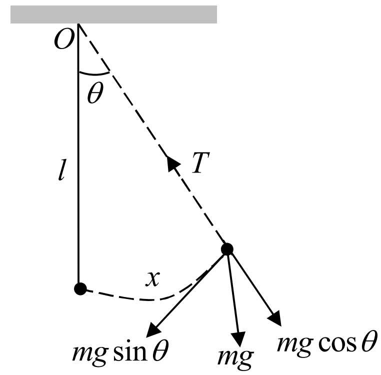
(a) Simple Pendulum

According to Hooke's law $F=-kx \Rightarrow ma=-kx \Rightarrow a=-\frac{k}{m}x=-\omega^2 x \Rightarrow \omega=\sqrt{\frac{k}{m}}$

$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{k}}$



At equilibrium, $T=mg$

After displacement of θ, $T\cos(\theta)=mg$

Restoring force $F_R=-mg\sin(\theta)$

If θ is very small, then $\sin(\theta)=\theta=\frac{x}{l}$

$$F_R=-mg\frac{x}{l}=ma \Rightarrow a=g\frac{x}{l}=-\omega^2 x \Rightarrow \omega=\sqrt{\frac{g}{l}} \Rightarrow T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{l}{g}}$$

(b) Angular Simple Harmonic Motion

Angular Oscillation is the oscillation when a body is allowed to rotate freely about a given axis.

We know that $x=A\sin(\omega t+\phi)$

For angular simple harmonic motion

$$\Rightarrow l\theta=l\theta_0\sin(\omega t+\phi),\ Where \quad -\theta_0\leq\theta\geq\theta_0$$

The angular speed can be written as

$$\dot{\theta}=\frac{d\theta}{dt}=\omega\theta_0\cos(\omega t+\phi),$$

$$\frac{d^2\theta}{dt^2}=-\omega^2\theta_0\sin(\omega t+\phi)=-\omega^2\theta \Rightarrow equation\ of\ motion$$

The kinetic energy $KE=\frac{I}{2}\dot{\theta}^2=\frac{I}{2}\dot{\theta}^2=\frac{I}{2}\omega^2\theta_0^2\cos^2(\omega t+\phi)=\frac{I}{2}\omega^2\left(\theta_0^2-\theta^2\right)$

The maximum kinetic energy will be when $\theta=0$, KE_{max}= $\frac{I}{2}\omega^2\theta_0^2$ =Total energy (E_T)

The potential energy PE=E_T-KE= $\frac{I}{2}\omega^2\theta^2$, PE= $\frac{I}{2}\omega^2\theta_0^2\sin^2(\omega t+\phi)$

$$\langle KE\rangle=\frac{I}{4}\omega^2\theta_0^2=\langle PE\rangle$$

(c) Compound Pendulum

Any rigid body suspended from a fixed support constitutes a physical pendulum . A circular ring suspended thorough on a nail in a wall, a heavy metallic rod suspended through hole in it are examples of compound pendulum.

The body rotates about a horizontal axis through O and perpendicular to the plane of motion. Let this axis be OA. Suppose the angular displacement of the body is θ at time t.

If l is the separation between point of suspension and the centre of mass, then torque can be written as

$$\tau = -mgl\sin(\theta)$$

Now if I is the moment of inertia about OA, and α is the angular acceleration, then

$$\tau = I\alpha = -mgl\theta \Rightarrow \alpha = \frac{-mgl\theta}{I} = -\omega^2\theta \Rightarrow \omega^2 = \frac{mgl}{I}$$

Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgl}}$

(d) Block in fluid

A Cylindrical wooden block of density σ cross section area A and length L floating up right when immersed to a depth d in liquid of density ρ as shown in figure. In the equilibrium Position the weight of block is balanced by upthrust force due to buoyancy. Block is slightly pressed vertically and released. show that motion of block is oscillatory.

