Jest 2025

Q1. The Fraunhofer diffraction pattern formed by an elliptical aperture will be

(a) elliptical with the semi-major axis parallel to that of the aperture.

(b) elliptical with the semi-major axis perpendicular to that of the aperture.

(c) circular.

(d) hyperbolic.

Q2. The electric dipole moment of a charge distribution is independent of the choice of the origin of coordinates only if

(a) The charge distribution is discrete.

(b) There is no magnetic field present.

(c) The total charge adds up to zero.

(d) The charge distribution is not time-dependent.

Q3. A boat is floating in a pond with still water. There is a heavy stone on the boat. If the stone is dropped gently into the water, what happens to the water level in the pond after the stone sinks completely?

(a) The level goes up or down depending on the size of the stone.

(b) The level remains the same.

(c) The level goes up.

(d) The level goes down.

Q4. A capacitor with capacitance C is connected in series with a resistor of resistance R and an ideal DC source with voltage V_S. At one instant during the charging of the capacitor if the resistor is replaced by a wire of zero resistance, which of the following statements is true?

(a) The capacitor immediately attains the source voltage V_S.

(b) None of the others is true.

(c) The voltage across the capacitor will increase slowly.

(d) The voltage across the capacitor will drop immediately to zero.

Q5. The Lagrangian of a two-dimensional system is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - k(x^2 + y^2)^{-1.5}$$

Which of the following is/are the constant(s) of motion?

(a) A. Energy and angular momentum (b) None of the others

(c) Energy only (d) Angular momentum only

Q6. Three observers successively measure the spin of a given proton along z-axis, x-axis and again z-axis, respectively. The first observer finds the spin projection to be $+\frac{1}{2}$. Assuming no other factors, what is the probability that the third observer finds the spin projection to be $-\frac{1}{2}$?

(a) 0.5 (b) 1

(c) None of the others (d) 0

Q7. Given the differential operator: $D \equiv \frac{d^2}{dx^2} + P\frac{d}{dx} + Q$, where P and Q are constants, what is the eigenvalue corresponding to the eigenfunction $y = e^x$?

(a) $(1 + P + Q)$ (b) $(1 + Q)$ (c) $(P + Q - 1)$ (d) $(P + Q)$

Q8. Consider a quantum system that is evolved sequentially with a finite sequence of Hermitian Hamiltonians $\{H_0, H_1, \dots H_n\}$. The full evolution operator is written as:

$$\mathcal{O} = U_n U_{n-1} \dots U_1 U_0 = e^{-i\mathcal{H}}, \text{ with } U_j = e^{-iH_j} \text{ and, } j = 0,1, \dots n$$

Then $\mathcal{H}$ is

(a) None of the others. (b) A Hermitian operator.

(c) undefined. (d) A unitary operator.

Q9. Match the following statements.

A1	Photoelectric effect	B1	Involves loosely bound or free electrons
A2	Compton scattering	B2	Inverse photoelectric effect
A3	Pair production	B3	Needs a minimum of 1.02 MeV of energy for the incident radiation
A4	Bremsstrahlung	B4	Involves bound electrons and depends on the specifics of the material

(a) A1-B4, A2-B1, A3-B3, A4-B2 (b) A1-B3, A2-B4, A3-B2, A4-B1

(c) A1-B4, A2-B3, A3-B2, A4-B1 (d) A1-B3, A2-B2, A3-B1, A4-B4

Q10. For a relativistic point particle the momentum is $\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1-v^2/c^2}}$, where $\vec{v}$ is its velocity as measured by an inertial observer. Then the acceleration is in the same direction as the applied force

(a) only when force is neither parallel nor perpendicular to the velocity.

(b) always.

(c) never.

(d) only when force is parallel or perpendicular to the velocity.

Q11. For a plane electromagnetic wave propagating with wave vector $\vec{k}$ in a homogeneous and isotropic medium, which of the following holds?

