PREVIOUS YEAR'S SOLUTION

IIT-JAM 2025

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IIT-JAM Physics Course

Section A: Q. 1 - Q. 10 Carry ONE mark each.

Q1. Consider a volume V enclosed by a closed surface S having unit surface normal $\hat{n}$. For $\mathbf{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, the value of the surface integral

$$\frac{1}{9}\oiint \mathbf{r}\cdot\hat{n}dS$$

is

(A) V (B) 3 V (C) $\frac{V}{3}$ (D) $\frac{V}{9}$

Ans.: (c)

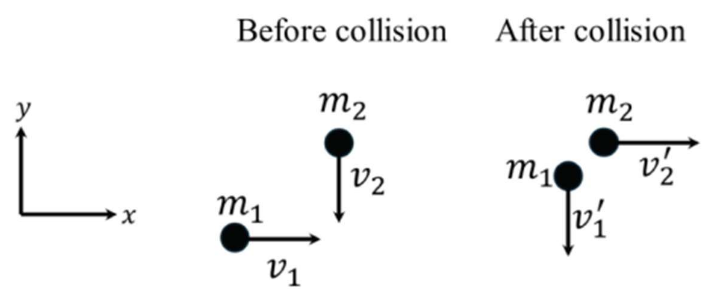
Solution: $\frac{1}{9}\oint \vec{r}\cdot\hat{n}ds$, $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$$= \frac{1}{9}\int_V (\vec{\nabla}\cdot\vec{r})dt = \frac{1}{9}\int_V 3dt = \frac{V}{3}$$

Topic - Mathematical physics

Sub topic: Vectors

Q2. Two point-particles having masses m_1 and m_2 approach each other in perpendicular directions with speeds v_1 and v_2, respectively, as shown in the figure below. After an elastic collision, they move away from each other in perpendicular directions with speeds v_1' and v_2', respectively.



The ratio $\frac{v_2'}{v_1'}$ is

(A) $\frac{m_1^2 v_1}{m_2^2 v_2}$ (B) $\frac{m_1 v_1}{m_2 v_2}$ (C) $\frac{m_1^2 v_2}{m_2^2 v_1}$ (D) $\frac{m_1 v_2}{m_2 v_1}$

Topic – Mechanics and properties of matter

Sub topic: Conservation of momentum

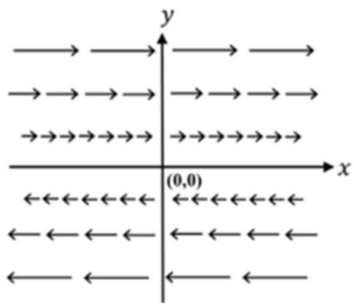
Ans.: (a)

Solution: - $m_1 v_1 = m_2 v_2'$ and $m_2 v_2 = m_1 v_1'$ after dividing equation $\frac{v_2'}{v_1'} = \frac{m_1^2 v_1}{m_2^2 v_2}$

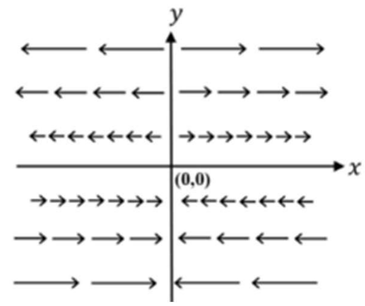
Q3. Which one of the following figures represents the vector field $A = y\hat{\imath}$?

($\hat{\imath}$ is the unit vector along the x-direction)

(a)



(b)



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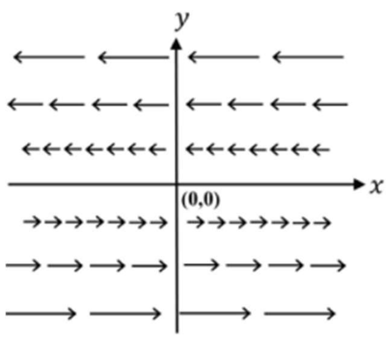

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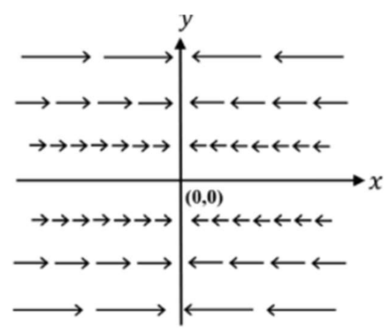


Topic - Mathematical physics

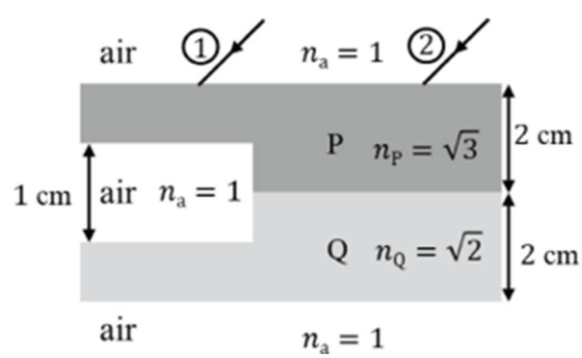
Sub topic – Vectors

Ans.: (a)

Solution: $A = y\hat{\imath}$

Vector is $\hat{\imath}$ cap direction increases with y changes direction with $y > 0, y < 0$

Q4. Two parallel light rays (1) and (2) are incident from air on a system consisting of media P, Q, and air, as shown in the figure below. The incident angle is 45°. Ray (1) passes through medium P, air and medium Q and ray (2) passes through media P and Q before leaving the system. After passing through the system, the angular deviation (in radians) between the two rays is [The dimensions of the media and their refractive indices $(n_a, n_P$ and $n_Q)$ are shown in the figure].



