IIT-JAM 2019

SECTION - A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q1. – Q10. carry one mark each.

Q1. The function $f(x)=\dfrac{8x}{x^2+9}$ is continuous everywhere except at

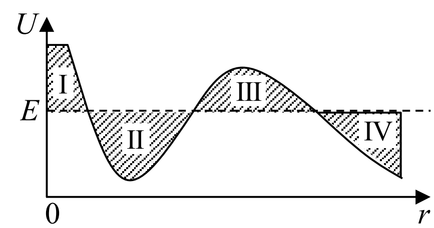
(a) $x=0$ (b) $x=\pm 9$ (c) $x=\pm 9i$ (d) $x=\pm 3i$

Ans. : (d)

Solution: We know that a rational function is discontinuous at a point where the denominator is 0.

Therefore, $x^2+9=0 \Rightarrow x=\pm 3i$

Q2. A classical particle has total energy E. The plot of potential energy (U) as a function of distance (r) from the centre of force located at $r=0$ is shown in the figure. Which of the regions are forbidden for the particle?



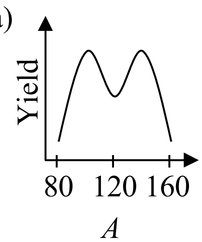
(a) I and II (b) II and IV

(c) I an IV (d) I and III

Ans. : (d)

Solution: In the region I and III potential energy is more than total energy .

Q3. In the thermal neutron induced fission of ^{235}U, the distribution of relative number of the observed fission fragments (Yield) versus mass number (A) is given by

(a)


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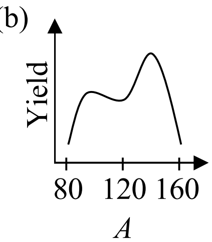

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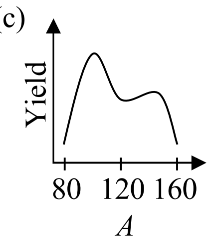
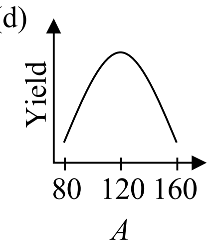
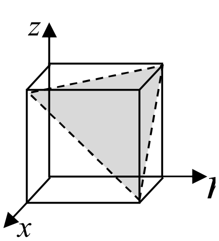
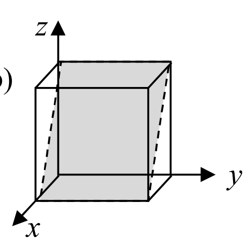


(d)


Ans. : (a)

Q4. Which one of the following crystallographic planes represent (101) Miller indices of a cubic unit cell?

(a)


(b)


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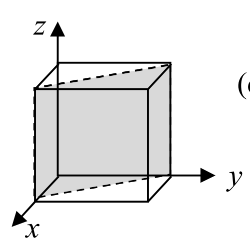
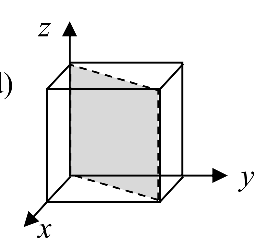


(d)


Ans. : (b)

Solution: Plane intercepts

$$x:y:z=\frac{a}{h}:\frac{b}{k}:\frac{c}{l}=\frac{a}{1}:\frac{b}{0}:\frac{c}{1}=a:\infty:c$$

$$\therefore\ x=a,\ y=\infty,\ z=c$$

Plane is parallel to y - axis and intersecting x and z - axis at a and c. Thus, option (b) is correct.

Q5. The Fermi-Dirac distribution function $\left[n(\varepsilon)\right]$ is

(k_B is the Boltzmann constant, T is the temperature and ε_F is the Fermi energy)

(a) $n(\varepsilon)=\dfrac{1}{e^{\frac{\varepsilon-\varepsilon_F}{k_BT}}-1}$ (b) $n(\varepsilon)=\dfrac{1}{e^{\frac{\varepsilon_F-\varepsilon}{k_BT}}-1}$

(c) $n(\varepsilon)=\dfrac{1}{e^{\frac{\varepsilon-\varepsilon_F}{k_BT}}+1}$ (d) $n(\varepsilon)=\dfrac{1}{e^{\frac{\varepsilon_F-\varepsilon}{k_BT}}+1}$

Ans. : (c)

Q6. If $\phi(x,y,z)$ is a scalar function which satisfies the Laplace equation, then the gradient of ϕ is

(a) Solenoidall and irrigational (b) Solenoidall but not irrotational

(c) Irrotational but not solenoid (d) Neither Solenoidall nor irrotational

Ans. : (a)

Solution: $\nabla^2\phi=0\Rightarrow\rho=0\Rightarrow\vec{E}=\vec{\nabla}\phi=0,\vec{\nabla}\cdot\vec{E}=0,\vec{\nabla}\times\vec{E}=0$

Q7. In a heat engine based on the Carnot cycle, heat is added to the working substance at constant

(a) Entropy (b) Pressure

(c) Temperature (d) Volume

Ans. : (c)

Q8. Isothermal compressibility is given by

(a) $\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ (b) $\frac{1}{P}\left(\frac{\partial P}{\partial V}\right)_T$ (c) $-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ (d) $-\frac{1}{P}\left(\frac{\partial P}{\partial V}\right)_T$

Ans. : (c)

Q9. For using a transistor as an amplifier, choose the correct option regarding the resistances of base-emitter (R_{BE}) and base-collector (R_{BC}) junctions

(a) Both R_{BE} and R_{BC} are very low (b) Very low R_{BE} and very high R_{BC}

(c) Very high R_{BE} and very low R_{BC} (d) Both R_{BE} and R_{BC} are very high

Ans. : (b)

Q10. A unit vector perpendicular to the plane containing $\vec{A}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}+\hat{k}$ is

(a) $\frac{1}{\sqrt{26}}\left(-\hat{i}+3\hat{j}-4\hat{k}\right)$

(b) $\frac{1}{\sqrt{19}}\left(-\hat{i}+3\hat{j}-3\hat{k}\right)$

(c) $\frac{1}{\sqrt{35}}\left(-\hat{i}+5\hat{j}-3\hat{k}\right)$

(d) $\frac{1}{\sqrt{35}}\left(-\hat{i}-5\hat{j}-3\hat{k}\right)$

Ans. : (d)

Solution: $\vec{A}\cdot\hat{n}=0$ and $\vec{B}\cdot\hat{n}=0$

Verify option (d): $\vec{A}\cdot\hat{n}=\frac{1}{\sqrt{35}}(-1-5-6)=0$

$$\vec{B}\cdot\hat{n}=\frac{1}{\sqrt{35}}(-2+5-3)=0$$

Q11. – Q30. carry two marks each.