(a) A. $\vec{E} \cdot \vec{B} = 0$ (b) $\vec{E} \times \vec{B} = \vec{0}$

(c) $\vec{k} \cdot (\vec{E} \times \vec{B}) = 0$ (d) None of the others.

Q12. Consider two identical charged balls, each of mass m and charge q. One of them is initially held fixed on a frictionless insulating horizontal surface and the other is carefully placed above the first one at a height h from the surface, such that the gravitational force on it is balanced by the Coulomb repulsion. The upper ball is now shifted horizontally by a distance $d(d \ll h)$ to the right and then both the balls are released. Which way will the balls move immediately after this?

(a) Both balls remain static at their new positions.

(b) Ball on the surface moves towards right and ball above moves upwards.

(c) Both balls oscillate around their original positions.

(d) Ball on the surface moves towards left, ball above moves downwards.

Q13. A wire of length 0.01 mm is placed perpendicular to the axis of a thin convex lens, 30 cm from its center. If the focal length of the lens is 20 cm , what is the length of the image of the wire?

(a) 1 mm (b) 0.03 mm (c) 0.02 mm (d) 0.01 mm

Q14. The number of independent real numbers that parameterize any (3×3) Hermitian matrix is

(a) 6 (b) 8 (c) 3 (d) 9

Q15. Suppose the mass of the Sun is reduced to half of its original value very slowly, e.g., over a billion years, what will be the effect of this on the Earth's orbit?

(a) Remains elliptical, but the mean radius changes.

(b) The Earth flies away.

(c) Orbit remains closed but not elliptical.

(d) Remains elliptical with the same mean radius.

Q16. One mole of ideal gas with a constant C_V undergoes a reversible adiabatic expansion. Which one of the following equations is valid? [$\gamma = \frac{C_P}{C_V}$ for the gas]

(a) $P^{1-\gamma}T^{\gamma} =$ constant (b) $P^{\gamma-1}T^{\gamma} =$ constant

(c) $V^{\gamma}T =$ constant (d) $VT^{\gamma} =$ constant

Q17. A particle is moving under the force field given by $\vec{F} = k\vec{r}$, where k is a positive constant. The difference in work done (in arbitrary units) if the particle moves from point $\mathrm{A}(-1,0,0)$ to point $\mathrm{B}(1,0,0)$ following semi-circular paths in the clockwise and anti-clockwise directions on the X-Y plane will be

(a) 0 (b) $\frac{1}{2}\pi k$ (c) πk (d) $2\pi k$

Q18. Consider the standard notation of discrete finite groups with $\mathbb{Z}_n$ corresponding to the rotation by $2\pi/n$ about a given axis, S_n corresponding to the permutation group of the set S having n elements, i.e. $S = \{1,2,3, \dots, n\}$, and the Dihedral group D_n corresponding to the reflection and rotation symmetries of a regular polygon with n number of sides. Which of the following is the smallest non-abelian group?

A. $\mathbb{Z}_3$ (b) D_4 (c) S_4 (d) S_3

Q19. A quantum mechanical system is spanned by the eigenstates $|a_1\rangle$ and $|a_2\rangle$ of a Hermitian operator A with eigenvalues a_1 and a_2 respectively. If there is no degeneracy, what is the expectation value of the operator $(A - a_1)(A - a_2)$ in the state $\frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}}$?

(a) $(a_2 - a_1)(a_1 - a_2)$ (b) $\frac{(a_2-a_1)(a_1-a_2)}{2}$

(c) 0 (d) 1

Q20. Which of the following thermodynamic variables is not a function of state for an ideal gas?

(a) None of the others (b) Pressure

(c) Temperature (d) Entropy

Q21. Consider a 2×2 matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ which has eigenvalues $\lambda_1 = \frac{1+\sqrt{5}}{2}$ and $\lambda_2 = \frac{1-\sqrt{5}}{2}$. For any natural number n which of the following is correct ?

