Q1(a) Show that the following operator B is unitary, given that A is skew-Hermitian.

(6 marks)

$$B = (I + A)(I + A)^{-1}$$

Q1(b) Find the value of the following complex integral

(4 marks)

$$\oint 2z^4 \exp\left(\frac{2}{z}\right) dz$$

where the contour is taken over a circle in the complex plane with $|z| = 3$, traversed once in an anti- clockwise manner.

Q2(a) For an n-channel enhancement mode MOSFET, the drain current $I_D = 4\,mA$ for $V_{GS} = 8\,V$. If the threshold voltage is $4\,V$, determine the V_{GS} to obtain a drain current of $25\,mA$

(3 marks)

Q 2(b) Write down the 3D Hamiltonian for a particle of mass m, moving with a momentum $\mathbf{p}$ in a potential $V(x)$. Show that

$$\frac{d}{dt}\langle \hat{x} \cdot \hat{p} \rangle - \left\langle \frac{\hat{p}^2}{m} \right\rangle - \langle \hat{x} \cdot \nabla V \rangle$$

(7 marks)

Q 3(a) Consider N number of spin-one, non-interacting, distinguishable dipoles each having a magnetic moment of magnitude μ and placed in an external magnetic field $\vec{H} = H\hat{z}$ at temperature T. The i^{th} dipole has an energy $-\mu H s_i$, where s_i can take the values $-1, 0$ and 1. Find the average magnetization per dipole $\langle m \rangle$ for this system. What would be the behaviour of $\langle m \rangle$ for $k_B T \gg \mu H$?

(6 marks)

Q 3(b) Write down the energy eigen states of a two-dimensional quantum harmonic oscillator of angular frequency ω. Further, compute the change in entropy when the oscillator makes a transition from $E = 6\hbar\omega$ to $E = 3\hbar\omega$.

(4 marks)

Q 4(a) Find the Laurent series for $f(z) = \frac{z}{z^2+1}$ (5 marks)

around $z = i$.

Q 4(b) Simplify the Boolean function $f = \Sigma m(1,3,5,7,9) + \Sigma d(6,12,13)$ and draw the circuit diagram using NAND-NAND logic. (5 marks)

Q 5(a) A classical diatomic molecule is free to move in a volume V at temperature T having only rotational degrees of freedom. Assuming the molecule is non-interacting, write down the Lagrangian for rotation assuming it as a rigid body with two degrees of rotation about the two axes normal to the axis joining the atoms. Also determine the rotational partition function and rotational average energy of the molecule. (7 marks)

Q 5(b) In a scattering experiment an incident plane wave of wave number k undergoes a purely p-wave scattering with a total scattering cross section of $\frac{3\pi}{k^2}$. Find the phase shift δ_1. (3 marks)

Q 6. The Lagrangian for a system can be written as

$$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - k\sqrt{x^2+y^2}$$

where a, b, c, f, g, k are constants. Write the Hamiltonian of this system. What quantities are conserved for this system? (10 marks)

Q 7. The spectral line of wavelength $21\,cm$ is used to map the concentration of atomic hydrogen in our galaxy. What is the origin of this line? Determine the energy gap between the upper and the lower levels of this transition in terms of a coupling constant A.

Q 8. An ion beam of Ne^{9-} is accelerated through a potential of $200\,kV$. Post-acceleration the ion Ne^{8-} is selected by applying a magnetic field normal to the direction of the ion beam. Determine the value of the magnetic field that will bend the ion through a radius of $460\,mm$. A Hall probe is used to measure this magnetic field. Determine the Hall voltage when a current of $25\,mA$ is passed through the probe. The thickness of the probe is $2\,mm$ and the electron concentration is $5.16 \times 10^{20}\,m^{-3}$. Given mass of Neon is $20\,amu$.

Q9. A photon of energy $0.7\,\mathrm{MeV}$ is scattered by an electron at rest. If the photon is scattered at an angle of 60°, determine the momentum of the recoiling electron in units of (MeV/c). (10 marks)

Q10. Show that the Faraday's law can be obtained by using the following four-vector form of Maxwell's equations $\dfrac{\partial G^{\mu\nu}}{\partial x^{\nu}} = 0$ (10 marks)

Here $G^{\mu\nu}$ represents the dual tensor of the field tensor $F^{\mu\nu}$.

Q11. Write an expression for the power radiated by an accelerated charged particle moving at relativistic speed. A circular electron storage ring is used to store electrons of $0.5\,\mathrm{GeV}$ energy. The radius of the circular motion is $9.1\,m$ and power is continuously supplied to keep the electrons at constant energy. How much power is required to maintain this circular orbit? (10 marks)

Q12. Which type of spectroscopy is used to determine the bond lengths of the diatomic molecules CO and N_2? If the spacing between two adjacent lines of the spectrum (which is used to determine the bond length) is $3.844\,cm^{-1}$ for CO and $7.992\,cm^{-1}$ for N_2, determine the bond lengths of these molecules. (10 marks)

Q 13. Find the frequency of normal modes of vibration of a linear CO_2 molecule. (10 marks)

Q 14. A piece of metal with $\mu = 1$ is placed in a static magnetic field $\vec{B} - B_0\hat{z}$. The metal can be treated as a free electron gas with a scattering time τ and number density n. Derive an expression for resistivity tensor of this metal. (10 marks)

Q15(a) Use two ideal operational amplifiers to implement the following summing function

$$V_0 - V_1 + 2V_2 - 3V_3 - 5V_4 \cdot$$

(5 marks)

Q15(b) Show whether the following reactions/decays are possible or not and state which interaction is involved. (5 marks)

i. $p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^0 + \pi^+ + \pi^-$

ii. $p + K^- \rightarrow \Sigma^- + \pi^- + \pi^- + \pi^- + \pi^0$

iii. $p + \pi^- \rightarrow \Lambda^0 + \bar{\Sigma}^0$

iv. $\bar{\nu}_\mu + p \rightarrow \mu^- + n$

v. $\Sigma^0 \rightarrow \Lambda^0 + \gamma$