Q11. A thin lens of refractive index $\frac{3}{2}$ is kept inside a liquid of refractive index $\frac{4}{3}$. If the focal length of the lens in air is 10 cm, then the focal length inside the liquid is

(a) 10 cm (b) 30cm (c) 40cm (d) 50cm

Ans. : (c)

Solution: $\frac{1}{f_a}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{f_l}=\left(\frac{3/2}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{f_l}{f_a}=\frac{\left(\frac{3}{2}-1\right)}{\left(\frac{9}{8}-1\right)}=4$$

$$f_l=4f_a=4\times10=40\,cm$$

Q12. The eigenvalues of $\begin{pmatrix}3 & i & 0\\ -i & 3 & 0\\ 0 & 0 & 6\end{pmatrix}$ are

(a) $2,4$ and 6 (b) $2i,4i$ and 6 (c) $2i,4$ and 8 (d) $0,4$ and 8

Ans. : (a)

Solution: For calculation of eigenvalues

$$\begin{vmatrix}3-\lambda & i & 0\\ -i & 3-\lambda & 0\\ 0 & 0 & 6-\lambda\end{vmatrix}=0$$

$$\Rightarrow(3-\lambda)\left[(3-\lambda)(6-\lambda)\right]-i\left[-i(6-\lambda)\right]=0$$

$$\Rightarrow(3-\lambda)(3-\lambda)(6-\lambda)-(6-\lambda)=0$$

or $(6-\lambda)\left[(\lambda-3)^2-1\right]=0$

or $(6-\lambda)\left[\left(\lambda^2-6\lambda+8\right)\right]=0$

or $(6-\lambda)(\lambda-2)(\lambda-4)=0$. Therefore, $\lambda=6$ or 2 or 4.

Q13. For a quantum particle confined inside a cubic box of side L, the ground state energy is given by E_0. The energy of the first excited state is

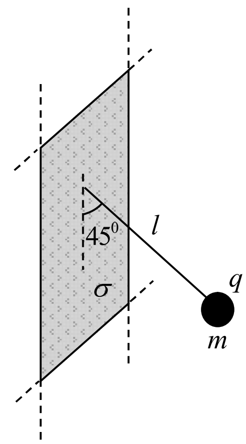
(a) $2E_0$ (b) $\sqrt{2}E_0$ (c) $3E_0$ (d) $6E_0$

Ans. : (d)

Solution: $E_{n_x,n_y,n_z} = \dfrac{\left(n_x^2+n_y^2+n_z^2\right)\pi^2\hbar^2}{2ma^2} = \left(n_x^2+n_y^2+n_z^2\right)E_0$

$$E_{2,1,1} = E_{1,2,1} = E_{1,1,2} = \frac{(4+1+1)\pi^2\hbar^2}{2ma^2} = 6E_0$$

Q14. A small spherical ball having charge q and mass m, is tied to a thin massless non-conducting string of length l. The other end of the string is fixed to an infinitely extended thin non-conducting sheet with uniform surface charge density σ. Under equilibrium the string makes an angle 45° with the sheet as shown in the figure. Then σ is given by (g is the acceleration due to gravity and ε_0 is the permittivity of free space)



(a) $\dfrac{mg\varepsilon_0}{q}$

(b) $\sqrt{2}\dfrac{mg\varepsilon_0}{q}$

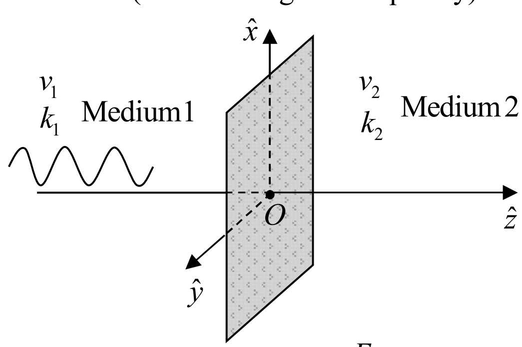
(c) $2\dfrac{mg\varepsilon_0}{q}$

(d) $\dfrac{mg\varepsilon_0}{q\sqrt{2}}$

Ans. : (c)

Solution: $\tan\theta = \dfrac{F}{mg} \Rightarrow \tan\theta = \dfrac{qE}{mg} = \dfrac{q\sigma}{2\varepsilon_0 mg} \Rightarrow \sigma = \dfrac{2mg\varepsilon_0}{q}\tan\theta \Rightarrow \sigma = \dfrac{2mg\varepsilon_0}{q}\tan 45^0 = \dfrac{2mg\varepsilon_0}{q}$

Q15. Consider the normal incidence of a plane electromagnetic wave with electric field given by $\vec{E} = E_0 \exp[k_1 z - \omega t]\hat{x}$ over an interface at $z=0$ separating two media [wave velocities v_1 and v_2 $(v_2 > v_1)$ and wave vectors k_1 and k_2, respectively] as shown in figure. The magnetic field vector of the reflected wave is (ω is the angular frequency)