(a) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} - \lambda_2^{n-1} & \lambda_1^n - \lambda_2^n \\ \lambda_1^n - \lambda_2^n & \lambda_1^{n+1} - \lambda_2^{n+1} \end{bmatrix}$ (b) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} + \lambda_2^{n-1} & \lambda_1^n - \lambda_2^n \\ \lambda_1^n - \lambda_2^n & \lambda_1^{n+1} + \lambda_2^{n+1} \end{bmatrix}$

(c) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} - \lambda_2^{n-1} & \lambda_1^n + \lambda_2^n \\ \lambda_1^n + \lambda_2^n & \lambda_1^{n+1} - \lambda_2^{n+1} \end{bmatrix}$ (d) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} + \lambda_2^{n-1} & \lambda_1^n + \lambda_2^n \\ \lambda_1^n + \lambda_2^n & \lambda_1^{n+1} + \lambda_2^{n+1} \end{bmatrix}$

Q22. Consider the time-independent Schrödinger equation with a real potential and suppose $\psi(x)$ is a solution of this equation. Which of the following is true?

(a) ψ^* is a solution of the same equation only if the potential is symmetric about $x = 0$. B.

(b) ψ^* is a solution of the same equation only if the potential vanishes at infinity.

(c) ψ^* is a solution of the same equation.

(d) ψ^* is never a solution of the same equation.

Q23. A slide calipers instrument has smallest main scale division of 0.4 mm and 40 vernier divisions match with 38 main scale divisions. The vernier constant of this instrument is

(a) 0.05 mm (b) 0.02 mm (c) 0.1 mm (d) 0.01 mm

Q24. Calculate the partition function for two indistinguishable bosonic particles at a temperature T, which can be distributed in two single-particle energy levels ϵ_1 and ϵ_2. Consider $\beta = \frac{1}{k_B T}$.

(a) $\left(e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2}\right)^2$ (b) $e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-\beta(\epsilon_1+\epsilon_2)}$

(c) $\frac{1}{2!}\left(e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2}\right)^2$ (d) $e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-2\beta(\epsilon_1+\epsilon_2)}$

Q25. If the lattice contribution to the C_V of a solid crystal at temperature $2K$ is found to be $0.5\ \text{mJ mole}^{-1}\ K^{-1}$, what will be the corresponding contribution at temperature $4K$?

(a) $8\ \text{mJ mole}^{-1} K^{-1}$ (b) $1\ \text{mJ mole}^{-1} K^{-1}$ (c) $4\ \text{mJ mole}^{-1} K^{-1}$ (d) $2\ \text{mJ mole}^{-1} K^{-1}$

Part B: 3-Mark MCQ

Q1. Evaluate $\vec{\nabla}\cdot(r^4\vec{r})$, where $\vec{r}$ represents a three dimensional position vector.

(a) $7r^4$ (b) $4r^4$ (c) 0 (d) $5r^4$

Q2. A silicon p-n junction diode operates at $27°$C. The current I is doubled when the forward bias is increased. The increase in the forward bias is closest to:

[Assume $I > I_s$, where I_s is the reverse saturation current and the emission coefficient $\eta_{\text{SI}} = 2$.]

(a) 54 mV (b) 18 mV (c) 72 mV (d) 36 mV

Q3. Consider the group S_4 corresponding to the permutations of the set S having four elements, say $S = \{1,2,3,4\}$. How many non-identity self-inverse (i.e. order 2) elements does S_4 have?

