PREVIOUS YEAR'S SOLUTION

GATE 2025

Scan For Video Solutions

CSIR-NET Physics Course

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General Aptitude

Q.1 – Q.5 Carry ONE mark Each

Q1. Is there any good show _______ television tonight?

Select the most appropriate option to complete the above sentence.

(A) in (B) at (C) within (D) on

Ans.: (D)

Q2. As the police officer was found guilty of embezzlement, he was ______dismissed from the service in accordance with the Service Rules.

Select the most appropriate option to complete the above sentence.

(A) sumptuously (B) brazenly (C) unintentionally (D) summarily

Ans.: (D)

Q3. The sum of the following infinite series is:

$$\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\cdots$$

(A) π (B) $1+e$ (C) $e-1$ (D) e

Ans.: (C)

Solution: We need to find the sum:

$$\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots$$

The exponential series is:

$$e=\sum_{n=0}^{\infty}\frac{1}{n!}=\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\cdots$$

$$1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\cdots-1=e-1$$

Q4. A thin wire is used to construct all the edges of a cube of 1 m side by bending, cutting and soldering the wire. If the wire is 12 m long, what is the minimum number of cuts required to construct the wire frame to form the cube?

(A) 3 (B) 4 (C) 6 (D) 12

Ans.: (A)

Solution: There are 8 corners, so we need to provide wire ends to satisfy these odd junctions.

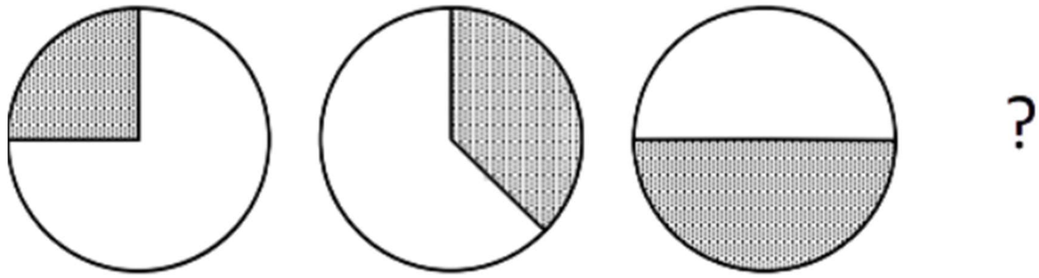
Each wire piece has 2 ends, so minimum number of wire pieces needed:

$$\text{pieces} \geq \frac{8}{2}=4$$

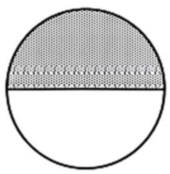
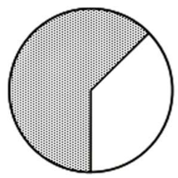
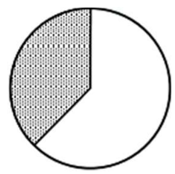
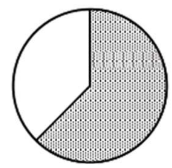
If you start with one wire, to make **4** pieces you need:

cuts $=4-1=3$, So cuts ≥ 3.

Q5. The figures I, II and III are parts of a sequence. Which one of the following options comes next in the sequence at IV?



I II III IV

(a)  (b)  (c)  (d) 

Ans.: (B)

Solution: Rotate clockwise and increase 25% . So Ans is B

Q.6 – Q.10 Carry TWO marks Each

Q6. "Why do they pull down and do away with crooked streets, I wonder, which are my delight, and hurt no man living? Every day the wealthier nations are pulling down one or another in their capitals and their great towns; they do not know why they do it; neither do I. It ought to be enough, surely, to drive the great broad ways which commerce needs and which are the life-channels of a modern city, without destroying all history and all the humanity in between: the islands of the past."

(From Hilaire Belloc's "The Crooked Streets")

Based only on the information provided in the above passage, which one of the following statements is true?

(A) The author of the passage takes delight in wondering.

(B) The wealthier nations are pulling down the crooked streets in their capitals.

(C) In the past, crooked streets were only built on islands.

(D) Great broad ways are needed to protect commerce and history.

Ans.: (B)

Q7. Rohit goes to a restaurant for lunch at about 1 PM. When he enters the restaurant, he notices that the hour and minute hands on the wall clock are exactly coinciding. After about an hour, when he leaves the restaurant, he notices that the clock hands are again exactly coinciding. How much time (in minutes) did Rohit spend at the restaurant?

(A) $64\frac{6}{11}$ (B) $66\frac{5}{12}$ (C) $65\frac{5}{11}$ (D) $66\frac{6}{13}$

Ans.: (C)

Solution: The hour and minute hands coincide once every

$$\frac{65}{11}\text{ minutes } = 65\frac{5}{11}\text{ minutes}$$

This interval is constant, irrespective of the time on the clock.

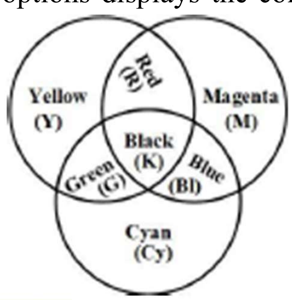
Rohit notices:

Coinciding hands when he enters ($\approx$ 1 PM)

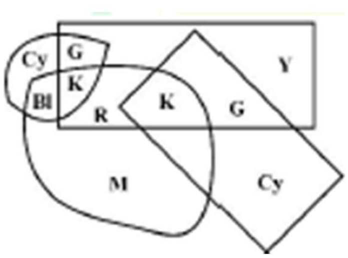
Coinciding hands again when he leaves ($\approx$ one hour later)

Q8. A color model is shown in the figure with color codes: Yellow (Y), Magenta (M), Cyan (Cy), Red (R), Blue (Bl), Green (G), and Black (K).

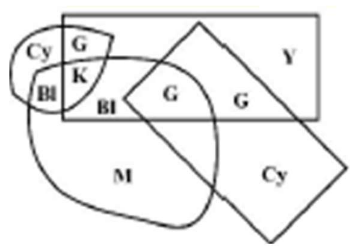
Which one of the following options displays the color codes that are consistent with the color model?



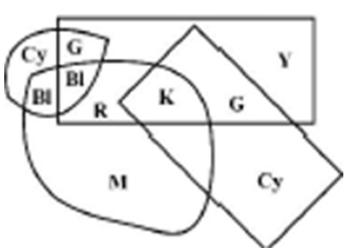
(a)



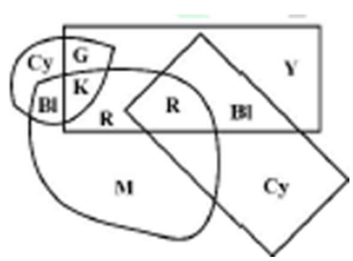
(b)



(c)



(d)



Ans.: (A)

Solution: The hour and minute hands coincide once every

$$\frac{65}{11} \text{ minutes } = 65\frac{5}{11} \text{ minutes}$$

This interval is constant, irrespective of the time on the clock.

Rohit notices:

- Coinciding hands when he enters ($\approx$ 1 PM)
- Coinciding hands again when he leaves ($\approx$ one hour later)

Black (K) is exactly at the three-way overlap Y, M, Cy are in the correct pairwise intersections

R, G, BI appear only in their correct single or overlapping regions.

Overall layout matches the reference color-code model perfectly

Overlap	Color
$R \cap G$	Yellow (Y)
R ∩ BI	Magenta (M)
G ∩ BI	Cyan (Cy)
R ∩ G ∩ BI	Black (K)

So the logical constraints are:

1. Black (K) must lie in the common intersection of R, G, and BI
2. Yellow (Y) must lie in $R \cap G$ only
3. Magenta (M) must lie in $R \cap Bl$ only
4. Cyan (Cy) must lie in G ∩ Bl only
5. No color label should appear outside its correct overlap region

Q9. A circle with center at $(x, y) = (0.5,0)$and radius $= 0.5$intersects with another circle with center at $(x, y) = (1,1)$and radius $= 1$at two points. One of the points of intersection $(x, y)$is:

(A) (0, 0) (B) (0.2, 0.4) (C) (0.5, 0.5) (D) (1, 2)

Ans.: (B)

Solution: The circles are

$$(x - 0.5)^2 + y^2 = 0.25, (x - 1)^2 + (y - 1)^2 = 1.$$

Substitute the options. For (0.2,0.4):

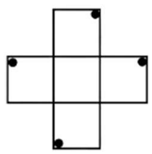
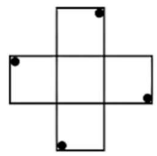
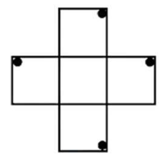
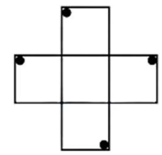
$$(0.2 - 0.5)^2 + 0.4^2 = 0.09 + 0.16 = 0.25$$
$$(0.2 - 1)^2 + (0.4 - 1)^2 = 0.64 + 0.36 = 1$$

It satisfies both equations. Hence, (0.2,0.4).

Q10. An object is said to have an n-fold rotational symmetry if the object, rotated by an angle of $\frac{2\pi}{n}$, is identical to the original.

Which one of the following objects exhibits 4 -fold rotational symmetry about an axis perpendicular to the plane of the screen?

Note: The figures shown are representative.

(a)  (b)  (c)  (d) 

Ans.: (B)

Solution:

Q.11 – Q.35 Carry ONE mark Each

Q11. For a two-dimensional hexagonal lattice with lattice constant a, the atomic density is

(a) $\frac{1}{\sqrt{3}a^2}$ (b) $\frac{1}{\sqrt{6}a^2}$ (c) $\frac{4}{3\sqrt{3}a^2}$ (d) $\frac{1}{3\sqrt{3}a^2}$

Topic: Condensed matter physics

Subtopic: Crystallography

Ans.: (C)

Solution: Primitive cell area:

$$A = \frac{3\sqrt{3}}{2}a^2$$

Atomic density:

$$\rho = \frac{N}{A} = \frac{2}{\frac{3\sqrt{3}}{2}a^2} = \frac{4}{3\sqrt{3}a^2}$$

Q12. Consider a crystal that has a basis of one atom. Its primitive vectors are

$$\vec{a}_1 = a\hat{\imath}, \vec{a}_2 = a\hat{\jmath}, \vec{a}_3 = \frac{a}{2}(\hat{\imath} + \hat{\jmath} + \hat{k})$$

where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are the unit vectors in the x, y, and z directions of the Cartesian coordinate system and a is a positive constant. Which one of the following is the correct option regarding the type of the Bravais lattice?