(a) $\dfrac{E_0}{v_1}\exp\left[i\left(k_1 z - \omega t\right)\right]\hat{y}$

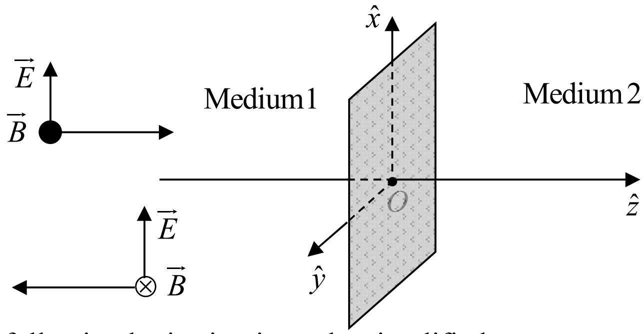
(b) $\dfrac{E_0}{v_1}\exp\left[i\left(-k_1 z - \omega t\right)\right]\hat{y}$

(c) $\dfrac{-E_0}{v_1}\exp\left[i\left(-k_1 z - \omega t\right)\right]\hat{y}$

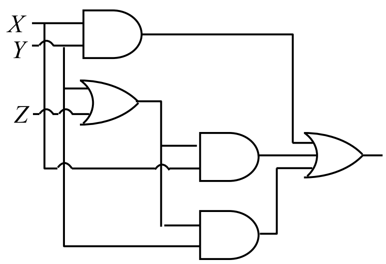
(d) $\dfrac{-E_0}{v_1}\exp\left[i\left(k_1 z - \omega t\right)\right]\hat{y}$

Ans. : (c)

Solution:



Q16. The output of following logic circuit can be simplified to



(a) $X+YZ$ (b) $Y+XZ$ (c) XYZ (d) $X+Y+Z$

Ans. : (b)

Solution: Output $=XY+X(Y+Z)+Y(Y+Z)=XY+XY+XZ+Y+YZ$

$$=XY+XZ+Y=Y(1+X)+XZ=Y+XZ$$

Q17. A red star having radius r_R at a temperature T_R and a white star having radius r_w at a temperature T_w, radiate the same total power. If these stars radiate as perfect black bodies, then

(a) $r_R > r_w$ and $T_R > T_w$ (b) $r_R < r_w$ and $T_R > T_w$

(c) $r_R > r_w$ and $T_R < T_w$ (d) $r_R < r_w$ and $T_R < T_w$

Ans. : (c)

Solution: $E=\sigma AT^4\,(\in=1) \Rightarrow \sigma\times 4\pi r_W^2 T_W^4=\sigma\times 4\pi\times r_R^2\times T_R^4$ as $r_W < r_R$

$$T_W=T_R\times\left(\frac{r_R}{r_W}\right)^2 \quad T_W > T_R$$

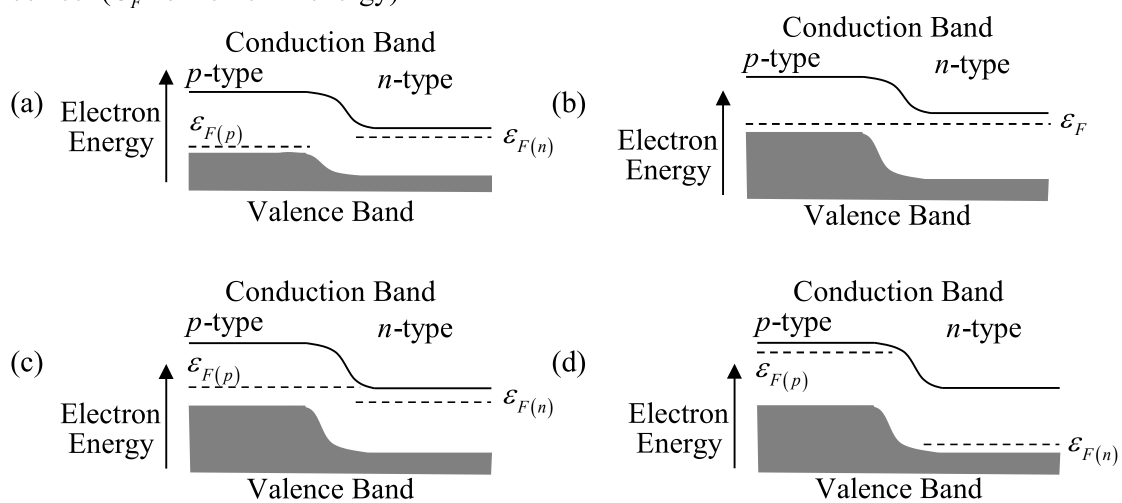
Q18. The mass per unit length of a rod (length 2m) varies as $\rho=3x$ kg/m. The moment of inertia (in kg m^2) of the rod about a perpendicular-axis passing through the tip of the rod (at $x=0$)

(a) 10 (b) 12 (c) 14 (d) 16

Ans. : (b)

Solution: $I=\int_0^l x^2\rho dx=\int_0^2 x^2 3x dx=\left.\frac{3x^4}{4}\right|_0^2=12$

Q19. For a forward biased p-n junction diode, which one of the following energy-band diagrams is correct (ε_F is the Fermi energy)



Ans. : (a)

Q20. The amount of work done to increases the speed of an electron nom $c/3$ to $2c/3$ is ($c = 3\times10^8$ m/s and rest mass of electron is 0..511 MeV)

(a) 56.50 keV (b) 143.58 keV (c) 168.20 keV (d) 511.00 keV

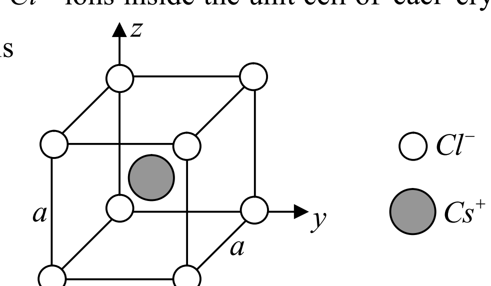
Ans. : (b)