(A) 0 (B) $\tan^{-1}\sqrt{\frac{3}{2}}$ (C) $\tan^{-1}\sqrt{\frac{2}{3}}$ (D) $\tan^{-1}\sqrt{\frac{1}{3}}$

Topic: Waves, oscillation and optics

Sub topic: Ray optics

Ans.: (a)

Solution: Let the incident angle in air be $\theta_0 = 45°$.

Ray (1): goes through $P \to$ air $\to Q$

At each interface,

$$n_a \sin\ \theta_0 = n_P \sin\ \theta_P = n_a \sin\ \theta_a = n_Q \sin\ \theta_Q = n_a \sin\ \theta_{\text{out},1}$$

Since the first and last media are the same air ($n_a = 1$),

$$\sin\ \theta_{\text{out},1} = \sin\ \theta_0 \Rightarrow \theta_{\text{out},1} = \theta_0 = 45°.$$

Ray (2): goes through $P \to Q$

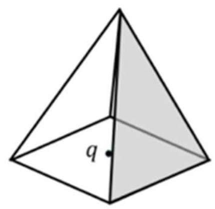
Similarly,

$$n_a \sin\ \theta_0 = n_P \sin\ \theta_P' = n_Q \sin\ \theta_Q' = n_a \sin\ \theta_{\text{out},2} \Rightarrow \theta_{\text{out},2} = \theta_0 = 45°.$$

Both rays emerge at the same angle as they entered (parallel entrance and exit faces guarantee this), so they remain parallel after the system. Any differences are only lateral shifts, not angular. So the angle of deviation is zero.

Q5. A charge q is placed at the centre of the base of a square pyramid. The net outward electric flux across each of the slanted faces is

(Consider permittivity as ε_0)



(A) $\frac{q}{\varepsilon_0}$ (B) $\frac{q}{2\varepsilon_0}$ (C) $\frac{q}{4\varepsilon_0}$ (D) $\frac{q}{8\varepsilon_0}$

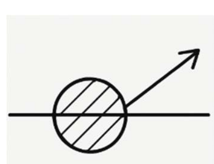
Topic: Electromagnetics theory

Sub topic: electrostatics

Ans.: (d)

Solution: $Q_{net} = \dfrac{q/2}{\varepsilon_0}$

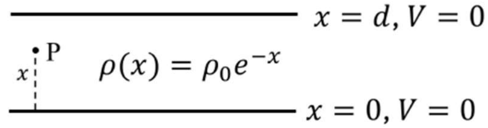
$$Q_{shaded} = \frac{1}{4} \cdot \frac{q}{2\varepsilon_0} = \frac{q}{8\varepsilon_0}$$



Charge above plane is $q/2$

($\therefore$ Base will get 0 flux)

Q6. Consider a parallel plate capacitor (distance between the plates d, and permittivity ε_0) as shown in the figure below. The space charge density between the plates varies as $\rho(x) = \rho_0 e^{-x}$. Voltage $V = 0$ both at $x = 0$ and $x = d$.



The voltage $V(x)$ at point P between the plates is

[ρ_0 is a constant of appropriate dimensions]

(A) $\frac{\rho_0}{\varepsilon_0}\left[e^{-x} + \frac{1-e^{-d}}{d}x - 1\right]$

(B) $\frac{2\rho_0}{\varepsilon_0}\left[e^{-x} + \frac{1-e^{-d}}{d}x - 1\right]$

(C) $\frac{\rho_0}{2\varepsilon_0}\left[e^{-x} + \frac{1-e^{-d}}{d}x - 1\right]$

(D) $\frac{3\rho_0}{\varepsilon_0}\left[e^{-x} + \frac{1-e^{-d}}{d}x - 1\right]$

Topic: Electromagnetic theory

Sub-topic: Electrostatics Poisson's Equation

Ans.: (a)

Solution: $\nabla \cdot E = \frac{\rho}{\epsilon_0}$

$$E = -\nabla V,\ \nabla^2 V = -\rho/\epsilon_0,\ \int \frac{\partial^2 V}{\partial x^2} = \int \frac{\rho_0 e^{-x}}{\epsilon_0},\ \frac{\partial V}{\partial x} = \frac{\rho_0 e^{-x}}{\epsilon_0} + c_1$$

$$V = -\frac{\rho_0 \bar{e}^{-x}}{\epsilon_0} + c_1 x + c_2$$

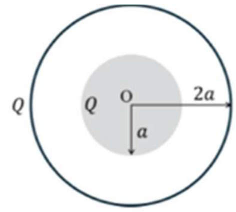
At $x = 0, V = 0$, $x = d, V = 0$

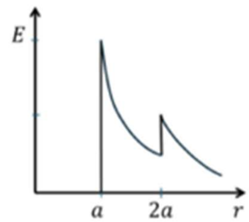
$$0 = -\frac{\rho}{\epsilon_0} + c_2, c_2 = \frac{\rho}{\epsilon_0}, V(x) = -\frac{\rho_0 e^{-x}}{\epsilon_0} + c_1 x + \frac{\rho}{\epsilon_0}, 0 = -\frac{\rho_0 e^{-d}}{\epsilon_0} + c_1 d + \frac{\rho}{\epsilon_0}$$