(A) It is BCC and the volume of the primitive cell is $\frac{a^3}{2}$

(B) It is FCC and the volume of the primitive cell is $\frac{a^3}{4}$

(C) It is BCC and the volume of the primitive cell is $\frac{a^3}{8}$

(D) It is FCC and the volume of the primitive cell is a^3

Topic: Condensed matter physics

Subtopic: Crystallography

Ans.: (A)

Solution: Primitive cell volume:

$$V = |\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)|$$

$$\vec{a}_2 \times \vec{a}_3 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & a & 0 \\ a/2 & a/2 & a/2 \end{vmatrix} = \frac{a^2}{2}\hat{\imath} - \frac{a^2}{2}\hat{k}$$

$$V = \left|(a\hat{\imath}) \cdot \left(\frac{a^2}{2}\hat{\imath} - \frac{a^2}{2}\hat{k}\right)\right| = \frac{a^3}{2}$$

Lattice points generated are

$$(x, y, z) = a\left(n_1 + \frac{n_3}{2}, n_2 + \frac{n_3}{2}, \frac{n_3}{2}\right)$$

So for n_3 even → (integer, integer, integer); for n_3 odd → $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ shift ⇒ BCC.

$$\text{BCC}, V_{\text{prim}} = \frac{a^3}{2}$$

Q13. A particle of mass m is in a potential

$$V(x) = \frac{1}{2}m\omega^2 x^2 \quad \text{for } x > 0$$

and

$$V(x) = \infty \quad \text{for } x \leq 0$$

where ω is the angular frequency. The ratio of the energies corresponding to the lowest energy level to the next higher level is

(A) $\frac{3}{7}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{3}{5}$

Topic: Quantum mechanics

Subtopic: Harmonic oscillator

Ans.: A

Solution: For semi parabolic potential $E_n = \left(n + \frac{1}{2}\right)\hbar\omega \; n = 1,3,5 .. \frac{E_1}{E_3} = \frac{\frac{3}{2}}{\frac{7}{2}} = \frac{3}{7}$

Q14. A particle is scattered from a potential $V(\vec{r}) = g\delta^3(\vec{r})$, where g is a positive constant. Using the first Born approximation, the angular (θ, ϕ) dependence of differential scattering cross section $\frac{d\sigma}{d\Omega}$ is

(A) Independent of θ but dependent on ϕ (B) Dependent on θ but independent of ϕ

(C) Dependent on both θ and ϕ (D) Independent of both θ and ϕ

Topic: Quantum Mechanics

Subtopic: Scattering

Ans.: D

Solution: Given $V(\mathbf{r}) = g\delta^3(\mathbf{r})$,

$$f \propto \int e^{-i\mathbf{q}\cdot\mathbf{r}} g\delta^3(\mathbf{r}) d^3r = g$$

So f is a constant, hence

$$\frac{d\sigma}{d\Omega} = |f|^2 = \text{ constant}$$

Q15. The Joule-Thomson expansion of a gas is

(A) Isentropic (B) Isenthalpic (C) Isobaric (D) Isochoric

Topic: Thermodynamics and statistical mechanics

Subtopic: Maxwell relation

Ans.: B

Solution: In Joule-Thomson expansion, a real gas flows through a porous plug/throttle valve from high pressure to low pressure without doing external work and without heat exchange with surroundings.

From the steady-flow energy equation:

$$H_{\text{initial}} = H_{\text{final}}$$

So the process occurs at constant enthalpy.

Q16. Which one of the following is correct for the phase velocity v_p and group velocity v_g ? (c is the speed of light in vacuum)

(A) For matter waves in the relativistic case, $v_p v_g > c^2$

(B) For electromagnetic waves in a medium, v_p represents the speed with which energy propagates

(C) For electromagnetic waves in a medium, both v_p and v_g can be more than c

(D) For matter waves in free space, $v_p \neq v_g$

Topic: Electromagnetic theory

Subtopic: Electromagnetic wave

Ans.: (D)

Solution: For matter wave

$$v_g = v, v_p = \frac{v}{2}$$

Q17. As per the Drude model of metals, the electrical resistance of a metallic wire of length L and crosssection area A is

(Consider τ as the relaxation time, m as electron mass, n as carrier concentration and e as electronic charge)

(A) $\frac{mL}{ne^2A\tau}$ (B) $\frac{2m}{ne^2A\tau}$ (C) $\frac{mL}{2ne^2A\tau}$ (D) $\frac{mL}{4ne^2A\tau}$

Topic: Condensed Matter

Subtopic: Electromagnetic wave

Ans.: A

Solution: Using Drude model,

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow R = \frac{L}{A\sigma} = \frac{mL}{ne^2A\tau}$$

Q18. Which one of the following baryons has strangeness quantum number $S = -1$?

(A) Σ^{*0} (B) n (C) Ξ^{*0} (D) Δ^0

Topic: Nuclear and particle physics

Subtopic: Particle physics

Ans.: A

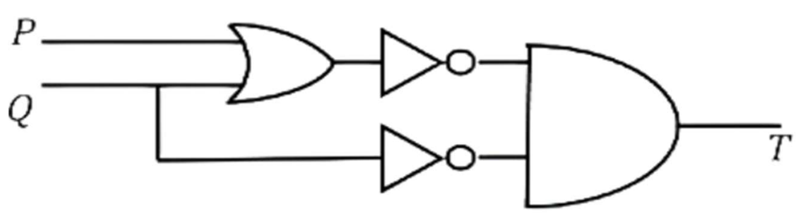
Solution: (A) Σ^{*0} : quark content uds → contains one s quark → $S = -1$

(B) n (neutron) : udd → no strange quark → $S = 0$

(C) Ξ^{*0}: uss → two strange quarks → $S = -2$

(D) Δ^0: udd → no strange quark → $S = 0$

Q19. A logic gate circuit is shown in the figure below. The correct combination for the input (P, Q) for which the output $T = 1$ is



(A) (0,0) (B) (0,1) (C) (1,1) (D) (1,0)

Topic: Electronics and digital electronics

Subtopic: Digital electronics (Logic gate)

Ans.: A

Solution: From the circuit,

$$T=\overline{(P+Q)}\cdot\bar{Q}=\bar{P}\bar{Q}$$

which is 1 only for $(P,Q)=(0,0)\Rightarrow$ option (A).

Q20. The nuclear energy levels of mirror nuclei are similar. Using this empirical fact alone, the nuclear force can be said to be independent of which one of the following properties of the nucleons?

(A) Mass (B) Spin (C) Charge (D) Parity

Topic: Nuclear and particle physics

Sub topic: Nuclear properties

Ans.: C

Solution: Since the only intrinsic difference between a proton and a neutron is electric charge, this shows that the nuclear force is independent of charge (apart from small Coulomb effects).

Mass ⟶ proton and neutron masses are already nearly equal, not the key inference

Spin ⟶ nuclear force is known to depend on spin

Parity ⟶ not related to mirror symmetry

Charge ⟶ correct inference

Q21. Consider the function

$$f(z)=\frac{1}{z^2(z-2)^3}$$

of a complex variable z. The residues of the function at $z=0$ and $z=2$, respectively, are

(A) $-\frac{3}{8}$ and $\frac{3}{8}$ (B) $\frac{3}{8}$ and $-\frac{3}{16}$ (C) $-\frac{3}{16}$ and $\frac{3}{16}$ (D) $-\frac{3}{8}$ and $\frac{3}{16}$

Topic: Mathematical physics

Subtopic: Complex variable

Ans.: C

Solution: At $z=0$, there is a pole of order 2 .

$$\operatorname{Res}(f,0)=\frac{1}{(2-1)!}\lim_{z\to0}\frac{d}{dz}[z^2f(z)]=\lim_{z\to0}\frac{d}{dz}\left(\frac{1}{(z-2)^3}\right)$$

$$\frac{d}{dz}(z-2)^{-3}=-3(z-2)^{-4}\Rightarrow\operatorname{Res}(f,0)=-\frac{3}{(-2)^4}=-\frac{3}{16}$$

At $z=2$, there is a pole of order 3 .

$$\operatorname{Res}(f,2)=\frac{1}{(3-1)!}\lim_{z\to2}\frac{d^2}{dz^2}[(z-2)^3f(z)]=\frac{1}{2}\lim_{z\to2}\frac{d^2}{dz^2}\left(\frac{1}{z^2}\right)$$

$$\frac{d}{dz}(z^{-2})=-2z^{-3},\frac{d^2}{dz^2}(z^{-2})=6z^{-4}\Rightarrow\operatorname{Res}(f,2)=\frac{1}{2}\cdot\frac{6}{2^4}=\frac{3}{16}$$

Q22. Consider one mole of a monovalent metal at absolute zero temperature, obeying the free electron model. Its Fermi energy is E_F. The energy corresponding to the filling of $\frac{N_A}{2}$ electrons, where N_A is the Avogadro number, is $2^n E_F$. The value of n is

(A) $-\frac{2}{3}$ (B) $+\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) -1

Topic: Condensed Matter physics

Subtopic: Free electron theory

Ans: A

Solution: In 3D free-electron model:

$$N(E) \propto E^{3/2} \Rightarrow E \propto N^{2/3}$$

For half the electrons:

$$E = \left(\frac{1}{2}\right)^{2/3} E_F = 2^{-2/3} E_F, n = -\frac{2}{3}$$

Q23. A paramagnetic material containing paramagnetic ions with total angular momentum $J = \frac{1}{2}$ is kept at absolute temperature T. The ratio of the magnetic field required for 80% of the ions to be in the lowest energy state to that required for having 60% of the ions to be in the lowest energy state at the same temperature is