(a) 6 (b) 12 (c) 9 (d) 8

Q4. For a particle in a one-dimensional box of width L, the uncertainty Δp in momentum in the n-th eigenstate of energy for large n is

(a) $\frac{2n\hbar}{L}$ (b) $\frac{\hbar}{n\pi L}$ (c) $\frac{n\pi\hbar}{L}$ (d) $\frac{2n\ \hbar}{L}$

Q5. Consider a circular disk of radius R and mass M in the $\mathrm{X}-\mathrm{Y}$ plane, with a surface mass density $\sigma(r) = \sigma_0 e^{-r^3/a^2}$, where r is the distance from the center of the disk. What is the moment of inertia around the Z-axis through the center of the disk? [consider $R \gg a$]

(a) $\frac{1}{3}Ma^2$ (b) $6Ma^2$ (c) $\frac{1}{2}Ma^2$ (d) Ma^2

Q6. A circular loop of radius a, carrying a current I in an anticlockwise direction (when seen downwards from the positive Z axis), is placed on the X-Y plane centered at the origin. What is the magnetic field on the X-Y plane at $r \gg a$?

(a) $\frac{\mu_0 I}{4\pi}\frac{a^2}{r^3}$ in the positive Z direction (b) 0

(c) $\frac{\mu_0 I}{4}\frac{a^2}{r^3}$ in the negative Z direction (d) $\frac{\mu_0 I}{4}\frac{a^2}{r^3}\hat{r}$

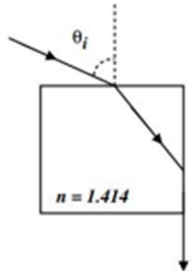
Q7. A ray of light is incident on a glass cube of refractive index 1.414 as shown in the figure. Find the angle of incidence θ_i, such that the ray grazes down the side of the glass cube.

(a) 0

(b) $\pi/3$

(c) $\pi/2$

(d) $\pi/4$



Q8. For a one-dimensional simple harmonic oscillator with mass m and angular frequency ω, consider a perturbation λx^4 in the Hamiltonian ($\lambda > 0$). What is the lowest order correction to the ground state energy?

[The position operator expressed in terms of the raising and lowering operators is $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^\dagger)$.]

(a) $\frac{3\lambda}{4}\left(\frac{\hbar}{m\omega}\right)^2$ (b) $\frac{5\lambda}{4}\left(\frac{\hbar}{m\omega}\right)^2$ (c) $\frac{3\lambda}{2}\left(\frac{\hbar}{m\omega}\right)^2$ (d) $\frac{5\lambda}{2}\left(\frac{\hbar}{m\omega}\right)^2$

Q9. Consider a two-dimensional Fermi gas at 0 K with Fermi energy ϵ_F. The average energy per particle of this gas is:

(a) $\frac{\epsilon_F}{2}$ (b) $\frac{\epsilon_F}{4}$ (c) $\frac{3\epsilon F}{5}$ (d) $\frac{\epsilon_F}{3}$

Q10. A particle is moving with velocity $v_x = v_y = v_z = c/2$ in frame S. The ratio of velocity component v_y to the velocity component $v_{y'}$ as measured in frame S' moving with velocity $c/2$ with respect to frame S along the common x direction is

(a) sin $(\pi/3)$ (b) cos $(\pi/6)$ (c) cos $(\pi/3)$ (d) sin $(\pi/6)$

Q11. The time averaged electrostatic potential of a neutral H-atom is given by

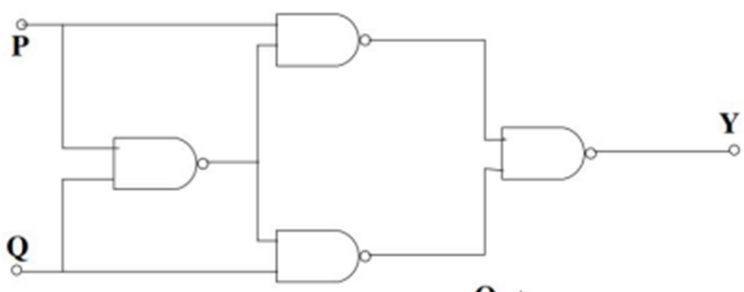
$$\Phi(\vec{r}) = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1 + \frac{\alpha r}{2}\right)$$