$$c_1 d = \frac{\rho_0 \mathrm{e}^{-d}}{\epsilon_0} - \frac{\rho}{\epsilon_0}\ V(x) = \frac{\rho}{\epsilon_0}\left[-e^{-x} + \frac{(e^{-d} - 1)}{d}x + 1\right]$$

$$c_1 = \frac{\rho_0 \bar{e}^{-d}}{\epsilon_0 d} - \frac{\rho}{\epsilon_0 d}\left[e^{-x} + \frac{(1-e^{-d})}{d} - 1\right], V(x) = -\frac{\rho_0 e^{-x}}{\epsilon_0} + \left[\frac{\rho_0 e^{-d}}{\epsilon_0 d} - \frac{\rho}{\epsilon_0 d}\right]x + \frac{\rho}{\epsilon_0}$$

Q7. Consider a metal sphere enclosed concentrically within a spherical shell. The inner sphere of radius a carries charge Q. The outer shell of radius $2a$ also has charge Q. The variation of the magnitude E of the electric field as a function of distance r from the center O is



(a)


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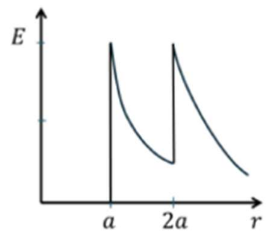

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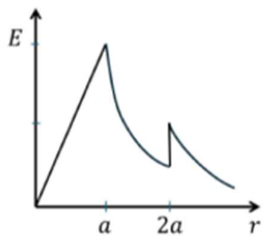

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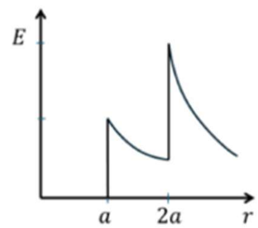


Topic – Electromagnetic theory

Sub-topic – Electrostatics

Ans.: (a)

Solution: $E = k\frac{Q}{r^2}\ a < r < 2a, E = 0\ r < a, E = k\frac{2\Omega}{r^2}\ r > 2a$

Q8. Consider radioactive decays $A \to B$ with half-life $\left(T_{1/2}\right)_A$ and $B \to C$ with half-life $\left(T_{1/2}\right)_B$. At any time t, the number of nuclides of B is given by

$$(N_B)_t = \frac{\lambda_A}{\lambda_B - \lambda_A}(N_A)_0\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$$

where $(N_A)_0$ is the number of nuclides of A at $t = 0$. The decay constants of A and B are λ_A and λ_B, respectively.

If $\left(T_{1/2}\right)_B < \left(T_{1/2}\right)_A$, then the ratio $\frac{(N_B)_t}{(N_A)_t}$ at time $t \gg \left(T_{1/2}\right)_A$ is [$(N_A)_t$ is the number of nuclides of A at time t]

(a) $\frac{\lambda_A}{\lambda_B - \lambda_A}$ (b) $\frac{\lambda_B}{\lambda_A}$ (c) $\frac{\lambda_A}{\lambda_B}$ (d) $\frac{\lambda_B}{\lambda_B - \lambda_A}$

Topic –Modern physics

Sub-topic – Nuclear physics (radioactivity)

Ans.: (a)

Solution: $N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_{A0}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right), N_A(t) = N_{A0} e^{-\lambda_A t}$

Take the ratio: $\frac{N_B(t)}{N_A(t)} = \frac{\lambda_A}{\lambda_B - \lambda_A} \frac{e^{-\lambda_A t} - e^{-\lambda_B t}}{e^{-\lambda_A t}} = \frac{\lambda_A}{\lambda_B - \lambda_A}\left(1 - e^{-(\lambda_B - \lambda_A)t}\right)$.

For $t \gg \left(T_{1/2}\right)_A$ (hence $t \gg 1/\lambda_A$), and since $\lambda_B > \lambda_A$, the exponential term decays to zero:

$$e^{-(\lambda_B - \lambda_A)t} \to 0.$$

Thus, $\frac{N_B(t)}{N_A(t)} \underset{t\to\infty}{\longrightarrow} \frac{\lambda_A}{\lambda_B - \lambda_A}$

Q9. For a non-relativistic free particle, the ratio of phase velocity to group velocity is

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{1}{4}$

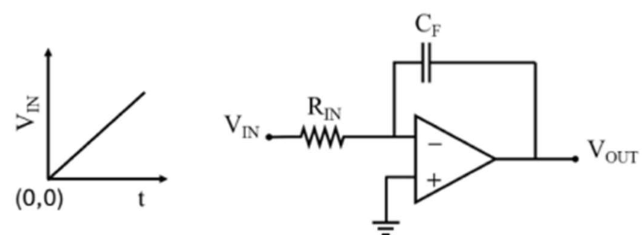
Topic –Modern physics

Sub-topic – Wave nature of particle (Group velocity and phase velocity)