(A) $\frac{2\ln 2}{\ln\left(\frac{3}{2}\right)}$ (B) $\frac{\ln 2}{\ln\left(\frac{3}{2}\right)}$ (C) $\frac{3\ln 2}{\ln\left(\frac{3}{2}\right)}$ (D) $\frac{\ln 3}{\ln\left(\frac{3}{2}\right)}$

Topic: Thermodynamics and Statistical Physics

Subtopic: Canonical ensemble (Paramagnetism)

Ans.: (a)

Solution: For $J = \frac{1}{2}$, there are only two Zeeman levels with energies,

$$E_{\pm} = \mp\frac{\Delta}{2}, \Delta = g\mu_B B.$$

This is a two-level system. Boltzmann populations give the fraction in the lowest state (using the concept of the partition function $e^{+\Delta/2k} + e^{-\Delta/2k}$):

$$P = \frac{e^{+\Delta/2kT}}{e^{+\Delta/2k} + e^{-\Delta/2kT}} = \frac{1}{1 + e^{-\Delta/kT}}$$

$$\frac{1-P}{P} = e^{-\Delta/kT} \Rightarrow \frac{\Delta}{kT} = \ln\left(\frac{P}{1-P}\right)$$

$$B \propto \ln\left(\frac{P}{1-P}\right).$$

$$\frac{B_{80}}{B_{60}} = \frac{\ln\left(\frac{0.8}{0.2}\right)}{\ln\left(\frac{0.6}{0.4}\right)} = \frac{\ln(4)}{\ln(3/2)} = \frac{2\ln 2}{\ln(3/2)}.$$

Q24. Which of the following option (s) is/are correct for the ground state of a hydrogen atom?

(A) Linear Stark effect is zero
(B) It has definite parity
(C) Spin-orbit coupling is zero
(D) Hyperfine splitting is zero

Topic: Atomic and molecular physics

Subtopic: Hydrogen Atom

Ans.: A,B and C

Solution: Hydrogen ground state has no linear Stark effect, definite parity, zero spin-orbit coupling, but non-zero hyperfine splitting.

Q25. Which of the following option(s) is/are correct for photons?

(A) Its rest mass is zero, but its energy is non-zero

(B) It carries non-zero linear momentum

(C) It carries zero spin angular momentum

(D) It has two linearly independent states of polarization

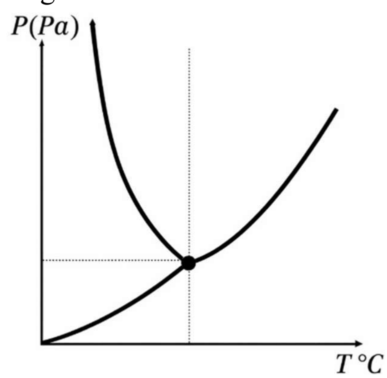
Topic: Nuclear and particle physics

Subtopic: Particle physics

Ans.: A,B and D

Solution: Photons have zero rest mass, non-zero energy & momentum, spin-1, and two polarization states.

Q26. A schematic Pressure-Temperature diagram of water is shown in the figure. Which of the following option(s) is/are correct?



(A) Clausius-Clapeyron equation is valid across the melting curve and the vaporization curve

(B) Melting curve has the highest slope

(C) The critical point exists only for the vaporization curve

(D) Clausius-Clapeyron equation is not valid across the melting curve and the vaporization curve

Topic: Thermodynamics and statistical mechanics

Subtopic: Phase transition

Ans.: A,B and C

Solution: Clausius–Clapeyron applies to all first-order phase transitions, so it is valid for both melting and vaporization → (A) true.

For water, the melting curve is very steep (small Δv), giving the largest $|dP/dT| \rightarrow$ (B) true.

A critical point exists only on the liquid–vapour curve → (C) true.

Hence (D) false.

Q27. Which of the following consideration(s) is/are showing that nuclear beta decay, $n \rightarrow p + e^- + \bar{\nu}_e$, has to be a three-body decay?

(A) Continuous distribution of the electron energy

(B) Spin of the final state

(C) Mass of the electron

(D) Mass of the proton

Topic: Nuclear and particle physics

Subtopic: Particle physics

Ans.: A and B

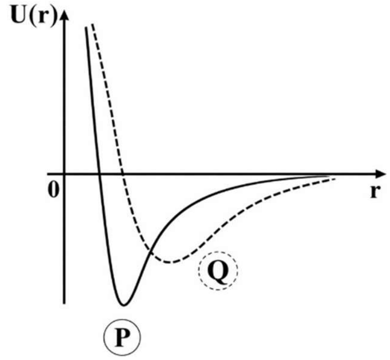
Solution: If it were a two-body decay $n \rightarrow p + e^-$, energy-momentum conservation would fix the electron energy to a single discrete value.

Experimentally, the beta electron shows a continuous spectrum, so a third particle must carry away variable energy/momentum.

Neutron has spin $1/2$. If only p and e^-were produced, angular momentum constraints (together with observed beta-decay features) indicate the need for an additional spin- $1/2$ particle to conserve angular momentum properly.

The neutrino provides that missing spin degree of freedom.

Q28. Potential energy of two diatomic molecules P and Q of the same reduced mass is shown in the figure. According to this diagram, which of the following option(s) is/are correct?



(A) The equilibrium inter-nuclear distance of Q is more than that of P

(B) The total energy $E = 0$ separates bound and unbound states of the molecules

(C) The lowest vibrational frequency of P is larger than that of Q

(D) Dissociation energy of Q is more than that of P

Topic: Atomic and molecular physics

Subtopic: Molecular physics (vibrational spectra)

Ans.: A,B and C

Solution: The equilibrium bond length r_e is at the minimum of $U(r)$.

The dissociation limit is $U(\infty) = 0$.

For small vibrations, $\omega \approx \sqrt{k/\mu}$ where $k = \left.\frac{d^2U}{dr^2}\right|_{r_e}$ and μ is same for P and Q , so ω depends only on curvature at the minimum.

Q29. Nuclear radiation emitted from a ^{60}Co radioactive source is detected by a photomultiplier tube (PMT) coupled to a scintillator crystal. Which of the following option(s) is/are correct?

(A) γ radiation from ^{60}Co will directly hit the photocathode of the PMT without interacting with the scintillator crystal and produce a signal

(B) β radiation from ^{60}Co source interacts with the scintillator crystal, producing γ radiation, which will hit the photocathode of the PMT and produce a signal

(C) A mu-metal shield is put all around the PMT to nullify the effect of external electric fields

(D) A mu-metal shield is put all around the PMT to nullify the effect of external magnetic fields

Topic: Nuclear and particle physics

Subtopic: Particle physics

Ans.: B and D

Solution: In scintillation detectors, γ-rays → scintillator → light → PMT, and mu-metal shields PMTs from magnetic fields.

Q30. One mole of an ideal monatomic gas at absolute temperature T undergoes free expansion to double its original volume, so that the entropy change is ΔS_1. An identical amount of the same gas at absolute temperature $2T$ undergoes isothermal expansion to double its original volume, so that the entropy change is ΔS_2. The value of $\frac{\Delta S_1}{\Delta S_2}$ (in integer) is ___

Topic: Thermodynamics and Statistical Mechanics

Subtopic: Second law of thermodynamics

Ans.: 1

Solution: Case 1: Free expansion $V \to 2V$ at temperature T

Even though the process is irreversible, compute ΔS using a reversible path:

$$\Delta S_1 = nR\ln\left(\frac{V_f}{V_i}\right) = (1)R\ln\ 2$$

Case 2: Isothermal expansion $V \to 2V$ at temperature $2T$

For isothermal reversible expansion:

$$\Delta S_2 = nR\ln\left(\frac{V_f}{V_i}\right) = (1)R\ln\ 2$$

So,

$$\frac{\Delta S_1}{\Delta S_2} = \frac{R\ln\ 2}{R\ln\ 2} = 1$$

Q31. A linear dielectric sphere of radius R has a uniform frozen-in polarization along the z-axis. The center of the sphere initially coincides with the origin, about which the electric dipole moment is $\vec{p}_1$. When the sphere is shifted to the point $(2R, 0,0)$, the corresponding dipole moment with respect to the origin is $\vec{p}_2$. The value of $\frac{|\vec{p}_1|}{|\vec{p}_2|}$ (in integer) is_____

Topic: Electromagnetic theory

Subtopic: Electrostatic (Dielectric)

Ans.: 1

Solution: For a localized neutral polarization distribution, shifting the whole object by a changes the dipole moment as

$$\mathrm{p}' = \mathrm{p} + \mathrm{a}Q_{\text{tot}}\,.$$

Here, the "frozen-in" uniform polarization gives only bound charges, and the total bound charge is zero:

$$Q_{\text{tot}} = \int \rho_b dV + \oint \sigma_b dA = 0$$

So the dipole moment is independent of origin shift:

$$\mathrm{p}_2 = \mathrm{p}_1 \Rightarrow \frac{|\mathrm{p}_1|}{|\mathrm{p}_2|} = 1.$$

Q32. The effective magnetic moment (in units of Bohr magneton) for the ground state of an isolated $4f$ ion with 6 unpaired electrons in the $4f$ shell according to Hund's rules is (in integer) _____

Topic: Atomic and molecular physics

Subtopic: Zeeman effect

Ans.: 0

Solution: Maximize spin

$$S = 6 \times \frac{1}{2} = 3$$

For $4f^6$, ground term is ${}^{2S+1}L = {}^7F \Rightarrow$

$$L = 3$$

For less than half-filled shell:

$$J = |L - S| = |3 - 3| = 0$$

Magnetic moment:

$$\mu_{\text{eff}} = g_J\sqrt{J(J+1)}\mu_B = 0$$

Q33. Powder X-ray diffraction pattern of a cubic solid with lattice constant a has the (111) diffraction peak at $\theta = 30°$. If the lattice expands such that the lattice constant becomes $1.25a$, the angle (in degrees) corresponding to the (111) peak changes to $\sin^{-1}\left(\frac{1}{n}\right)$. The value of n (rounded off to one decimal place) is______