Solution: Change in kinetic energy is equal to work done

$$W = \left(\frac{m_0c^2}{\sqrt{1-\frac{v_2^2}{c^2}}} - m_0c^2\right) - \left(\frac{m_0c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} - m_0c^2\right) = \frac{m_0c^2}{\sqrt{1-\frac{v_2^2}{c^2}}} - \frac{m_0c^2}{\sqrt{1-\frac{v_1^2}{c^2}}}$$

put $v_1 = c/3, v_2 = 2c/3 \quad m_0c^2 = 0.511, \qquad W = 143.58$ keV

Q21. The location of Cs^+ and Cl^- ions inside the unit cell of cacl crystal is shown in the figure. The Bravais lattice of $CaCl$ is



(a) Simple cubic (b) Body centred orthorhomble

(c) Face centred cubic (d) Base centred orthorhombic

Ans. : (a)

Solution: Cesium-Chloride is made of two interpenetrating simple cubic lattices are displaced diagonally by half of the diagonal length. Thus, Bravais lattice of $CsCl$ is simple cubic. The correct option is (a).

Q22. A γ -ray photon emitted from a ^{137}Cs source collides with an electron at rest. If the Compton shift of the photon is $3.25\times10^{-13}\text{ m}$, then the scattering angle is closets to (Planck's constant $h=6.626\times10^{-34}\text{ Js}$, electron mass $m_\theta=9.109\times10^{-31}\text{ kg}$ and velocity of light in free space $c=3\times10^{8}\text{ m/s}$)

(a) 45^{o} (b) 60^{o} (c) 30^{o} (d) 90^{o}

Ans. : (c)

Solution: $\Delta\lambda=\dfrac{h}{m_ec}(1-\cos\theta)\Rightarrow\cos\theta=1-\dfrac{\Delta\lambda.m_ec}{h}$

$$=1-\frac{3.25\times10^{-13}\times9.109\times10^{-31}\times3\times10^{8}}{6.6\times10^{-34}}=0.866=\frac{\sqrt{3}}{2}$$

$\theta=30^{0}$

Q23. During free expansion of an ideal gas under adiabatic condition, the internal energy of the gas.

(a) Decreases (b) Initially decreases and then increases

(c) Increases (d) Remains constant

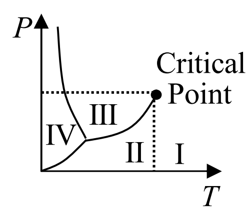
Ans. : (d)

Solution: As $W=\Delta U+Q$

$Q=0\Rightarrow W=\Delta U$

Work is done at the expense of internal energy.

Q24. In the given phase diagram for a pure substance regions I, II, III, IV, respectively represent



(a) Vapour, Gas, Solid, Liquid (b) Gas, Vapour, Liquid, solid

(c) Gas, Liquid, Vapour, solid (d) Vapour, Gas, Liquid, Solid

Ans. : (b)

Solution: IV – Solid

III – Liquid

II – Vapour

I – Gas (superheated dry vapour)

Q25. Light of wavelength λ (in free space) propagates through a dispersive medium with refractive index $n(\lambda)=1.5+0.6\lambda$. The group velocity of a wave travelling inside this medium in units of 10^{8} m/s is

(a) 1.5 (b) 2.0 (c) 3.0 (d) 4.0

Ans. : (b)

Solution: $v_g = \frac{d\omega}{dk} = \frac{d\omega}{d\lambda}\frac{d\lambda}{dk} \quad \because k = \frac{2\pi}{\lambda}$

$$= -\frac{\lambda^2}{2\pi}\frac{d\omega}{d\lambda} \qquad \frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2}$$

$$= -\frac{\lambda^2}{2\pi}\frac{d}{d\lambda}\left(\frac{c\,2\pi}{n\lambda}\right) \qquad \because\ n = \frac{c}{v_p} = \frac{ck}{\omega} = -c\lambda^2\frac{d}{d\lambda}\left(\frac{1}{n\lambda}\right)$$

$$= -c\lambda^2\frac{(n\lambda)\cdot 0 - 1\cdot\frac{d}{d\lambda}(n\lambda)}{n^2\lambda^2} \qquad = -c\lambda^2\frac{-\left[1\cdot n + \lambda\frac{d}{d\lambda}n\right]}{n^2\lambda^2}$$

$$= c\frac{n+\lambda(0.6)}{n^2} = c\frac{1.5+1.2\lambda}{(1.5+2.6\lambda)^2} \simeq \frac{c}{1.5} \qquad \because\ \lambda \sim 10^{-7}\,m \qquad \simeq \frac{2}{3}c \simeq 2\times 10^8$$

Q26. The maximum number of intensity minima that can be observed I the Fraunhofer diffraction pattern of a single slit (width $10\,\mu m$) illuminated by a laser bean (wavelength $0.630\,\mu m$) will be

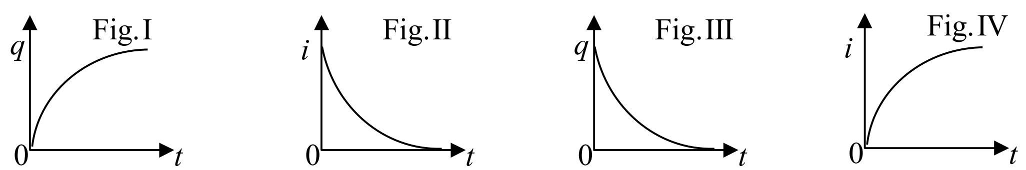
(a) 4 (b) 7 (c) 12 (d) 15

Ans. : (d)

Solution: $e\sin\theta = n\lambda$

$$n_{\max} = \frac{e}{\lambda} = \frac{10\,\mu m}{0.63\,\mu m} = 15.87 \approx 15$$

Q27. During the charging of a capacitor C in a series RC circuit, the typical variations in the magnitude of the charge $q(t)$ deposited on one of the capacitor plates, and the current $i(t)$ in the circuit, respectively are best represented by



(a) Figure I and figure II
(b) Figure I and Figure IV
(c) Figure III and figure II
(d) Figure III and figure IV

Ans. : (a)

Q28. Which one of the following is an impossible magnetic field $\vec{B}$?

