N_A$$

$$P = \frac{m}{VM} N_A kT \Rightarrow \frac{P}{\rho} = \frac{RT}{M} \Rightarrow v_{sound} = \sqrt{\frac{\gamma RT}{M}}$$

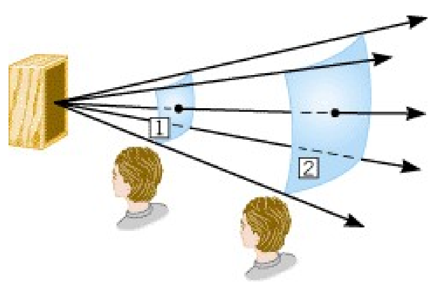
If medium is same but different temperature

$$\frac{v_{sound1}}{v_{sound1}} = \sqrt{\frac{T_1}{T_2}}$$

If medium is different

$$\frac{v_{sound_1}}{v_{sound_2}} = \sqrt{\frac{\gamma_1 T_1 M_2}{\gamma_2 T_2 M_1}}$$

Power of wave (P)



The power is energy transported through a perpendicular surface of area a in unit time.

The expression for power can be written as follows

$$P = 2\pi^2 n^2 A^2 \rho v s$$

$n = Frequency$ of wave

A =Amplitude of wave

ρ =Density of medium

v =Velocity

s =Surface area

Again, energy stored per unit volume $= 2\pi^2 n^2 A^2 \rho$

Power transported by string wave

$$P = 2\pi^2 n^2 A^2 \rho v s$$

$$\rho s = \frac{m}{ls} s = m/l = \mu \text{ Linear mass density}$$

$$P = 2\pi^2 n^2 A^2 \mu v$$

Energy transported along string in one period

$$E = PT = 2\pi^2 n^2 A^2 \mu v \frac{1}{n} \Rightarrow E = PT = 2\pi^2 n A^2 \mu v$$

Wave intensity

Power transported per unit correctional area is called intensity

$$I = \frac{P}{s} = 2\pi^2 n^2 A^2 \rho v$$

Representing sound wave as a pressure wave

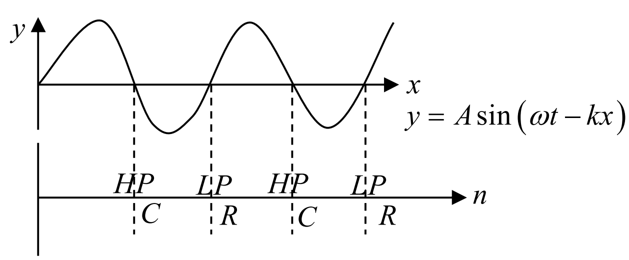
We know that the displacement of wave can be written as follows

$$y = A\sin(\omega t - kx)$$

$$dy = -kA\cos(\omega t - kx)dx$$

Volume compression $\Delta V = -kA\cos(\omega t - kx)dx$

$$\frac{\Delta V}{V} = -kA\cos(\omega t - kx)$$



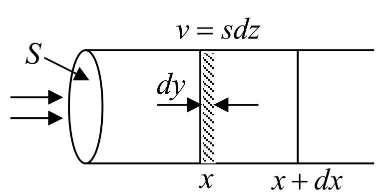
Now, $B = -V\frac{\Delta P}{\Delta V} = BkA\cos(\omega t - kx)$

$$\Delta p = BkA\cos(\omega t - kx)$$

$$\Delta p = \Delta p_0 \cos(\omega t - kx);$$

$\Delta p_0 =$ Pressure amplitude

$$\Delta p_0 = BKA = BA\frac{2\pi}{\lambda} \Rightarrow A = \frac{\lambda \Delta p_0}{2\pi B}$$



Intensity of sound wave

$$I = 2\pi^2 n^2 A^2 \rho v; \;\; v = n\lambda; \; v = \sqrt{\frac{B}{\rho}} \Rightarrow B = \rho v^2$$

$$I = 2\pi^2 \left(\frac{v}{\lambda}\right)^2 \left(\frac{\lambda \Delta p_0}{2\pi B}\right)^2 \rho v; \; B = \rho v^2$$

$$I = \frac{(\Delta \rho_0)^2}{2\rho v}; \; v = Wave\,velocity$$