Topic: Condensed Matter Physics

Subtopic: Crystallography Bragg's diffraction

Ans.: 2.5

Solution: Using Bragg's law $2d\sin\,\theta = \lambda$ with $d_{111} \propto a$, expansion $a \to 1.25a$ gives

$\sin\,\theta' = \sin\,30°/1.25 = 0.4 = 1/n$, hence

$$n = 2.5$$

Q34. Consider a monatomic chain of length 30 cm . The phonon density of states is 1.2×10^{-4} s. Assuming the Debye model, the velocity of sound in m/s (rounded off to one decimal place) is_______

Topic: Condense Matter Physics

Subtopic: Band theory

Ans.: 794.0 to 797.0

Solution: For a 1D monatomic chain (Debye model), the phonon density of states is

$$g(\omega) = \frac{L}{\pi v}$$

Given:

- $L = 30 \text{ cm} = 0.30 \text{ m}$
- $g(\omega) = 1.2 \times 10^{-4}$ s

Solve for sound velocity v : $v = \frac{L}{\pi g(\omega)} = \frac{0.30}{\pi \times 1.2 \times 10^{-4}}$

$$v \approx \frac{0.30}{3.7699 \times 10^{-4}} \approx 795.8 \text{ m/s}$$

Q35. The Δ^+baryon with spin $\frac{3}{2}$, at rest, decays to a proton and a pion ($\Delta^+ \rightarrow p + \pi^0$). The Δ^+has positive intrinsic parity and π^0 has negative intrinsic parity. The orbital angular momentum of the proton-pion system (in integer) is ___

Topic: Nuclear and particle physics

Subtopic: Particle physics

Ans.: 1

Solution: For the decay $\Delta^+ \left(J^P = \frac{3}{2}^+\right) \rightarrow p + \pi^0$:

- Intrinsic parities: $P_\Delta = +1, P_p = +1, P_{\pi^0} = -1$.
- Final parity:

$$P_f = P_p P_{\pi^0} (-1)^L = (+1)(-1)(-1)^L = -(-1)^L$$

Parity conservation $P_f = P_i = +1$ gives

$$-(-1)^L = +1 \Rightarrow (-1)^L = -1 \Rightarrow L = \text{ odd}$$

Angular momentum: proton spin $1/2$, pion spin $0 \Rightarrow S_f = 1/2$. To get total $J = 3/2$, with L odd, the smallest allowed is $L = 1$ (since $L = 1$ with $S = \frac{1}{2}$ can produce $J = \frac{3}{2}$).

Q.36 – Q.55 Carry TWO marks Each

Q36. The screened nuclear charge of neutral Helium atom is given as 1.7 e , where **e** is the magnitude of the electronic charge. Assuming the Bohr model of the atom for which the energy levels are

$$E_n = -\frac{Z^2}{2}\frac{1}{n^2}$$

atomic units (Z is the atomic number), the first ionization potential of Helium in atomic units is

(A) 0.89 (B) 1.78 (C) 0.94 (D) 3.16

Topic: Atomic and molecular physics

Subtopic: Bohr model

Ans.: A

Solution: Ground-state energy of neutral He

Both electrons are in $n = 1$:

$$E(\text{He}) = 2\left(-\frac{(1.7)^2}{2}\right) = -(1.7)^2 = -2.89 \text{ a.u.}$$

2. Ground-state energy of He^+

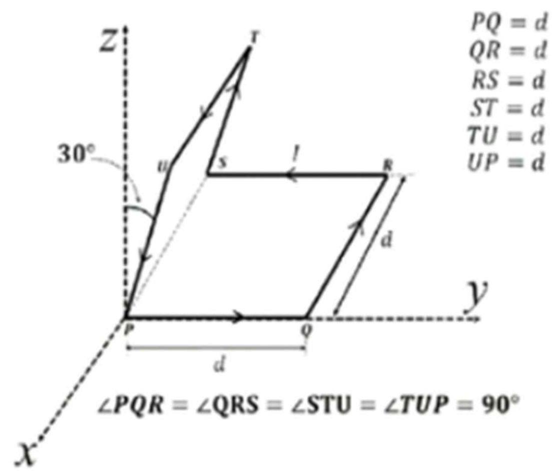
After removing one electron, He^+ is hydrogen-like with $Z = 2, n = 1$:

$$E(\text{He}^+) = -\frac{2^2}{2} = -2 \text{ a.u.}$$

3. First ionization potential

$$I_1 = E(\text{He}^+) - E(\text{He}) = (-2) - (-2.89) = 0.89 \text{ a.u.}$$

Q37. The wire loop shown in the figure carries a steady current I. Each straight section of the loop has length d. A part of the loop lies in xy plane and the other part is tilted at 30° with respect to the xz plane. The magnitude of the magnetic dipole moment of the loop (in appropriate units) is



(A) $\sqrt{2}Id^2$ (B) $2Id^2$ (C) $\sqrt{3}Id^2$ (D) Id^2

Topic: Electromagnetic theory

Subtopic- Magneto state

Ans.: (d)

Solution: Pass a current I from $P \rightarrow S$ and from $S \rightarrow P$ so that two closed loops are formed. Each loop acts as a magnetic dipole.

The magnetic moment of each loop is, $\mu = Id^2$.

The two magnetic moments make an angle of $120°$ with each other. Therefore, the resultant magnetic moment is obtained using the cosine law:

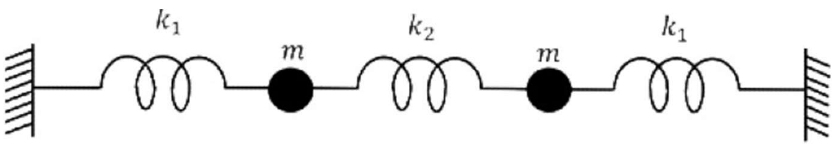
$$\mu_{\text{res}} = \sqrt{\mu^2 + \mu^2 + 2\mu^2 \cos\, 120°}.$$

Since $\cos\, 120° = -\frac{1}{2}$

$$\mu_{\text{res}} = \sqrt{2\mu^2 - \mu^2} = \mu = Id^2.$$

$$\mu_{\text{res}} = Id^2$$

Q38. The figure shows a system of two equal masses m and three massless horizontal springs with spring constants k_1, k_2, k_1. Ignore gravity. The masses can move only in the horizontal direction and there is no dissipation. If $m = 1, k_1 = 2$ and $k_2 = 3$ (all in appropriate units), the frequencies of the normal modes of the system in the same system of units are



(A) $\sqrt{2}, \sqrt{8}$ (B) $\sqrt{2}, \sqrt{6}$ (C) $\sqrt{3}, \sqrt{10}$ (D) $\sqrt{3}, \sqrt{8}$

Topic: Classical Mechanics

Subtopic: Small Oscillation

Ans.: (a)

Solution: Normal modes satisfy, $\det(K - \omega^2 M) = 0$

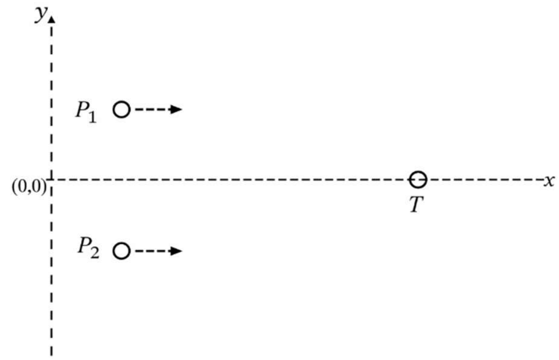
i.e. $\det\begin{pmatrix} k_1 + k_2 - m\omega^2 & -k_2 \\ -k_2 & k_1 + k_2 - m\omega^2 \end{pmatrix} = 0$

$\Rightarrow (k_1 + k_2 - m\omega^2)^2 - k_2^2 = 0 \Rightarrow k_1 + k_2 - m\omega^2 = \pm k_2$

$\omega_1^2 = \frac{k_1}{m}, \omega_2^2 = \frac{k_1 + 2k_2}{m},$

Put $m = 1, k_1 = 2, k_2 = 3, \quad \omega_1 = \sqrt{2}, \omega_2 = \sqrt{8}$

Q39. Two projectile protons P_1 and P_2 both with spin up (along the $+z$ direction) are scattered from another fixed target proton T with spin up at rest in the xy plane, as shown in the figure. They scatter one at a time. The nuclear interaction potential between both the projectiles and the target proton is $\lambda\vec{L}\cdot\vec{S}$, where $\vec{L}$ is the orbital angular momentum of the system with respect to the target, $\vec{S}$ is the spin angular momentum of the system and λ is a negative constant in appropriate units. Which one of the following is correct?