(a) $\vec{B} = 3x^2z^2\hat{x} - 2xz^3\hat{z}$

(b) $\vec{B} = -2xy\hat{x} + yz^2\hat{y} + \left(2yz - \frac{z^3}{3}\right)\hat{z}$

(c) $\vec{B} = (xz+4y)\hat{x} - yx^3\hat{y} + \left(x^3z - \frac{z^2}{2}\right)\hat{z}$

(d) $\vec{B} = -6xz\hat{x} + 3yz^2\hat{y}$

Ans. : (d)

Solution: Check that $\vec{\nabla}\cdot\vec{B}\neq 0$

(a) $\vec{\nabla}\cdot\vec{B}=6xz^2-6xz^2=0$

(b) $\vec{\nabla}\cdot\vec{B}=-2y+z^2+\left(2y-z^2\right)=0$

(c) $\vec{\nabla}\cdot\vec{B}=z-x^3+\left(x^3-z\right)=0$

(d) $\vec{\nabla}\cdot\vec{B}=-6z+3z^2\neq 0$

Q29. If the motion of a particle is described by $x=5\cos\left(8\pi t\right), y=5\sin\left(8\pi t\right)$ and $z=5t$, then the trajectory of the particle is

(a) Circular (b) Elliptical (c) Helical (d) Spiral

Ans. : (c)

Solution: $x=5\cos\left(8\pi t\right), y=5\sin\left(8\pi t\right)$ and $z=5t$, $\Rightarrow x^2+y^2=5^2, z=5t$ motion is Helical

Q30. A ball of mass m is falling freely under gravity through a viscous medium in which the drag force is proportional to the instantaneous velocity v of the ball. Neglecting the buoyancy force of the medium, which one of the following figures best describes the variation of v as a function of time t?

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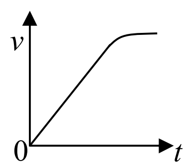

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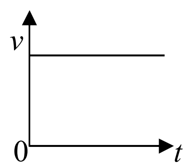

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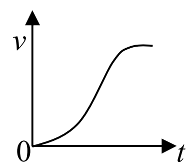

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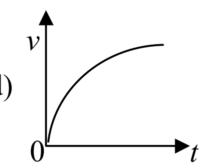


Ans. : (d)

Solution: $F\propto V$

$ma=KV$

$\frac{mdV}{dt}=KV\Rightarrow V\propto t^2$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q31. – Q40. carry two marks each.

Q31. The relation between the nuclear radius (R) and the mass number (A), given by $R = 1.2\, A^{1/3}$ fm, implies that

(a) The central density of nuclei is independent of A

(b) The volume energy per nucleon is a constant

(c) The attractive part of the nuclear force has a long range

(d) The nuclear force is charge dependent

Ans. : (a), (b), (d)

Q32. Consider an object moving with a velocity $\vec{v}$ in a frame which rotates with a constant angular velocity $\vec{\omega}$. The Coriolis force experienced by the object is

(a) Along $\vec{v}$

(b) Along $\vec{\omega}$

(c) Perpendicular to both $\vec{v}$ and $\vec{\omega}$

(d) always directed towards the axis of rotation

Ans. : (c)

Solution: $F_c = -2m(\vec{\omega} \times \vec{v})$

Q33. The gradient of scalar field $S(x, y, z)$ has the following characteristic(s)

(a) Line integral of a gradient is path-independent

(b) Closed line integral of a gradient is zero

(c) Gradient of S is a measure of the maximum rate of change in the field S

(d) Gradient of S is a scalar quantity

Ans.: (a), (b), (c)

Q34. A thermodynamic system is described by the P, V, T coordinates. Choose the valid expression(s) for the system.

(a) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial P}{\partial T}\right)_V$

(b) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial P}{\partial T}\right)_V$

(c) $\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -\left(\frac{\partial V}{\partial P}\right)_T$

(d) $\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = \left(\frac{\partial V}{\partial P}\right)_T$

Ans. : (a), (c)

Q35. Which of the following statement(s) is/are true?

(a) Newton's laws of motion and Maxwell's equations are both invariant under Lorentz transformations

(b) Newton's laws of motion and Maxwell's equations are both invariant under Galilean transformations

(c) Newton's laws of motion are invariant under Galilean transformations and Maxwell's equations are invariant under Lorentz transformations

(d) Newton's laws of motion are invariant under Lorenz transformations and Maxwell's equations are invariant under Galilean transformations

Ans. : (c)

Q36. For an under damped harmonic oscillator with velocity $v(t)$

(a) Rate of energy dissipation varies linearly with $v(t)$

(b) Rate of energy dissipation varies as square of $v(t)$

(c) The reduction in the oscillator frequency, compared to the undamped case, is independent of $v(t)$

(d) For weak damping, the amplitude decays exponentially to zero

Ans. : (b), (c), (d)

Solution: Displacement $x = Ae^{-rt}\sin(\omega t+\phi)$

Velocity $$v=\frac{dx}{dt}\cong A\omega e^{-rt}\cos(\omega t+\phi)$$

Energy $$E=\frac{1}{2}mv^2+\frac{1}{2}m\omega^2x^2=\frac{1}{2},\omega^2A^2e^{-2rt}$$

Power dissipation, $$P=\frac{dE}{dt}=\frac{1}{2}m\omega^2A^2e^{-2rt}(-2r)$$

$$P\propto v^2$$

Power dissipation is proportional to v^2, thus option (a) is wrong and option (b) is correct.

Also, displacement $x = Ae^{-rt}\sin(\omega t+\phi)$ decays exponentially to zero, thus option (d) is also correct.

The damped oscillation frequency is

$$\omega=\sqrt{\omega_0^2-r^2}$$

It is independent of $v(t)$. Thus option (c) is also correct.