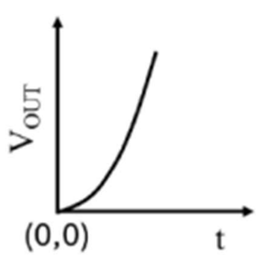
Ans.: (b)

Solution: $E = \frac{p^2}{2m}$ phase velocity $v_p = \frac{E}{p} = \frac{p}{2m}$ and group velocity $v_g = \frac{dE}{dp} = \frac{p}{m}$ $\frac{v_p}{v_g} = \frac{1}{2}$

Q10. If the input voltage waveform V_{IN} is a ramp function (as shown in the V_{IN}-t plot below), then the output wave form (V_{OUT}) for the given circuit diagram having an ideal operational amplifier (Op-Amp) is



(a)



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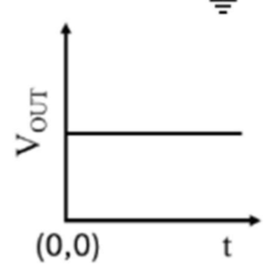
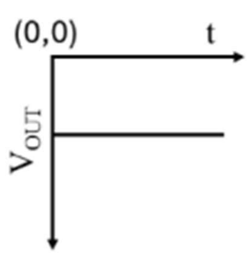
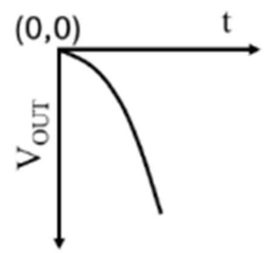


(c)



(d)



Topic – Solid state and Electronics

Sub-topic – Operational amplifier

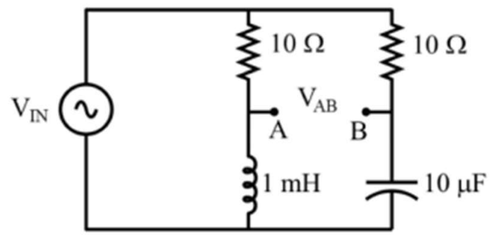
Ans.: (d)

Solution: $\frac{V_{ui}-0}{R_{in}} = 0 - C_F \frac{dV_0}{dt}$

$\frac{dV_0}{dt} = -\frac{1}{R_{in}C_F}kt,\ dV_0 = -\frac{1}{R_{in}C_F}ktdt,\ V_0 = -\frac{1}{R_{in}C_F}R\frac{t^2}{2}$

Section A: Q. 11 - Q. 30 Carry TWO marks each.

Q11. In the circuit given below, the frequency of the input voltage V_{IN} is $\omega = 10^4 \text{rad/s}$. The output voltage V_{AB} leads V_{IN} by



(A) $0°$ (B) $45°$ (C) $90°$ (D) $-90°$

Topic – Solid state and Electronics

Sub-topic – AC circuit

Ans.: (c)

Solution: $I_L = I_0 \sin(\omega t + \pi/4), I_L = I_0 \sin(\omega t + \pi/4)$

$$V_a - V_b = \frac{\pi}{4} - (-\pi/4) = \frac{\pi}{2}$$

Q12. Given a function $f(x,y) = \frac{x}{a}e^y + \frac{y}{b}e^x$, where $x = at$ and $y = bt$ (a and b are non-zero constants, the value of $\frac{df}{dt}$ at $t = 0$ is

(A) -1 (B) 0 (C) 1 (D) 2

Topic: Mathematical physics

Sub topic: Differential equation

Ans.: (d)

Solution: $f(x,y) = \frac{x}{a}e^y + \frac{y}{b}e^x$

$x = at, y = bt$

$df = \left(\frac{\partial f}{\partial x}\right)dx + \left(\frac{\partial f}{\partial y}\right)dy,\ df = \left(\frac{e^y}{a} + \frac{y}{b}\right)dx + \left(\frac{x}{a}e^y + \frac{1}{b}e^x\right)dy$

$\frac{df}{dt} = \left(\frac{e^y}{a} + \frac{y}{b}\right)\frac{dx}{dt} + \left(\frac{x}{a}e^y + \frac{1}{b}e^x\right)\frac{dy}{dt}, \frac{df}{dt} = \left(\frac{e^{bt}}{a} + \frac{bt}{b}\right)a + \left(\frac{x}{a}e^{bt} + \frac{1}{b}e^{at}\right)b$

$\left.\frac{df}{dt}\right|_{t=0} = \left(\frac{1}{a} + \frac{D}{b}\right)a + \left(\frac{1}{b}\right)\cdot b = 2$

Q13. If the system of linear equations

$$x + my + az = 0$$
$$2x + ay + mz = 0$$
$$ax + 2y - z = 0$$

with m and a as non-zero constants, admits a non-trivial solution, then which one of the following conditions is correct?