(A) P_1 will be scattered in the $+y$ direction (upward) and P_2 will be scattered in the $-y$ direction (downward)

(B) P_1 will be scattered in the $+y$ direction (upward) and P_2 will be scattered in the $+y$ direction (upward)

(C) P_1 will be scattered in the $-y$ direction (downward) and P_2 will be scattered in the $+y$ direction (upward)

(D) P_1 will be scattered in the $-y$ direction (downward) and P_2 will be scattered in the $-y$ direction (downward)

Topic: Quantum mechanics

Subtopic: Spin

Ans.: (b)

Solution: For P1 S_z is +ve so $\lambda\vec{S}.\vec{L}=\lambda S_z L_z$ where $L_z = y\hat{j}\times p_x\hat{i} = -yp_x\hat{z}$ so potential is $V=-\lambda S_z p_x\hat{y}$

So $\dot{p}_y = -\lambda\dfrac{\partial V}{\partial y} = -\lambda(-p_x) = \lambda p_x$ hence λ is negative force is attractive

For P2 S_z is +ve so $\lambda\vec{S}.\vec{L}=\lambda S_z L_z$ where $L_z = -y\hat{j}\times p_x\hat{i} = yp_x\hat{z}$ so potential is $V=\lambda S_z p_x\hat{y}$

So $\dot{p}_y = -\lambda\dfrac{\partial V}{\partial y} = -\lambda(p_x) = -\lambda p_x$ hence λ is negative force is repulsive

Q40. A thin circular ring of radius R lies in the xy plane with its centre coinciding with the origin. The ring carries a uniform line charge density λ. The quadrupole contribution to the electrostatic potential at the point $(0,0,d)$, where $d \gg R$, is

(A) $-\dfrac{\lambda R^3}{4\varepsilon_0 d^3}$ (B) 0 (C) $\dfrac{\lambda R^3}{4\varepsilon_0 d^3}$ (D) $-\dfrac{\lambda R^3}{2\varepsilon_0 d^3}$

Topic: Electromagnetic Theory

Subtopic: Electrostatics (Electric multipole)

Ans.: (a)

Solution: Total charge on the ring:

$$Q = 2\pi R\lambda$$

Electrostatic potential on the axis of a uniformly charged ring at $(0,0,d)$ is

$$V(d) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{d^2+R^2}}$$

For $d \gg R$, expand using binomial approximation:

$$\frac{1}{\sqrt{d^2+R^2}} = \frac{1}{d}\left(1+\frac{R^2}{d^2}\right)^{-1/2} \approx \frac{1}{d}-\frac{R^2}{2d^3}+\cdots$$

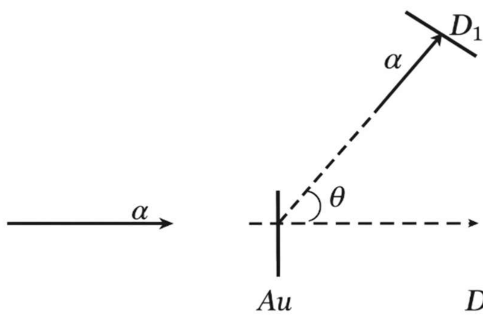
Hence, $V(d) \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{d}-\frac{QR^2}{2d^3}\right)$

First term $\longrightarrow$ monopole

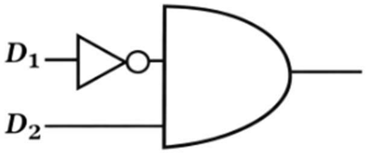
Second term $\longrightarrow$ quadrupole contribution

Quadrupole term: $V_Q = -\frac{1}{4\pi\varepsilon_0}\frac{QR^2}{2d^3} = -\frac{1}{4\pi\varepsilon_0}\frac{(2\pi R\lambda)R^2}{2d^3}$, $V_Q = -\frac{\lambda R^3}{4\varepsilon_0 d^3}$

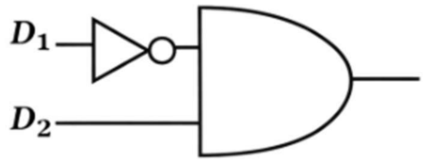
Q41. An α particle is scattered from an Au target at rest as shown in the figure. D_1 and D_2 are the detectors to detect the scattered α particle at an angle θ and along the beam direction, respectively, as shown. The signals from D_1 and D_2 are converted to logic signals and fed to logic gates. When a particle is detected, the signal is 1 and is 0 otherwise. Which one of the following circuits detects the particle scattered at the angle θ only?



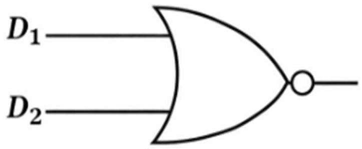
(a)



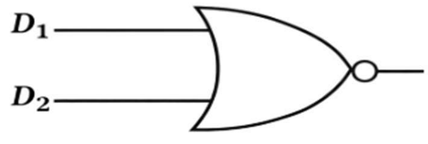
(b)



(c)



(d)



Topic: Electronics

Subtopic: Digital electronics (Logic gate)

Ans.: A

Solution: So the required logic is: Output $= D_1 \wedge \overline{D_2}$

Q42. Consider a two-level system with energy states $+\varepsilon$ and $-\varepsilon$. The number of particles at $+\varepsilon$ level is N_+ and the number of particles at $-\varepsilon$ level is N_-. The total energy of the system is E and the total number of particles is $N = N_+ + N_-$. In the thermodynamic limit, the inverse of the absolute temperature of the system is (Given: $\ln N! \simeq N\ln N - N$)

(a) $\frac{k_B}{2\varepsilon}\ln\left[\frac{N-\frac{E}{\varepsilon}}{N+\frac{E}{\varepsilon}}\right]$ (b) $\frac{k_B}{\varepsilon}\ln N$ (c) $\frac{k_B}{2\varepsilon}\ln N$ (d) $\frac{k_B}{\varepsilon}\ln\left[\frac{N-\frac{E}{\varepsilon}}{N+\frac{E}{\varepsilon}}\right]$

Topic: Thermodynamics and statistical mechanics

Subtopic: Microcanonical ensemble

Ans.: A

Solution: For two levels $+\varepsilon$ and $-\varepsilon$,

$$E = \varepsilon(N_+ - N_-), N = N_+ + N_-$$

$$\Rightarrow N_+ = \frac{1}{2}\left(N + \frac{E}{\varepsilon}\right), N_- = \frac{1}{2}\left(N - \frac{E}{\varepsilon}\right)$$

Multiplicity:

$$\Omega = \frac{N!}{N_! N_-!}, S = k_B \ln \Omega$$

Using Stirling $\ln n! \simeq n\ln n - n$:

$$S \simeq k_B[N\ln N - N_+\ln N_+ - N_-\ln N_-]$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N$$

with $\frac{dN_+}{dE} = \frac{1}{2\varepsilon}, \frac{dN_-}{dE} = -\frac{1}{2\varepsilon}$, giving

$$\frac{1}{T} = \frac{k_B}{2\varepsilon}\ln\left(\frac{N_-}{N_+}\right) = \frac{k_B}{2\varepsilon}\ln\left[\frac{N-\frac{E}{\varepsilon}}{N+\frac{E}{\varepsilon}}\right]$$

Q43. Let $|m\rangle$ and $|n\rangle$ denote the energy eigenstates of a one-dimensional simple harmonic oscillator. The position and momentum operators are $\hat{X}$ and $\hat{P}$, respectively. The matrix element $\langle m|\hat{P}\hat{X}|n\rangle$ is non-zero when

(a) $m = n \pm 2$ only

(b) $m = n$ or $m = n \pm 2$

(c) $m = n \pm 3$ only

(d) $m = n \pm 1$ only

Topic: Quantum mechanics

Subtopic: Harmonic Oscillator

Ans.: B

Solution: $\hat{p}\hat{x} \propto (a^\dagger - a)(a + a^\dagger) = a^\dagger a + a^\dagger a^\dagger - aa - aa^\dagger$

Now check how each term changes $|n\rangle$:

$a^\dagger a$ and $aa^\dagger$ keep n same $\Rightarrow \Delta n = 0$

$a^\dagger a^\dagger$ raises by 2 $\Rightarrow \Delta n = +2$

aa lowers by 2 $\Rightarrow \Delta n = -2$

Hence $\langle m|\hat{p}\hat{x}|n\rangle \neq 0$ only for

$$m = n \text{ or } m = n \pm 2$$

Q44. A two-level quantum system has energy eigenvalues E_1 and E_2. A perturbing potential

$$H' = \lambda\Delta\sigma_x$$

is introduced, where Δ is a constant having dimensions of energy, λ is a small dimensionless parameter, and

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The magnitudes of the first and the second order corrections to E_1 due to H', respectively, are

(A) 0 and $\frac{\lambda^2\Delta^2}{|E_1-E_2|}$ (B) $\frac{|\lambda\Delta|}{2}$ and $\frac{\lambda^2\Delta^2}{|E_1-E_2|}$ (C) $|\lambda\Delta|$ and $\frac{\lambda^2\Delta^2}{|E_1-E_2|}$ (D) 0 and $\frac{1}{2}\frac{\lambda^2\Delta^2}{|E_1-E_2|}$

Topic: Quantum Mechanics

Subtopic: Non-degenerate perturbation

Ans.: A

Solution: We assume original Hamiltonian is diagonalise

$$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}, H' = \lambda\Delta\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

First order correction to E_1'

$$E_1^{(1)} = \langle 1|H'|1\rangle = \lambda\Delta(\sigma_x)_{11} = 0$$

Second order correction to E_1

Only $n = 2$ contributes:

$E_1^{(2)} = \frac{|\langle 2|H'|1\rangle|^2}{E_1-E_2}, \langle 2|H'|1\rangle = \lambda\Delta(\sigma_x)_{21} = \lambda\Delta$

$E_1^{(2)} = \frac{|\langle 2|H'|1\rangle|^2}{E_1-E_2}, \langle 2|H'|1\rangle = \lambda\Delta(\sigma_x)_{21} = \lambda\Delta$

So magnitude: $\left|E_1^{(2)}\right| = \frac{\lambda^2\Delta^2}{|E_1-E_2|}, \lambda^2\Delta^2$

Q45. An electron with mass m and charge q is in the spin up state $\binom{1}{0}$ at time $t = 0$. A constant magnetic field is applied along the y-axis, $B = B_0\hat{j}$, where B_0 is a constant. The Hamiltonian of the system is $H = -\hbar\omega\sigma_y$, where $\omega = \frac{qB_0}{2m} > 0$ and $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. The minimum time after which the electron will be in the spin down state along the x-axis, i.e., $\frac{1}{\sqrt{2}}\binom{1}{-1}$, is

(A) $\frac{\pi}{8\omega}$ (B) $\frac{\pi}{4\omega}$ (C) $\frac{\pi}{2\omega}$ (D) $\frac{\pi}{\omega}$

Topic: Quantum mechanics

Subtopic: Spin

Ans.: (b)

Solution: $|\psi(0)\rangle = \binom{1}{0} = \frac{1}{\sqrt{2}}(| + y\rangle + | - y\rangle)$

Time evolution:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{i\omega t}| + y\rangle + e^{-i\omega}\ | - y\rangle\right) = \begin{pmatrix} \cos\ \omega t \\ -\sin\ \omega t \end{pmatrix}$$

Target state: $| - x\rangle = \frac{1}{\sqrt{2}}\binom{1}{-1}$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{i\omega t}| + y\rangle + e^{-i}\quad | - y\rangle\right) = \begin{pmatrix} \cos\ \omega t \\ -\sin\ \omega t \end{pmatrix}$$

Target state:

$$| - x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Equating gives

$$\cos\ \omega t = \sin\ \omega t = \frac{1}{\sqrt{2}} \Rightarrow \omega t = \frac{\pi}{4}.$$

Q46. A system of three non-identical spin $\frac{1}{2}$ particles has the Hamiltonian

$$H = \frac{A}{\hbar^2}(\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_3,$$

where $\vec{S}_1, \vec{S}_2$ and $\vec{S}_3$ are the spin operators of particles labelled 1,2 and 3 respectively and A is a constant with appropriate dimensions.