Solution: The restoring force must be equal to weight of displaced liquid

$$F = -\rho Agy \Rightarrow m\frac{d^2y}{dt^2} = -\rho gAy$$

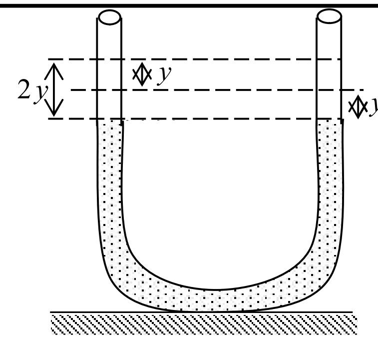
Where m is mass of block $m = \sigma Adg$ so equation of motion is given by $\sigma ALg\frac{d^2y}{dt^2} = -\rho Agy$

$\frac{d^2y}{dt^2} + \frac{\rho}{\sigma L}y = 0$ compare the equation with $\frac{d^2x}{dt^2} + \omega^2 x = 0$ the value of $\omega = \sqrt{\frac{\rho}{\sigma L}}$.

(e) Capillary

Consider a U-shaped tube filled with liquid of density ρ up to height as shown in figure. initially the liquid is in equilibrium. The height of liquid in both arm of tube is L. If height of liquid is changing in one arm of tube with small distance y, then show that motion of oscillatory in nature and find angular frequency of oscillation.

If hight of liquid is increasing with height y in right arm of tube then it is also decreasing the y length in left arm. So, restoring force acting on liquid is $F = -\rho gA(2y)$ where A is area of cross section.



The equation of motion is given by $m\frac{d^2y}{dt^2} = -\rho gA(2y)$.

where m is mass of liquid

$m = \rho Ag(2L)$ so, equation of motion will reduce to

$$\rho gA(2l)\frac{d^2y}{dt^2} + \rho gA(2y) = 0$$

$\rho gA(2l)\frac{d^2y}{dt^2} + \rho gA(2y) = 0 \Rightarrow \frac{d^2y}{dt^2} + \frac{g}{L}y = 0$ comparing this equation with $\frac{d^2x}{dt^2} + \omega^2 x = 0$

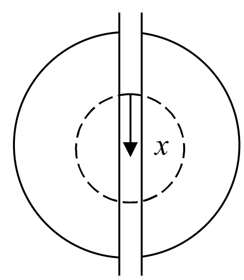
So $\omega = \sqrt{\frac{g}{L}}$ and motion is considered as oscillatory.

(f) Ball in tunnel of earth

There is a tunnel along the Diameter of earth. A ball of mass m is placed in the tunnel. Prove that motion of ball is equivalent to Simple harmonic motion find the angular frequency ball. Assume mass of earth is M and radius of earth is R.

Let us consider the mass density of earth is ρ if M is mass of earth then mass of inner part of sphere is

$$M_1 = \frac{M\rho\frac{4\pi}{3}x^3}{\frac{4\pi}{3}\rho R^3} = \frac{Mx^3}{R^3}$$



$$F = -\frac{GM_1 m}{x^2} = -\frac{GMx^3}{R^3.x^2} = -\frac{GMx}{R^3} \Rightarrow m\frac{d^2x}{dt^2} = -\frac{GM}{R^3}.x \Rightarrow \frac{d^2x}{dt^2} = -\frac{GM}{mR^3}x \text{ so } \omega = \sqrt{\frac{GM}{mR^3}}$$

(g) LC Circuit:

A capacitor of capacitance C is fully charged .it discharges through as inductor of inductance L connected through parallel with capacitor.

Prove that the discharging of charge in capacitor follow differential equation of small oscillation. Find the frequency of oscillation.

Solution let us assume current i is flowing in the circuit so the emf equation is given by

$$L\frac{di}{dt} + \frac{q}{c} = 0 \Rightarrow L\frac{d^2q}{dt^2} + \frac{q}{c} = 0 \Rightarrow \frac{d^2q}{dt^2} + \frac{q}{LC} = 0$$

If we compare above equation with $\frac{d^2x}{dt^2} + \omega^2 x = 0$, $\omega = \sqrt{\frac{1}{LC}}$.