(A) $m^2 - a^2 = 3$ (B) $m^2 - a^2 = -3$ (C) $a^2 - 2m^2 = -3$ (D) $m^2 - 2a^2 = 3$

Topic - Mathematical physics

Sub topic – Matrices and Determinant

Ans.: (b)

Solution: $\begin{bmatrix} 1 & m & a \\ 2 & a & m \\ a & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

For non trivial solutions det of matrix $= 0$

$$1(-a - 2m) - m(-2 - ma) + a(4 - a^2) = 0$$

$$-a - 2m + 2m + m^2a + 4a - a^3 = 0$$

$$am^2 + 3a - a^3 = 0, m^2 + 3 - a^2 = 0, m^2 - a^2 = -3$$

Q14. If $\left(\frac{1-i}{1+i}\right)^{\frac{n}{2}} = -1$, where $i = \sqrt{-1}$, one possible value of n is

(A) 2 (B) 4 (C) 6 (D) 8

Topic - Mathematical physics

Sub topic – Complex number

Ans.: (b)

Solution: $\left(\frac{1-i}{1+i}\right)^{n/2} = -1 \quad , \left(\frac{e^{-i\pi/4}}{e^{i\pi/4}}\right)^{n/2} = e^{i\pi}$

$$\left(e^{-i\ \ /2}\right)^{n/2} = e^{i\pi}, e^{-i\frac{\pi}{4}\cdot n} = e^{i\pi}, n = 4, e^{-i\hbar} = e^{i\pi}$$

$$\left(\frac{\frac{1}{\sqrt{2}}e^{-i\pi/4}}{\frac{1}{4}e^{i\pi/4}}\right)^{n/2} = e^{i\pi}, \quad e^{-\frac{i\pi}{2}\cdot\frac{n}{2}} = e^{-i\pi} \text{ (For } n > 0), \quad i\frac{n\pi}{4} = i\pi, n = 4$$

Q15. In Cartesian coordinates, consider the functions $u(x, y) = \frac{1}{2}(x^2 - y^2)$ and $v(x, y) = xy$. If (r, θ) are the polar coordinates, the Jacobian determinant $\left|\frac{\partial(u,v)}{\partial(r,\theta)}\right|$ is

(A) r (B) $\frac{1}{r}$ (C) r^2 (D) r^3

Topic: Mathematical physics

Sub topic: Jacobian

Ans.: (d)

Solution: $u=\frac{1}{2}(x^2-y^2)$

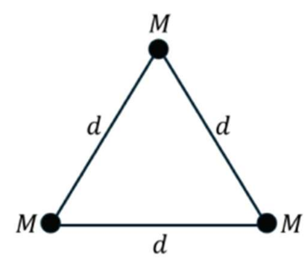
$$v=xy, x=r\cos\ \theta, y=r\sin\ \theta$$

$$u=\frac{1}{2}r^2[\cos^2\ \theta-x^2\theta]=\frac{1}{2}r^2\cos\ 2\theta$$

$$J=\left|\frac{\partial\ (u,v)}{\partial(r,\theta)}\right|, v=\frac{r^2}{2}\sin\ 2\theta$$

$$J=\begin{vmatrix}\frac{\partial u}{\partial r} & \frac{\partial u}{\partial\theta}\\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial\theta}\end{vmatrix}=\begin{vmatrix}r\cos\ 2\theta & \frac{1}{2}r^2(-\sin\ 2\theta)\cdot 2\\ r\sin\ 2\theta & \frac{\partial^2}{2}\cos\ 2\theta\cdot 2\end{vmatrix}=r^3[\cos\ 2\theta+\sin^2\ 2\theta]=r^3$$

Q16. Three particles of equal mass M, interacting via gravity, lie on the vertices of an equilateral triangle of side d, as shown in the figure below. The whole system is rotating with an angular velocity ω about an axis perpendicular to the plane of the system and passing through the center of mass. The value of ω, for which the distance between the masses remains d, is (G is the universal gravitational constant)



(A) $\sqrt{\frac{2GM}{d^3}}$ (B) $\sqrt{\frac{3GM}{d^3}}$ (C) $\sqrt{\frac{GM}{3d^3}}$ (D) $\sqrt{\frac{GM}{d^3}}$

Topic: Mechanics and properties of matter

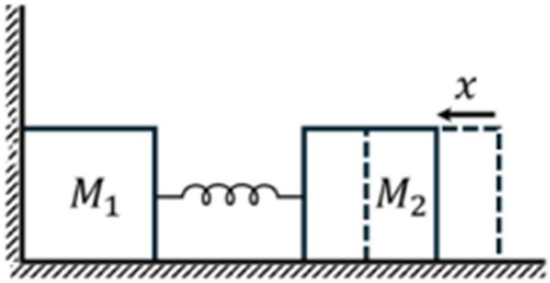
Sub Topic: Central force problem

Ans.: (b)

Solution: Centripetal force must be equal to gravitational force in direction of radius

$$M\omega^2 r=2\frac{GMM}{d^2}\cos 30 \text{ where } r=\frac{d}{\sqrt{3}}\ \omega=\sqrt{\frac{3GM}{d^3}}$$

Q17. Two masses, M_1 and M_2, are connected through a massless spring of spring constant k, as shown in the figure below. The mass M_1 is at rest against a rigid wall. Both M_1 and M_2 are on a frictionless surface. The mass M_2 is pushed towards M_1 by a distance x from its equilibrium position and then released. After M_1 leaves the wall, the speed of the center of mass of the composite system is



(A) $\sqrt{\frac{k}{M_2}}x$ (B) $\sqrt{\frac{k}{M_1+M_2}}x$ (C) $\frac{\sqrt{kM_2}}{M_1+M_2}x$ (D) $\frac{\sqrt{kM_1}}{M_1+M_2}x$