Q37. Out of the following statements, choose the correct option(s) about a perfect conductor.

(a) The conductor has an equipotential surface

(b) Net charge, if any, resides only on the surface of conductor

(c) Electric field cannot exist inside the conductor

(d) Just outside the conductor, the electric field is always perpendicular to its surface

Ans.: (a), (b), (c), (d)

Q38. In the X-ray diffraction pattern recorded for a simple cubic solid (lattice) parameter $a = 1\mathring{A}$) using X-rays of wavelength $1\mathring{A}$, the first order diffraction peak(s) would appear for the

(a) (100) planes (b) (112) planes (c) (210) planes (d) (220) planes

Ans. : (a)

Solution: In simple cubic cell, planes are present. The first order diffraction peak would appear for the first plane (100).

Q39. Consider a classical particle subjected to an attractive inverse-square force field. The total energy of the particle is E and the eccentricity is ε. The particle will follow a parabolic orbit if

(a) $E > 0$ and $\varepsilon = 1$ (b) $E < 0$ and $\varepsilon < 1$

(c) $E = 0$ and $\varepsilon = 1$ (d) $E < 0$ and $\varepsilon = 1$

Ans. : (c)

Solution: $\varepsilon = \sqrt{1 + \frac{2EJ^2}{mk^2}}$ for parabolic orbit $E = 0$ and $\varepsilon = 1$

Q40. An atomic nucleus X with half-life T_X decays to a nucleus Y, which has half-life T_Y. The condition (s) for secular equilibrium is (are)

(a) $T_X \simeq T_Y$ (b) $T_X < T_Y$ (c) $T_X \ll T_Y$ (d) $T_x \gg T_Y$

Ans. : (d)

SECTION - C

NUMERICAL ANSWER TYPE (NAT)

Q41. – Q50. carry one mark each.

Q41. In a typical human body, the amount of radioactive ^{40}K is 3.24×10^{-5} percent of its mass. The activity due to ^{40}K in a human body of mass $70\,\text{kg}$ is ______ kBq.

(Round off to 2 decimal places)

(Half-life of $^{40}K = 3.942\times10^{16}\,\text{S}$, Avogadro's number $N_A = 6.022\times10^{23}\,\text{mol}^{-1}$

Ans. : 6.0

Solution: $\left|\frac{dN}{dt}\right| = \lambda N$

$$= \frac{0.693}{3.942\times10^{6}(s)}\times\frac{\left(70\times10^{3}\right)}{40}\times\frac{3.24\times10^{-5}}{100}\times6.022\times10$$

$= 6.0\times10^{13}$ disintegrations /s

$= 6.0\times10^{13}$ Bq $= 6.0\times10^{10}$ kBq

Q42. Sodium (Na) exhibits body-centred cubic (BCC) crystal structure with atomic radius 0.186 nm.

The lattice parameter of Na unit cell is______ nm.

Ans. : 0.43

Solution: For BCC, $\sqrt{3}a = 4r$

$$a = \frac{4r}{\sqrt{3}} = \frac{4\times0.186}{\sqrt{3}} \Rightarrow a = 0.43\,nm$$

Q43. Light of wavelength 680 nm is incident normally on a diffraction grating having 4000 lines/cm.

The diffraction angle (in degrees) corresponding to the third-order maximum is ______

(Round off to 2 decimal places)

Ans. : 55^0

Solution: $(e+d)\sin\theta = n\lambda$

$$\frac{10^{-2}}{4000}\times\sin\theta = 3\times680\times10^{-9}$$

$$\theta = \sin^{-1}(0.82)\approx55^0$$

Q44. Two gases having molecular diameters D_1 and D_2 and mean free paths λ_1 and λ_2, respectively, are trapped separately in identical containers. If $D_2 = 2D_1$, then $\frac{\lambda_1}{\lambda_2} =$_______.

(Assume there is no change in other thermodynamic parameters)

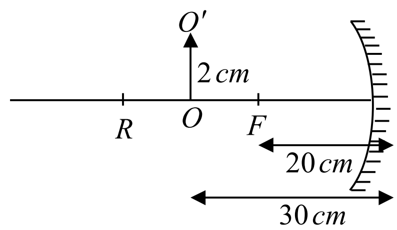
Ans. : 4

Solution: $x \propto \frac{1}{d^2} \Rightarrow \frac{x_1}{x_2} = \left(\frac{d_2}{d_1}\right)^2 = 4$

Q45. An object of $2\,\text{cm}$ height is placed at a distance of $30\,\text{cm}$ in front of a concave mirror with radius of curvature $40\,\text{cm}$. The height of the image is ________cm.

Ans. : 4

Solution: $u = -30\,cm$



$f = -20\,cm$

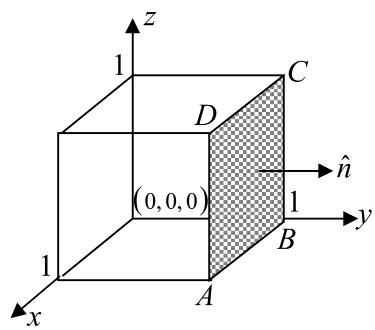
$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-20} - \frac{1}{-30}$

$\frac{1}{v} = -\frac{1}{60} \Rightarrow v = -60\,cm$

$m = \frac{I}{O} = -\frac{v}{u},\ I = -\frac{(-60)}{(-30)} \times 2\,cm = -4\,cm$

Q46. The flux of the function $\vec{F} = (y^2)\hat{x} + (3xy - z^2)\hat{y} + (4yz)\hat{z}$ passing through the surface $ABCD$ along $\hat{n}$ is_________

(Round off to 2 decimal places)