The set of possible energy eigenvalues of the system is

(A) $0, \frac{A}{2}, -A$ (B) $0, \frac{A}{2}, -\frac{A}{2}$ (C) $0, \frac{3A}{2}, -\frac{A}{2}$ (D) $0, -\frac{3A}{2}, \frac{A}{2}$

Topic- Quantum mechanics

Subtopic- Identical particle

Ans.: A

Solution: $H = \frac{A}{\hbar^2}(\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_3$

$\vec{S}_{12} = \vec{S}_1 + \vec{S}_2, \qquad (\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_3 = \frac{1}{2}(\vec{S}^2 - \vec{S}_{12}^2 - \vec{S}_3^2)$

where $\vec{S} = \vec{S}_{12} + \vec{S}_3$.

Allowed quantum numbers

Each particle has spin $1/2$

$S_{12} = 0,1$

For $S_{12} = 0: S = \frac{1}{2}$

For $S_{12} = 1: S = \frac{1}{2}, \frac{3}{2}$

1. $S_{12} = 0, S = \frac{1}{2}$

$$E = \frac{A}{2}\left(\frac{3}{4} - 0 - \frac{3}{4}\right) = 0$$

2. $S_{12} = 1, S = \frac{1}{2}$

$$E = \frac{A}{2}\left(\frac{3}{4} - 2 - \frac{3}{4}\right) = -A$$

3. $S_{12} = 1, S = \frac{3}{2}$

$$E = \frac{A}{2}\left(\frac{15}{4} - 2 - \frac{3}{4}\right) = \frac{A}{2}$$

Q47. Which of the following option(s) is/are correct for a Type I superconductor?

(A) The phase transition to the normal state in the absence of a magnetic field is of second order

(B) With increase in temperature, the critical magnetic field decreases linearly to zero

(C) Below the critical temperature, the entropy in the superconducting state is less than that in the normal state

(D) The phase transition to the normal state in the presence of a magnetic field is of first order

Topic- Thermodynamics and statistical mechanics

Subtopic- Phase transition

Ans.: A,C and D

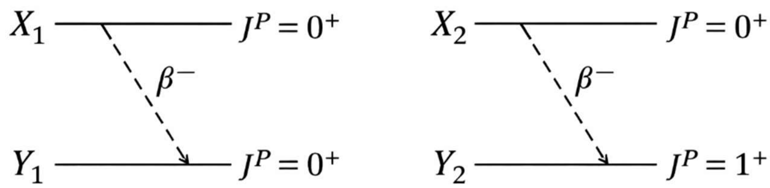
Solution: (B) False.

The thermodynamic critical field varies approximately as

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

i.e. it is not linear over the full range (only near T_c it looks roughly linear).

Q48. Consider two hypothetical nuclei X_1 and X_2 undergoing β decay, resulting in nuclei Y_1 and Y_2, respectively. The decay scheme and the corresponding J^P values of the nuclei are given in the figure. Which of the following option(s) is/are correct? (J is the total angular momentum and P is parity)



(A) $X_1 \rightarrow Y_1$ is Fermi transition and $X_2 \rightarrow Y_2$ is Fermi transition

(B) $X_1 \rightarrow Y_1$ is Fermi transition and $X_2 \rightarrow Y_2$ is Gamow-Teller transition

(C) $X_1 \rightarrow Y_1$ is Gamow-Teller transition and $X_2 \rightarrow Y_2$ is Fermi transition

(D) $X_1 \rightarrow Y_1$ is Gamow-Teller transition and $X_2 \rightarrow Y_2$ is Gamow-Teller transition

Topic- Nuclear physics

Subtopic- Beta decay

Ans.: (B)

Solution: For allowed β-decay ($L = 0$), parity does not change.

Fermi (F): $\Delta J = 0$ (allowed for $0 \rightarrow 0$)

Gamow-Teller (GT): $\Delta J = 0, \pm 1$ but $0 \rightarrow 0$ forbidden

From the figure:

1. $X_1: 0^+ \rightarrow Y_1: 0^+$

   $\Delta J = 0$, parity same $\Rightarrow$ Fermi transition

2. $X_2: 0^+ \rightarrow Y_2: 1^+$

   $\Delta J = 1$, parity same $\Rightarrow$ Gamow-Teller transition

Q49. A point charge q is placed at a distance d above an infinite, grounded conducting plate placed on the xy plane at $z = 0$. The electrostatic potential in $z > 0$ region is given by

$$\phi = \phi_1 + \phi_2$$

Where, $\phi_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{x^2+y^2+(z-d)^2}}$ and $\phi_2 = -\frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{x^2+y^2+(z+d)^2}}$.

Which of the following option (s) is/are correct?

(A) The magnitude of the force experienced by the point charge q is $\frac{1}{16\pi\epsilon_0}\frac{q}{d^2}$

(B) The electrostatic energy of the system is $\frac{1}{8\pi\epsilon_0}\frac{q^2}{d}$

(C) The induced surface charge density on the plate is proportional to $\frac{1}{\sqrt{x^2+y^2+d^2}}$

(D) The electrostatic potential ϕ_1 satisfies Poisson's equation for $z > 0$

Topic- Electromagnetic theory

Subtopic- image problem

Ans: D

Solution: (A) The magnitude of the force experienced by the point charge q is $\frac{1}{8\pi\epsilon_0}\frac{q}{d^2}$

(B) The electrostatic energy of the system is $-\frac{1}{16\pi\epsilon_0}\frac{q^2}{d}$

(C) The induced surface charge density on the plate is proportional to $\sigma(x,y) = \frac{-qd}{2\pi(x^2+y^2+d^2)^{3/2}}$.

(D) The electrostatic potential ϕ_1 satisfies Poisson's equation for $z > 0$ Correct due to presence of charge q

Q50. In coordinates (t,x), a contravariant second rank tensor A has non-zero diagonal components $A^{tt} = P$ and $A^{xx} = Q$, with all other components vanishing, and P,Q being real constants. Here, t is time and x is space coordinate. Consider a Lorentz transformation $(t,x) \rightarrow (t',x')$ to another frame that moves with relative speed v in the $+x$ direction, so that $A \rightarrow A'$. If A'^{tt} and A'^{xx} are the diagonal components of A', then setting the speed of light $c = 1$, and with

$$\gamma = \frac{1}{\sqrt{1-v^2}},$$

which of the following option(s) is/are correct?

(A) $A'^{tt} = \gamma^2 P + \gamma^2 v^2 Q$

(B) $A^{tt} = \gamma^2 v^2 P + v^2 Q$

(C) $A'^{xx} = \gamma^2 v^2 P + \gamma^2 Q$

(D) $A'^{x} = v^2 P + \gamma^2 Q$

Topic- Classical mechanics

Subtopic: Four vectors

Ans.: A and C

Solution: For a boost along $+x$,

$$\Lambda^t_t = \gamma, \Lambda^t_x = -\gamma v, \Lambda^x_t = -\gamma v, \Lambda^x_x = \gamma$$

Contravariant tensor transforms as

$$A^{\mu\nu} = \Lambda^\mu_\alpha \Lambda^\nu_\beta A^{\alpha\beta}$$

Given only $A^{tt} = P, A^{xx} = Q$ are nonzero:

Given only $A^{tt} = P, A^{xx} = Q$ are nonzero:

A'^{tt} :

$$A'^{tt} = (\Lambda^t_t)^2 P + (\Lambda^t_x)^2 Q = \gamma^2 P + \gamma^2 v^2 Q$$

matches (A)

A'^{xx} :

$$A'^{x} = (\Lambda^x_t)^2 P + (\Lambda^x_x)^2 Q = \gamma^2 v^2 P + \gamma^2 Q$$

Q51. The Lagrangian of a particle of mass m and charge q moving in a uniform magnetic field of magnitude $2B$ that points in the z direction, is given by

$$L=\frac{m}{2}v^2+qB\left(xv_y-yv_x\right)$$

where v_x, v_y, v_z are the components of its velocity $\vec{v}$.

If p_x, p_y, p_z denote the conjugate momenta in the x, y, z directions and H is the Hamiltonian, which of the following option(s) is/are correct?

(A) $\frac{dx}{dt}=\frac{1}{m}(p_x-qBy)$

(B) $\frac{dp_x}{dt}=\frac{qB}{m}\left(p_y-qBx\right)$

(C) $\frac{dp_y}{dt}=-\frac{qB}{m}(p_x+qBy)$

(D) $H=\frac{1}{2m}\left[(p_x+qBy)^2+\left(p_y-qBx\right)^2+p_z^2\right]$

Topic: Classical mechanics

Subtopic: Hamiltonian

Ans.: B,C and D

Solution: $L=\frac{m}{2}\left(v_x^2+v_y^2+v_z^2\right)+qB\left(xv_y-yv_x\right)$,

The conjugate momenta are

$$p_x=\frac{\partial L}{\partial v_x}=mv_x-qBy, p_y=\frac{\partial L}{\partial v_y}=mv_y+qBx, p_z=mv_z.$$

Hence

$$v_x=\dot{x}=\frac{p_x+qBy}{m}, v_y=\dot{y}=\frac{p_y-qBx}{m}, v_z=\dot{z}=\frac{p_z}{m}.$$

So (A) is false (wrong sign).