Topic: Mechanics and properties of matter

Subtopic: Centre of Mass

Ans. : (c)

Solution: Center of mass is $X_{cm} = \frac{M_1.0 + M_2(L+x)}{M_1+M_2} \Rightarrow \dot{X}_{cm} = \frac{M_2\dot{x}}{M_1+M_2}$

$\frac{1}{2}M_2\dot{x}^2 = \frac{1}{2}kx^2 \Rightarrow \dot{x} = \sqrt{\frac{k}{M_2}}x$ so where $\omega = \sqrt{\frac{k}{M_2}}$ because M_1 is attached to rigid support

So $\dot{x}_{cm} = \frac{\sqrt{kM_2}}{M_1+M_2}x$

Q18. One end of a long chain is lifted vertically from flat ground to a height H with constant speed v by a force of magnitude F. Assume that the length of the chain is greater than H and that it has a uniform mass per unit length ρ. The magnitude of the force F at height H is

(g is the acceleration due to gravity)

(A) $\rho(gH+v^2)$ (B) $\rho(gH+2v^2)$ (C) $\rho(2gH+v^2)$ (D) $\frac{\rho}{2}(gH+v^2)$

Topic: Mechanics and properties of matter

Subtopic –Variable mass

Ans. : (a)

Solution: Force $F = \frac{dP}{dt} = \frac{dmv}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$

$m\frac{dv}{dt} = mg = \rho gh$ and $v\frac{dm}{dt} = v\rho\frac{dl}{dt} = v\rho v = \rho v^2$

So, force is $\rho gh + \rho v^2$

Q19. For a two-slit Fraunhofer diffraction, each slit is 0.1 mm wide, and the separation between the two slits is 0.8 mm . The total number of interference minima between the first diffraction minima on both sides of the central maxima is

(A) 16 (B) 18 (C) 8 (D) 9

Topic: Wave oscillation and optics

Subtopic: Diffraction

Ans.: (a)

Solution: Here the double-slit has slit width $a = 0.1$ mm and slit separation $d = 0.8$ mm.

Diffraction envelope (single slit):

First minima at

$$a\sin\theta = \pm\lambda \Rightarrow |\sin\theta| < \frac{\lambda}{a}$$

inside the central diffraction maximum.

Interference minima (two slits): $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda, m \in \mathbb{Z}$

To lie inside the central diffraction maximum:

$$\left|\left(m + \frac{1}{2}\right)\frac{\lambda}{d}\right| < \frac{\lambda}{a} \Rightarrow \left|m + \frac{1}{2}\right| < \frac{d}{a} = \frac{0.8}{0.1} = 8.$$

Thus, $-8 < m + \frac{1}{2} < 8 \Rightarrow -8.5 < m < 7.5$,

so m can take the integer values

$$m = -8, -7, \ldots, 6, 7,$$

which is 16 values (minima), counting both sides of the central maximum.

Q20. Consider the superposition of two orthogonal simple harmonic motions $y_1 = a\cos 2\omega t$ and $y_2 = b\cos(\omega t + \phi)$. If $\phi = \pi$, the resultant motion will represent

(a, b and ω are constants with appropriate dimensions)

(A) a parabola (B) a hyperbola (C) an ellipse (D) a circle

Topic: Wave oscillation and optics

Subtopic: Lissajous figure

Ans.: (a)

Solution: Given two perpendicular SHMs:

$$x = a\cos 2\omega t, y = b\cos(\omega t + \phi)$$

With $\phi = \pi \Rightarrow y = b\cos(\omega t + \pi) = -b\cos\omega t$.

Use the double-angle identity:

$$x = a\cos\ 2\omega t = a(2\cos^2\ \omega t - 1).$$

Let $u = \cos\ \omega t$. Then $y = -bu \Rightarrow u = -\frac{y}{b}$.

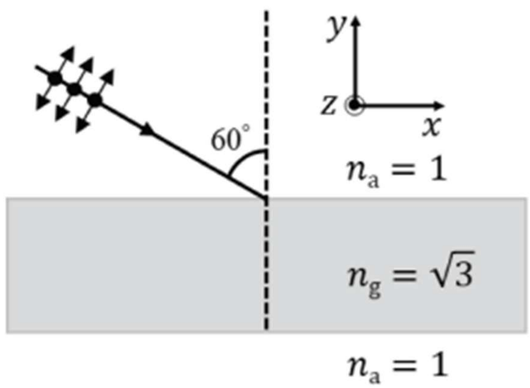
Substitute u into x :

$$x = a(2u^2 - 1) = a\left(2\frac{y^2}{b^2} - 1\right) = \frac{2a}{b^2}y^2 - a.$$

This is a quadratic relation in y and linear in x :

$$x = \frac{2a}{b^2}y^2 - a$$

Q21. An unpolarized light ray passing through air (refractive index $n_a = 1$) is incident on a glass slab (refractive index $n_g = \sqrt{3}$) at an angle of $60°$, as shown in the figure below. The amplitude of the in-plane ($x - y$) electric field component of the incident light is $4\ \mathrm{V/m}$ and amplitude of the out of plane (z) electric field component is $3\ \mathrm{V/m}$. After passing through the glass slab, the electric field amplitude (in V/m) of the light is



(A) 5 (B) 4 (C) 7 (D) 3

Topic: Electromagnetic Theory

Subtopic: Optics

Ans.: (b)

Solution: The slab at 60° (Brewster angle) as an ideal polarizer.