Ans. : 1.17

Solution: $y = 1$ plane

$$\int_S \vec{F} \cdot d\vec{a} = \iint \vec{F} \cdot (dxdz\hat{y}) = \iint (3xy - z^2)\,dxdz$$

$$= \int_0^1 \int_0^1 (3x - z^2)\,dxdz = \int_0^1 \left[3xz - \frac{z^3}{3}\right]_{z=0}^1 dx = \int_0^1 \left[3z - \frac{1}{3}\right] dx = \left[3\frac{x^2}{2} - \frac{x}{3}\right]_0^1 = \frac{3}{2} - \frac{1}{3} = \frac{9-2}{6} = \frac{7}{6} = 1.17$$

Q47. The electrostatic energy (in units of $\frac{1}{4\pi\varepsilon_0} J$) of a uniformly charged spherical shell of total charge $5\,C$ and radius $4\,m$ is_______. (Round off to 3 decimal places)

Ans.: 3.125

Solution: $W = \frac{q^2}{8\pi\varepsilon_0 R} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2R},\ W = \frac{1}{4\pi\varepsilon_0}\frac{25}{2 \times 4} Joules = \left(\frac{1}{4\pi\varepsilon_0} \times 3.125\right) Joules$

Q48. An infinitely long very thin straight wire carries uniform line charge density $8\pi\times10^{-2}C/m$. The magnitude of electric displacement vector at a point located 20 mm away from the axis of the wire is __________ C/m^2.

Ans. : 2

Solution: $\lambda = 8\pi\times10^{-2}c/m^2, \left|\vec{E}\right| = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow \left|\vec{D}\right| = \varepsilon_0\left|\vec{E}\right| = \frac{\lambda}{2\pi r}$

$$D = \frac{8\pi\times10^{-2}}{2\pi\times20\times10^{-3}} = \frac{4}{2}c/m^2 = 2C/m^2$$

Q49. The 7^{th} bright fringe in the Young's double slit experiment using a light of wavelength 550 nm shifts to the central maxima after covering the two slits with two sheets of different refractive indices n_1 and n_2 but having same thickness $6\ \mu m$. The value of $|n_1 - n_2|$ is _______.

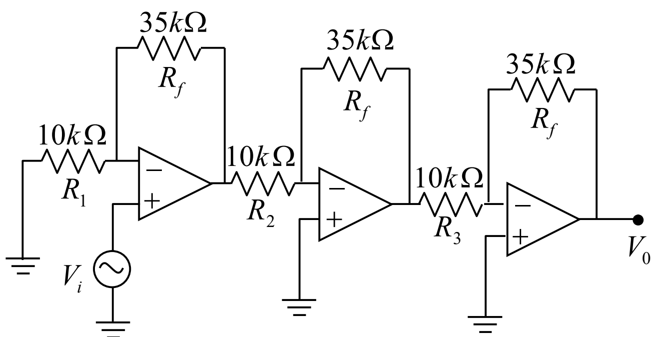
(Round off to 2 decimal places)

Ans. : 0.64

Solution: $(n_1-1)t-(n_2-1)t = 7\lambda$

$$(n_1-n_2) = \frac{7\lambda}{t} = \frac{7\times550\times10^{-9}}{6\times10^{-6}} = 0.64$$

Q50. For the input voltage $V_1(200\,mV)\sin(400t)$, the amplitude of the output voltage (V_0) of the given OPAMP circuit is __________ V. (Round off to 2 decimal places)



Ans. : 11.03

Solution: $v_{01} = \left(1+\frac{35}{10}\right)v_i = (4\cdot5\times200mV)\sin(400t)$

$$v_{02} = -\frac{35}{10}\times(4\cdot5\times200mV)\sin(400t)$$

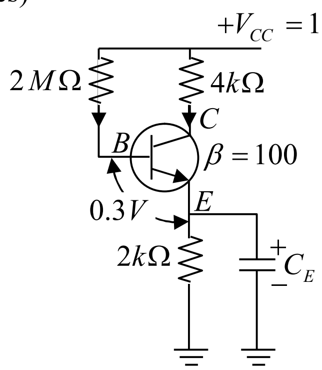
$$v_0 = -\frac{35}{10}\times\left(\frac{-35}{10}\right)(4\cdot5\times200mV)\sin(400t)$$

$$V_m = (3.5\times3.5\times4.5\times200)mV = 11.03\,Volts$$

Q51. – Q60. carry one mark each.

Q51. The value of emitter current in the given circuit is _______ μA.

(Round off to 1 decimal places)



Ans. : 444.9

Solution: $I_B = \dfrac{V_{CC} - V_{BE}}{R_B + (\beta+1)R_E}$

$$I_B = \frac{10 - 0\cdot 3}{2\times 10^6 + 101\times 2\times 10^3} = \frac{9.7}{2.202\times 10^6} A$$

$$I_E = (\beta+1)I_B = 101\times\frac{9.7}{2.202}\mu A = 444.9\mu A$$

Q52. The value of $\left|\int_0^{3+i} (\bar{z})^2 dz\right|^2$, along the line $3y = x$, where $z = x + iy$ is______

(Round off to 1 decimal places)

Ans. : 111.1

Solution: $\left|\int_0^{3+i} (\bar{z})^2 dz\right|^2 \qquad 3y = x$

$$z = x + iy$$

$$z = 3y + iy$$

$$\bar{z} = x - iy = 3y - iy = (3-i)y,\ dz = 3dy + idy \qquad = (3+i)dy$$

$$\left|\int_0^1 (3-i)(3+i)(3-i)y^2 dy\right|^2,\ 1000\left|\int_0^1 y^2 dy\right|^2 = \frac{1000}{9}\times 1 = 111.11$$

Q53. If the wavelength of $K\alpha z$ X-ray line of an element is $1.544\,\mathring{A}$. Then the atomic number (Z) of the element is________

(Rydberg constant $R = 1.097\times 10^7$ m^{-1} and velocity of light $c = 3\times 10^8$ m/s)

Ans. : 29

Solution: According to Mosely's formula, the frequency of $K\alpha$ X - ray line is related to atomic number by the formula

$$f(K\alpha) = \left(3.29\times10^{15}\right)\times\frac{3}{4}\times(z-1)^2\ Hz$$

or $\frac{c}{\lambda} = \left(3.29\times10^{15}\right)\times\frac{3}{4}\times(z-1)^2$ or $\frac{3\times10^8}{1.544\times10^{-10}} = \left(3.29\times10^5\right)\times\frac{3}{4}\times(z-1)^2$

Therefore, $z-1 = 28.06$, or $z = 29.06$

Since atomic number must be an integer $z = 29$

Q54. A proton is confined within a nucleus of size 10^{-13} cm . The uncertainty in its velocity is _________ $\times10^8$ m/s .