Hamiltonian:

$$H=\sum_i p_i v_i-L=\frac{1}{2m}\left[(p_x+qBy)^2+\left(p_y-qBx\right)^2+p_z^2\right],$$

Now Hamilton's equations:

$$\dot{p}_x=-\frac{\partial H}{\partial x}=-\frac{1}{2m}2\left(p_y-qBx\right)(-qB)=\frac{qB}{m}\left(p_y-qBx\right),$$

so (B) is correct.

$$\dot{p}_y=-\frac{\partial H}{\partial y}=-\frac{1}{2m}2(p_x+qBy)(qB)=-\frac{qB}{m}(p_x+qBy),$$

so (C) is correct.

Q52. A bead is constrained to move along a long, massless, frictionless horizontal rod parallel to the x axis. The rod itself is moving vertically upward along the z direction against gravity with a constant speed, starting from $z = 0$ at $t = 0$, and remains horizontal. The conjugate momenta are denoted by p_x, p_y, p_z and the Hamiltonian by H. Which of the following option(s) is/are correct?

(A) H is the total energy of the system and is conserved

(B) H is the total energy of the system and is not conserved

(C) H is not the total energy of the system, but it is conserved

(D) H is not the total energy of the system and is not conserved

Topic- Classical mechanics

Subtopic- Hamiltonian

Ans.: D

Solution: The Hamiltonian is $H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + mgvt$

The Hamiltonion H is explicit dependent on time so H is not total energy which is not conserve

Q53. In a one-dimensional Hamiltonian system with position q and momentum p, consider the canonical transformation $(q, p) \rightarrow \left(Q = \frac{1}{p}, P = qp^2\right)$, where Q and P are the new position and momentum, respectively. Which of the following option(s) regarding the generating function F is/are correct?

(A) $F = F_1(q, Q) = \frac{q}{Q}$

(B) $F = F_2(q, P) = \sqrt{Pq}$

(C) $F = F_3(p, Q) = 2\frac{p}{Q}$

(D) $F = F_4(p, P) = \frac{P}{p}$

Topic- classical mechanics

Subtopic- Generating function

Ans.: A and D

Solution: Given $Q = \frac{1}{p}, P = qp^2$.

For $F_1(q, Q) = q/Q$: $p = \partial F/\partial q = 1/Q \Rightarrow Q = 1/p$ and $P = -\partial F/\partial Q = q/Q^2 = qp^2$

For $F_4(p, P) = P/p$: $Q = \partial F/\partial P = 1/p$ and $q = -\partial F/\partial p = P/p^2 \Rightarrow P = qp^2$

Q54. The energy of a free, relativistic particle of rest mass m moving along the x axis in one dimension, is denoted by T. When moving in a given potential $V(x)$, its Hamiltonian is $H = T + V(x)$. In the presence of this potential, its speed is v, conjugate momentum p, and the Lagrangian L. Then, which of the following option(s) is/are correct?

(A) $H = c^2\sqrt{m^2 + \frac{p^2}{c^2}} + V(x)$ (B) $v = \frac{pc}{\sqrt{p^2+m^2c^2}}$

(C) $L = mc^2\sqrt{1-\frac{v^2}{c^2}} - V(x)$ (D) $L = -mc^2\sqrt{1-\frac{v^2}{c^2}} - V(x)$

Topic: Classical mechanics

Subtopic: Hamiltonian

Ans.: A, B and D

Solution: Relativistic energy $E = \sqrt{p^2c^2 + m^2c^4}$, so $H = E + V(x) = c^2\sqrt{m^2 + p^2/c^2} + V(x) \Rightarrow$ (A).

Using $v = \frac{\partial H}{\partial p} = \frac{pc^2}{E} = \frac{pc}{\sqrt{p^2+m^2c^2}} \Rightarrow$ (B).

Free relativistic $L_0 = -mc^2\sqrt{1 - v^2/c^2}$; with potential $L = L_0 - V \Rightarrow$ (D), not (C).

Q55. Consider the integral

$$I = \frac{1}{2\pi i}\oint \frac{z^4 - 1}{\left(z - \frac{a}{b}\right)\left(z - \frac{b}{a}\right)} dz,$$

where z is a complex variable and a, b are positive real numbers. The integral is taken over a unit circle with center at the origin. Which of the following option(s) is/are correct?

(A) $I = \frac{5}{8}$ when $a = 1, b = 2$ (B) $I = \frac{10}{3}$ when $a = 1, b = 3$

(C) $I = \frac{5}{8}$ when $a = 2, b = 1$ (D) $I = \frac{5}{8}$ when $a = 3, b = 2$

Topic- Mathematical physics

Subtopic- Contour integration

Ans.: A and C

Solution: Poles of the integrand are at $z = \frac{a}{b}$ and $z = \frac{b}{a}$.

On $|z| = 1$, only the pole with modulus < 1 contributes (simple pole).

For the pole $z = z_0$ inside the circle,

$$I = \text{Res}(f, z_0) = \frac{z_0^4 - 1}{z_0 - (\text{ other pole })}$$

Evaluating gives $I = \frac{5}{8}$ for $(a, b) = (1,2)$ and $(2,1)$; the other cases do not match.

Q56. A neutral conducting sphere of radius R is placed in a uniform electric field of magnitude E_0, that points along the z axis. The electrostatic potential at any point $\vec{r}$ outside the sphere is given by

$$V(r,\theta) = V_0 - E_0 r\left(1 - \frac{R^3}{r^3}\right)\cos\,\theta,$$

where V_0 is the constant potential of the sphere. Which of the following option(s) is/are correct?

(A) The induced surface charge density on the sphere is proportional to $\sin\,\theta$

(B) As $r \to \infty, \vec{E} = E_0 \cos\,\theta\hat{r}$

(C) The electric field at any point is curl free for $r > R$

(D) The electric field at any point is divergence free for $r > R$

Topic- Electromagnetic theory

Subtopic- Boundary value problem

Ans.: C and D

Solution: Given, $V(r,\theta) = V_0 - E_0\left(r - \frac{R^3}{r^2}\right)\cos\,\theta\ (r > R), \vec{E} = -\nabla V$.

(A) False. Induced surface charge $\sigma(\theta) = \varepsilon_0 E_r(R,\theta) \propto \cos\,\theta$, not $\sin\,\theta$.

(B) False. As $r \to \infty$, the field is uniform $\vec{E} \to E_0\hat{z}$, i.e.

$$E_r \to E_0 \cos\,\theta, E_\theta \to -E_0 \sin\,\theta,$$

so, it's not purely radial $E_0 \cos\,\theta\hat{r}$.

(C) True. Electrostatic field $\Rightarrow \nabla \times \vec{E} = 0$ for $r > R$.

(D) True. No charge in the region $r > R \Rightarrow \nabla \cdot \vec{E} = 0$ for $r > R$.

Correct options: (C) and (D)

Q57. A point charge $\boldsymbol{q}$ is placed at the origin, inside a linear dielectric medium of infinite extent, having relative permittivity ε_r. Which of the following option(s) is/are correct?

(A) The magnitude of the polarization varies as $\frac{1}{r^2}$

(B) The magnitude of the polarization varies as $\frac{1}{r^3}$

(C) The magnitude of the screened charge due to the dielectric medium is less than the magnitude of the point charge q for $\varepsilon_r > 1$

(D) The magnitude of the screened charge due to the dielectric medium is more than the magnitude of the point charge q for $\varepsilon_r = 1$

Topic- Electromagnetic theory

Subtopic- Electrostatics

Ans.: A and C

Solution: For an infinite, linear, homogeneous dielectric:

$$\mathrm{D} = \varepsilon \mathrm{E}, \varepsilon = \varepsilon_0 \varepsilon_r, \nabla \cdot \mathrm{D} = q\delta(\mathrm{r})$$

So for $r > 0$,

$$\mathrm{D} = \frac{q}{4\pi r^2}\hat{\mathrm{r}} \Rightarrow \mathrm{E} = \frac{\mathrm{D}}{\varepsilon} = \frac{q}{4\pi\varepsilon_0\varepsilon_r}\frac{1}{r^2}\hat{\mathrm{r}}$$

Polarization:

$$\mathrm{P} = \varepsilon_0(\varepsilon_r - 1)\mathrm{E} = \frac{\varepsilon_r - 1}{\varepsilon_r}\frac{q}{4\pi}\frac{1}{r^2}\hat{\mathrm{r}}$$

Hence $|\mathrm{P}| \propto 1/r^2$.

(A) true, × (B) false.

Bound (screening) charge: $\rho_b = -\nabla \cdot \mathrm{P} = 0$ for $r > 0$, but there is a delta-function bound charge at the origin. The effective screened charge seen in E is

$$q_{\text{eff}} = \frac{q}{\varepsilon_r}$$

So the "screening charge" magnitude is

$$|q_{\text{screen}}| = |q - q_{\text{eff}}| = |q|\left(1 - \frac{1}{\varepsilon_r}\right)$$

For $\varepsilon_r > 1$, this is less than $|q|$.

(C) true.

For $\varepsilon_r = 1, q_{\text{screen}} = 0$ (not more than $|q|$).

X (D) false.

Correct options: (A) and (C)

Q58. A linear magnetic material in the form of a cylinder of radius R and length L is placed with its axis parallel to the z axis. The cylinder has uniform magnetization $M\hat{k}$. Which of the following option(s) is/are correct?