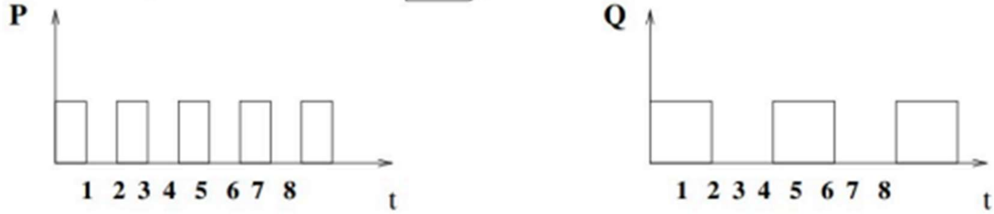
The classical charge distribution corresponding to this is

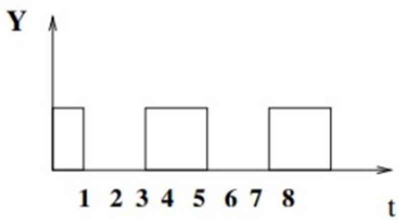
(a) $-\frac{q}{8\pi}\alpha^3 e^{-\alpha r} + q\delta^3(\vec{r})$ (b) $\frac{q}{8\pi}\alpha^3 e^{-\alpha}\left(1 + \frac{\alpha r}{2}\right) - q\delta^3(\vec{r})$

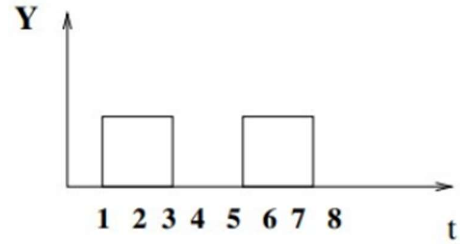
(c) $-\frac{q}{8\pi}\alpha^3 e^{-\alpha r}$ (d) $qe^{-\alpha r}\left(1 + \frac{\alpha r}{2}\right)$

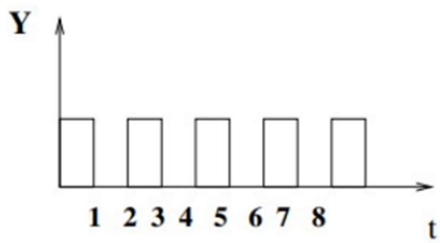
Q12. For the circuit and the inputs P and Q shown, which of the following is the correct output Y?

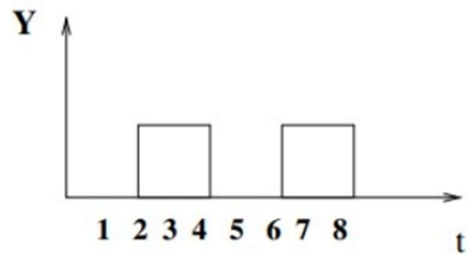




(a) 

(b) 

(c) 

(d) 

Q13. The volume of a nucleus, treated as a Fermi gas in three-dimensional space, is proportional to the number of fermions present in it. If the total number of fermions is changed from N to $2N$, the total energy of the system will

(a) Remain the same
(b) Be doubled
(c) Be 4 times its original value
(d) Behalf of its original value

Q14. Given an isolated thermodynamic system with a total energy E, total volume V and total number of particles N, the condition for stable thermal equilibrium, in terms of its entropy S under small changes ΔE and ΔV, is given by

(a) $-S(E+\Delta E, V+\Delta V, N)+S(E-\Delta E, V-\Delta V, N)-2S(E,V,N)<0$
(b) $S(E+\Delta E, V+\Delta V, N)+S(E-\Delta E, V-\Delta V, N)+2S(E,V,N)<0$
(c) $S(E+\Delta E, V+\Delta V, N)-S(E-\Delta E, V-\Delta V, N)-2S(E,V,N)<0$
(d) $S(E+\Delta E, V+\Delta V, N)+S(E-\Delta E, V-\Delta V, N)-2S(E,V,N)<0$

Q15. A block, suspended from a massless spring, is fully immersed in a liquid contained in a reservoir. What is the time period of small oscillations of the block?