(Round off to 2 decimal places)

(Planck's constant $h = 6.626\times10^{-34}\,J$ and proton mass $m_P = 1.672\times10^{-27}$ kg)

Ans. : 0.31

Solution: $\Delta p\Delta x \simeq \frac{h}{4\pi}$

$$\Delta v \simeq \frac{h}{4\pi m\Delta x} \simeq \frac{6.6\times10^{-34}}{4\times3.14\times1.672\times10^{-27}\times\left(10^{-15}\right)} \simeq 0.31\times10^8\ m/s$$

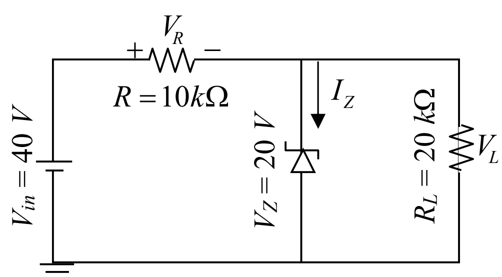
Q55. Given the wave function of a particle $\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{L}x\right) 0 < x < L$ and 0 elsewhere the probability of finding the particle between $x = 0$ and $x = \frac{L}{2}$ is _______.

(Round off to 1 decimal places)

Ans. : 0.5

Solution: $\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{L}x\right) 0 < x < L$, $p\left(0 \le x \le \frac{L}{2}\right) = \int_0^{L/2} |\psi|^2\, dx = \frac{1}{2}$

Q56. The Zener current I_z for the given circuit is ______ mA.



Ans. : 1

Solution: Open circuit voltage $V_i = \dfrac{20k}{20k+10k} \times 40V = \dfrac{2}{3} \times 40 = 26.7\,Volts$

$V_i > V_Z$, Zener "ON"

$I_L = \dfrac{V_Z}{R_L} = \dfrac{20}{20} = 1mA$ and $I_R = \dfrac{40-20}{10} = 2mA$

$I_Z = I_R - I_L = 1mA$

Q57. If the diameter of the Earth is increased by 4% without changing the mass, then the length of the day is _______ hours.

(Take the length of the day before the increment as 24 hours. Assume the Earth to be a sphere with uniform density)

(Round off to 2 decimal places)

Ans. : 25.95

Solution: $I_1\omega_1 = I_2\omega_2 \Rightarrow MR^2 \times \dfrac{2\pi}{T_1} = M\left(R + .04R\right)^2 \times \dfrac{2\pi}{T_2}$

$T_2 = T_1 \times \left(1.04\right)^2 = 24 \times \left(1.04\right)^2 = 25.95$

Q58. A di-atomic gas undergoes adiabatic expansion against the piston of a cylinder. As a result, the temperature of the gas drops from $1150\ K$ to $400\ K$. The number of moles of the gas required to obtain $2300\,J$ of work from the expansion is ______. (The gas constant $R = 8.314\,J\,\text{mol}^{-1}K^{-1}$.)

(Round off to 2 decimal places)

Ans. : 0.1475

Solution: $\gamma = \dfrac{7}{5}$

$W = \dfrac{nR\left(T_2 - T_1\right)}{1-V}, \Rightarrow 2300 = n \times 8.314 \times \dfrac{\left(400-1150\right)}{1-1.4} \Rightarrow n = 0.1475$

$\lambda = \dfrac{1}{\sqrt{2}\pi d^2 N/V}, \ \lambda \propto \dfrac{1}{d^2}$

Q59. The decimal equivalent of the binary number 110.101 is _______.

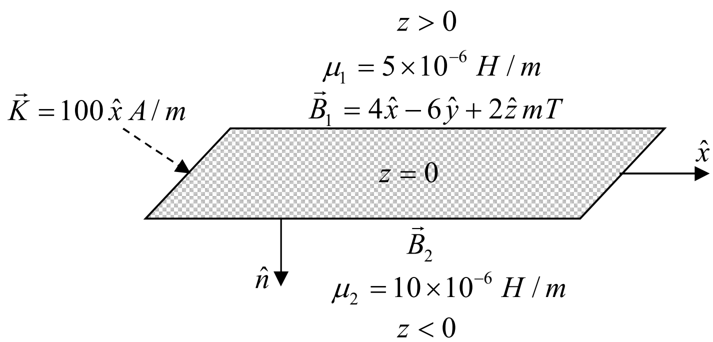
Ans. : 6.625

Solution: $110 \cdot 101 = 1^2 + 1 \times 2^1 + 0 \times 2^0 \cdot 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= \left(4+2+0\right) \cdot \left(\dfrac{1}{2} + 0 + \dfrac{1}{8}\right)$

$= 6 \cdot \left(0.5 + 0 + 0.125\right) = 6.625$

Q60. A surface current $\vec{K} = 100\hat{x}$ A/m flows on the surface $z = 0$, which separates two media with magnetic permeabilities μ_1 and μ_2 as shown in the figure. If the magnetic field in the region 1 is $\vec{B}_1 = 4\hat{x} - 6\hat{y} + 2\hat{z}\, mT$, then the magnitude of the normal component of $\vec{B}_2$ will be ________ mT



Ans. : 2

Solution: $B_2^{\perp} = B_1^{\perp} = 2\hat{z}mT$ (Since $B_1^{\perp} = 2\hat{z}mT$)