(A) The magnetic field at any point outside the cylinder can be expressed as the gradient of a scalar function

(B) The bound volume current density is zero

(C) The surface current density on the curved surface is non-zero

(D) The surface current densities on the flat surfaces (top and bottom) are non-zero

Topic- Electromagnetic theory

Subtopic- Magneto state

Ans.: (A), (B) and (C)

Solution: For uniform magnetization $\mathrm{M} = M\hat{k}$:

Bound volume current:

$$\mathrm{J}_b = \nabla \times \mathrm{M} = 0$$

(B) Bound surface current:

$$\mathrm{K}_b = \mathrm{M} \times \hat{n}$$

A curved surface has $\hat{n} = \hat{r}$, so

$$\mathrm{K}_b = M\hat{z} \times \hat{r} = M\hat{\phi} \neq 0$$

(C) Top/bottom faces have $\hat{n} = \pm\hat{z}$, so

$$\mathrm{K}_b = M\hat{z} \times (\pm\hat{z}) = 0$$

(D) Outside the magnet, $\mathrm{J}_{\text{free}} = 0$ and $\mathrm{J}_b = 0$ (it exists only as surface current), so in the region outside:

$$\nabla \times \mathrm{H} = 0 \Rightarrow \mathrm{H} = -\nabla\Phi_m$$

and since outside $\mathrm{M} = 0, \mathrm{B} = \mu_0\mathrm{H}$ is also derived from a scalar potential (in simply connected regions). A is correct.

Correct options: (A), (B), (C)

Q59. Cyclotrons are used to accelerate ions like deuterons (d) and α particles. Keeping the magnetic field same for both, d and α are extracted with energies 10 MeV and 20 MeV with extraction radii r_d and r_α, respectively. Taking the masses $M_d = 2000\mathrm{MeV}/c^2$ and $M_\alpha = 4000\mathrm{MeV}/c^2$, the value of $\frac{r_\alpha}{r_d}$ (in integer) is

Topic- Electromagnetic theory

Subtopic- Charge particle in magnetic field

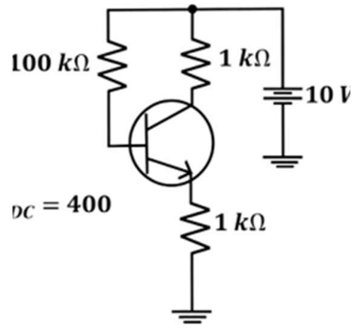
Ans.: 1

Solution: Cyclotron radius $r = \frac{p}{qB}$. For non-relativistic ions, $p = \sqrt{2mK}$, so

$$r \propto \frac{\sqrt{mK}}{q}.$$

$$\frac{r_\alpha}{r_d} = \frac{\sqrt{m_\alpha K_\alpha}/(2e)}{\sqrt{m_d K_d}/e} = \frac{\sqrt{4000\cdot 20}}{2\sqrt{2000\cdot 10}} = \frac{\sqrt{4}}{2} = 1, \frac{r_\alpha}{r_d} = 1$$

Q60. In the transistor circuit shown in the figure, $V_{BE} = 0.7$ V and $\beta_{DC} = 400$. The value of the base current in μA (rounded off to one decimal place) is ___



Topic- Electronics

Subtopic- Transistor

Ans.: 18 to 19

Solution: Given:

$V_{BE} = 0.7$ V

$\beta = 400$

Base resistor $R_B = 100\text{k}\Omega$

Emitter resistor $R_E = 1\text{k}\Omega$

Supply $V_{CC} = 10$ V

KVL from supply to ground (base-emitter loop)

$$10 = I_B(100000) + 0.7 + I_E(1000)$$

With, $I_E = (\beta + 1)I_B = 401I_B$

Substitute: $10 = 100000I_B + 0.7 + 401000I_B$

$10 - 0.7 = 501000I_B$, $9.3 = 501000I_B$, $I_B = 1.856 \times 10^{-5}$ A

Base current

$$I_B \approx 18.6\mu \text{ A}$$

(rounded to one decimal place)

Q61. Consider the set $\{1, x, x^2\}$. An orthonormal basis in $x \in [-1,1]$ is formed from these three terms, where the normalization of a function $f(x)$ is defined via $\int_{-1}^{1} x^2[f(x)]^2 dx = 1$. If the orthonormal basis set is $\left(\sqrt{\frac{3}{2}}, \sqrt{\frac{5}{2}}x, \frac{1}{2}\sqrt{\frac{21}{N}}(5x^2 - 3)\right)$, then the value of N (in integer) is ___

Topic- Mathematical physics

Subtopic- Polynomial

Ans.: 6

Solution: Weight $w(x) = x^2$, third basis $f(x) = \frac{1}{2}\sqrt{\frac{21}{N}}(5x^2 - 3)$.

Normalize: $1 = \int_{-1}^{1} x^2 f^2 dx = \frac{1}{4}\frac{21}{N}\int_{-1}^{1} x^2(5x^2-3)^2 dx$.

$$\int_{-1}^{1} x^2(5x^2-3)^2 dx = \int_{-1}^{1}(25x^6 - 30x^4 + 9x^2)dx = \frac{8}{7}$$

So $1 = \frac{1}{4}\frac{21}{N}\cdot\frac{8}{7} = \frac{6}{N} \Rightarrow N = 6$.

Q62. The Hamiltonian for a one dimensional system with mass m, position q and momentum p is $H(p,q) = \frac{p^2}{2m} + q^2 A(q)$, where $A(q)$ is a real function of q. If $m\frac{d^2q}{dt^2} = -5qA(q)$, then $\frac{dA(q)}{dq} = n\frac{A(q)}{q}$. The value of n (in integer) is ___

Topic-Classical mechanics

Subtopic- Hamiltonian

Ans.: 3

Solution: From Hamilton's equations:

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m} \Rightarrow p = m\dot{q}, \dot{p} = -\frac{\partial H}{\partial q} = -(2qA + q^2A')$$

But $\dot{p} = m\ddot{q}$. Given $m\ddot{q} = -5qA$, so

$$-(2qA + q^2A') = -5qA \Rightarrow 2A + qA' = 5A \Rightarrow qA' = 3A$$

$$\therefore \frac{dA}{dq} = \frac{3A}{q} \Rightarrow n = 3.$$

Q63. A system of five identical, non-interacting particles with mass m and spin $\frac{3}{2}$ is confined to a one-dimensional potential well of length L. If the lowest energy of the system is $N\frac{\pi^2\hbar^2}{2mL^2}$, the value of N (in integer) is ___

Topic- Quantum mechanics
Subtopic- identical particle

Ans.: 8

Solution: For a 1D infinite well, single-particle levels are

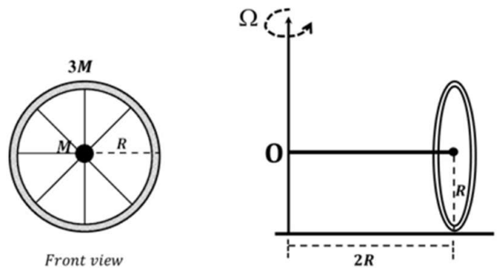
$$E_n = n^2\frac{\pi^2\hbar^2}{2mL^2} \equiv n^2 E_0$$

Spin $3/2 \Rightarrow (2s+1) = 4$ spin states, and since spin is half-integer the particles are fermions, so max **4** particles can occupy the same n.

Fill lowest levels for 5 particles: 4 in $n = 1$,1 in $n = 2$.

$$E_{\min} = 4(1^2E_0) + 1(2^2E_0) = (4+4)E_0 = 8E_0$$

Q64. A wheel of mass $4M$ and radius R is made of a thin uniform distribution of mass $3M$ at the rim and a point mass M at the center. The spokes of the wheel are massless. The center of mass of the wheel is connected to a horizontal massless rod of length $2R$, with one end fixed at O , as shown in the figure. The wheel rolls without slipping on horizontal ground with angular speed Ω. If $\vec{L}$ is the total angular momentum of the wheel about O, then the magnitude $\left|\frac{d\vec{L}}{dt}\right| = N(MR^2\Omega^2)$. The value of N (in integer) is



Topic- Classical mechanics

Subtopic- Rotational dynamics

Ans.: 6

Solution: Rolling constraint

Centre speed: $v_C = (2R)\Omega$ (radius $OC = 2R$)

Rolling: $\omega_{\text{spin}} = \frac{v_C}{R} = 2\Omega$.

Spin angular momentum about CM

Only rim mass $3M$ contributes: $I_{\text{CM}} = 3MR^2$.

So $L_s = I_{\text{CM}}\omega_{\text{spin}} = 3MR^2(2\Omega) = 6MR^2\Omega$.

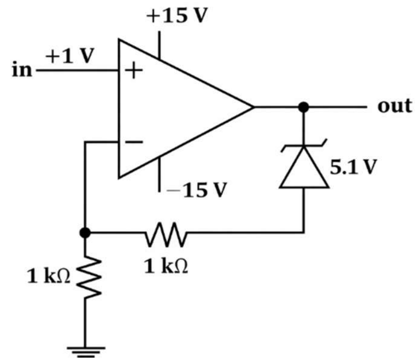
Use $\frac{d\mathbf{L}}{dt} = \Omega \times \mathrm{L}$

$\mathbf{L}_s$ is along the horizontal axle, and $\boldsymbol{\Omega}$ is vertical $\Rightarrow$ angle 90°.

$$\left|\frac{d\mathbf{L}}{dt}\right| = |\boldsymbol{\Omega} \times \mathbf{L}_s| = \Omega L_s \sin 90^\circ = \Omega(6MR^2\Omega) = 6MR^2\Omega^2.$$

Thus $|d\mathbf{L}/dt| = N(MR^2\Omega^2) \Rightarrow N = 6$.

Q65. The figure shows an op-amp circuit with a 5.1 V Zener diode in the feedback loop. The op-amp runs from ± 15 V supplies. If a +1 V signal is applied at the input, the output voltage (rounded off to one decimal place) is__________



Topic-electronics

Subtopic- operational amplifier

Ans.: 7.0 to 7.2

Solution: When +1 V is applied at the non-inverting input, the op-amp output initially rises. Negative feedback is established only after the 5.1 V Zener conducts.

In steady state, $V_{-} = V_{+} = 1 \text{ V}$

The inverting node is connected to ground through two equal resistors $1\text{k}\Omega - 1\text{k}\Omega$, hence:

$$V_{-} = \frac{V_x}{2} \Rightarrow V_x = 2 \text{ V}$$

With the Zener in breakdown, $V_{\text{out}} - V_x = 5.1 \text{ V}$

$$V_{\text{out}} = 5.1 + 2.0 = 7.1 \text{ V}$$

Output voltage = 7.1 V