For air $(n_a = 1) \rightarrow$ glass $(n_g = \sqrt{3})$, $\tan\ \theta_B = n_g/n_a = \sqrt{3} \Rightarrow \theta_B = 60°$.

At Brewster angle an ideal interface transmits only the $\mathbf{p}$-polarized (in-plane) component and completely suppresses the s-polarized (out-of-plane) compone

Given amplitudes: $E_p^{(i)} = 4\ \mathrm{V/m}$ (in-plane), $E_s^{(i)} = 3\ \mathrm{V/m}$ (out-of-plane).

After the slab (ideal polarizer assumption): $E_s^{(t)} = 0, E_p^{(t)} = E_p^{(i)} = 4\ \mathrm{V/m}$.

Resultant electric-field amplitude:

$$E_{\text{out}} = \sqrt{\left(E_p^{(t)}\right)^2 + \left(E_s^{(t)}\right)^2} = \sqrt{4^2 + 0^2} = 4\ \mathrm{V/m}.$$

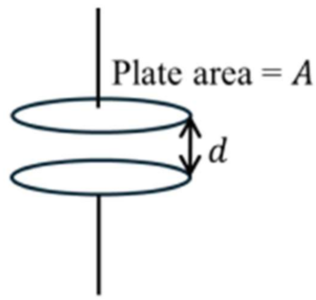
Q22. Consider a slowly charging parallel plate capacitor (distance between the plates is d) having circular plates each with an area A, as shown in the figure below. An electric field of magnitude $E=E_0\sin(\omega t)$ exists between the plates while charging. The associated magnitude of the magnetic field B at the periphery (outer edge) of the capacitor is (Neglect fringe effects)

(A) $\frac{1}{2c^2}\sqrt{\frac{A}{\pi}}E_0\omega\cos(\omega t)$

(B) $\frac{1}{2c^2}\sqrt{\frac{A}{\pi}}E_0\omega\sin(\omega t)$

(C) $\frac{1}{c^2}\sqrt{\frac{A}{\pi}}E_0\omega\cos(\omega t)$

(D) $\frac{1}{c^2}\sqrt{\frac{A}{\pi}}E_0\omega\sin(\omega t)$



Topic: Electromagnetic Theory

Subtopic: Maxwell's Equations (Displacement Current)

Ans.: (a)

Solution: $\oint \vec{B}\cdot d\vec{l}=\mu_0 I_{en}$ (Ampere's law)

$B\cdot 2\pi r=\mu_0 J_d A$

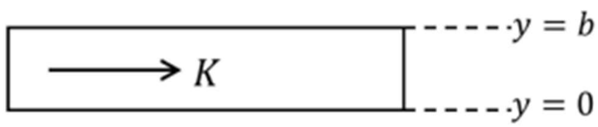
Displacement current $J_d=\epsilon_0\frac{\partial E}{\partial t}, J_d=\epsilon_0 E_0\omega\cos\ \omega t$

$\Rightarrow B\cdot 2\pi r=\mu_0 t_0 E_0\omega\cos\ \omega tA$

$\pi r^2=A, r=\sqrt{\frac{A}{\pi}}\ \mu_0 t_0=\frac{1}{c^2}$

$$B=\frac{1}{2c^2}\sqrt{\frac{A}{\pi}}E_0\omega\cos\ \omega t$$

Q23. A surface current density $K=ae^{-y}$ exists on a thin strip of width b, as shown in the figure below. The associated surface current is



(a is a constant of appropriate dimensions)

(A) $a(1-e^{-b})$ (B) $a(1+e^{-b})$ (C) $a(e^{-b}-1)$ (D) $a(e^{b}+e^{-b})$

Topic: Electromagnetic theory

Subtopic: Magnetostatics

Ans.: (a)

Solution: $K = \frac{dI}{dy}$

$$dI = kdy$$

$$I = \int_0^b ae^{-y}dy = \frac{a}{-1}[e^{-y}]_0^b = -a[e^{-b} - 1] = a[1 - e^{-b}]$$

Q24. For an electromagnetic wave, consider an electric field $E = E_0 e^{-i[a(x+y)-\omega t]}\hat{k}$. The corresponding magnetic field B is

(E_0, a, ω are constants of appropriate dimensions and c is the speed of light)

(A) $\frac{1}{c\sqrt{2}} E_0 e^{-i[a(x+y)-\omega t]}(\hat{\imath} - \hat{\jmath})$

(B) $\frac{1}{c\sqrt{2}} E_0 e^{-i[a(x+y)-\omega t]}(\hat{\imath} + \hat{\jmath})$

(C) $\frac{1}{c\sqrt{2}} E_0 e^{-i[a(x+y)-\omega t]}(-\hat{\imath} - \hat{\jmath})$

(D) $\frac{1}{c\sqrt{2}} E_0 e^{-i[a(x+y)-\omega \ ]}(-\hat{\imath} + \hat{\jmath})$

Topic: Electromagnetic theory

Subtopic: Maxwell's Equations

Ans.: (a)