[Given: Mass of the block m, density of the block ρ_b, natural length of the spring L, spring constant k, acceleration due to gravity g, density of the liquid ρ_l, damping coefficient of the liquid i.e., damping per unit mass per unit velocity γ.]

(a) $2\pi\sqrt{\frac{m}{k}}$
(b) $2\pi\sqrt{\frac{1}{k/m+\gamma^2/4}}$
(c) $2\pi\sqrt{\frac{1}{k/m-\gamma^2/4}}$
(d) $2\pi\sqrt{\frac{L}{(1-\rho_l/\rho_b)g}}$

Part C: 3-Mark Numerical

Q1. What is the value of the integral

$$I=\frac{3}{2\pi}\oint_{C}\frac{dz}{1+z^{2}}$$

where the contour $\mathcal{C}$ is a circle of radius 2 centered at the origin?

Q2. If a resistor of $10\mathrm{k}\Omega$ and a capacitor of 0.5μ F are connected in series across an AC supply of 220 V (rms) at 50 Hz , what is the average power (in mW , to the nearest integer) dissipated in the circuit?

Q3. The Fraunhofer diffraction intensity pattern for light of wavelength λ by a single slit of width a is given by

$$I=A_{0}^{2}\left(\frac{\sin\beta}{\beta}\right)^{2}$$

where A_0 is the intensity of the central maximum and $\beta=\frac{\pi a\sin\theta}{\lambda}$, where θ is the angle with the incident. beam. What is the angular separation in milli-radians, between the two first minima on two sides of the central beam, if $a=1\ \mathrm{mm}$ and $\lambda=5000\text{Å}$?

Q4. A current of 10 A is maintained for 1 s in a resistor of resistance 25Ω, which is thermally insulated. The initial temperature of the resistor is $23^{\circ}\mathrm{C}$. The resistor has a mass of 10 gm and a specific heat of $836\mathrm{Jkg}^{-1}\ \mathrm{K}^{-1}$. What is the entropy change of the resistor, rounding off to the nearest whole number in units of JK^{-1} ?

Q5. Given the mass of the proton $m_p\simeq 1836m_e$ and mass of the deuteron $m_d\simeq 3670m_e$, where m_e is the electron mass, find the fractional shift (in parts per million, to the nearest integer) of the ground state energy of the deuterium atom as compared to H-atom.

Q6. Suppose the wave function of a free particle in one dimension obeys $\frac{d^2\psi}{dx^2}=-4\psi$ in units where $\hbar=1$. What is the magnitude of the momentum of the particle?

Q7. A 3×3 matrix M satisfies $M^2-3M+2I=0$. Find the determinant of the matrix M if its trace is 6 .

Q8. A simple pendulum with effective length l and a bob of mass m has a time period T_1. Suppose now that the bob is given an electric charge $+Q$. It is made to oscillate just above a two dimensional infinite sheet with surface charge density $+\sigma$, where $\frac{Q\sigma}{mg\epsilon_o}=\frac{3}{2}$, ϵ_0 being the permittivity of free space and g being the acceleration due to gravity. If the period of oscillation in this case is T_2, determine $\frac{T_2}{T_1}$. [Neglect radiation from the charge.]

Q9. The average lifetime of a muon in its rest frame is 2200 ns . What will be the average distance (in meters, to the nearest integer) travelled by it, when created with a velocity of $\frac{1}{3}c$, before it decays? Here c is the speed of light.

Q10. A heat engine works between a high temperature source and a sink at $27°\mathrm{C}$. If the maximum efficiency possible for it to achieve is 50%, what is the temperature of the source in $°\mathrm{C}$?