Solution: $\vec{E} = E_0 e^{-i[a(x+y)-\omega_t]}\hat{k}$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_0 \end{vmatrix}_{a(x+y)-\omega t]} = -\frac{\partial \rho}{\partial t}$$

$$\hat{\imath}\left[E_0 e^{-i[a(x+y)-\omega t]}(+ia)\right] = -\frac{\partial B}{\partial t}, \vec{k} = \vec{k}_x + \vec{k}_y, = \sqrt{2}a$$

$$\hat{\jmath}\left[E_0 e^{-i(a(x+y)-\omega t)}\right][-ia] = -\frac{\partial \beta}{\partial t}, \frac{kc}{\omega} = 1, \sqrt{2}a = \frac{\omega}{c}$$

$$B = \frac{aE_0}{\omega} e^{-i[a(x+y)-\omega t]}[i - j] = \frac{E_0}{\phi} \cdot \frac{\omega}{\sqrt{2}c}$$

Q25. Consider Maxwell's relation $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$. The equation of state of a thermodynamic system is given as $P = \frac{AT}{V^2} + \frac{BT^3}{V}$, where A and B are constants of appropriate dimensions. Then $\left(\frac{\partial C_V}{\partial V}\right)_T$ of the system varies with temperature as (C_V is the heat capacity at constant volume)

(A) T^2 (B) T (C) T^{-1} (D) T^3

Topic: Kinetic theory and thermodynamics

Subtopic: Maxwell relation

Ans.: (a)

Solution: $\left(\frac{\partial S}{\partial V}\right)_T=\left(\frac{\partial P}{\partial T}\right)_V$, $P=\frac{AT}{V^2}+\frac{BT^3}{V}$, $T\left(\frac{\partial S}{\partial V}\right)_T=T\left(\frac{\partial P}{\partial T}\right)_V$

$\left(\frac{\partial Q}{\partial V}\right)_T=T\left(\frac{\partial P}{\partial T}\right)_V$, $\left(\frac{\partial \theta/\partial T}{\partial V}\right)_T=\left(\frac{\partial P}{\partial T}\right)_V$, $\left(\frac{\partial C}{\partial V}\right)_T=\left(\frac{\partial P}{\partial T}\right)_V=\frac{A}{V^2}+\frac{3BT^2}{V}$, αT^2

Q26. Consider a relativistic particle of rest mass $2m$ moving with a speed v along the x direction. It collides with another relativistic particle of rest mass m moving with the same speed but in the opposite direction. These two particles coalesce to form one particle whose rest mass M is ($\beta=\frac{v}{c}$, where c is the speed of light)

(A) $m\sqrt{\frac{9-\beta^2}{1-\beta^2}}$ (B) $2m\sqrt{\frac{3-\beta^2}{1-\beta^2}}$ (C) $\frac{m}{2}\sqrt{\frac{9-\beta^2}{2-\beta^2}}$ (D) $\frac{m}{4}\sqrt{\frac{1-\beta^2}{2-\beta^2}}$

Topic –Modern physics

Subtopic – STR (Mass Energy equivalence)

Ans.: (a)

Solution: Initial momentum of system is

$$p=\frac{2mv}{\sqrt{1-\frac{v^2}{c^2}}}-\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$ which is also momentum of composite mass

Using conservation of energy

$$\frac{2mc^2}{\sqrt{1-\frac{v^2}{c^2}}}+\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}=\sqrt{p^2c^2+M^2c^4}$$ where $p=\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$ and M is rest mass of composite mass

So $$\frac{9m^2c^2}{1-\frac{v^2}{c^2}}=\frac{m^2v^2c^2}{1-\frac{v^2}{c^2}}+M^2c^4\Rightarrow M=\sqrt{\frac{9m^2c^2-m^2v^2}{c^2-v^2}}=m\sqrt{\frac{9-\beta^2}{1-\beta^2}}$$ where $\beta=\frac{v}{c}$

Q27. A particle of mass m is subjected to a potential $V(x)$. If its wavefunction is given by

$$\psi(x,t)=\alpha x^2e^{-\beta x}e^{i\gamma t/\hbar}, x>0$$
$$\psi(x,t)=0, x\le 0$$

then $V(x)$ is

(α,β and γ are constants of appropriate dimensions)

(A) $-\gamma+\frac{\hbar^2}{2m}\left(\frac{2}{x^2}-\frac{4\beta}{x}+\beta^2\right)$ (B) $-\gamma+\frac{\hbar^2}{2m}\left(\frac{2}{x^2}+\frac{4\beta}{x}+\beta^2\right)$

(C) $-\gamma+\frac{\hbar^2}{2m}\left(\frac{2}{x^2}-\frac{4\beta}{x}-\beta^2\right)$ (D) $-\gamma+\frac{\hbar^2}{2m}\left(-\frac{2}{x^2}-\frac{4\beta}{x}+\beta^2\right)$

Topic –Modern physics

Subtopic –postulates of quantum mechanics

Ans.: (a)

Solution: Use the time-dependent Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\right]\psi.$$

$$i\hbar\frac{\partial\psi}{\partial t}=-\gamma\psi$$

Let $f(x)=x^2e^{-\beta}$. Then

$$f'=e^{-\beta x}(2x-\beta x^2), f''=e^{-\beta x}(2+\beta^2x^2-4\beta x)$$

Thus, $\frac{\psi''}{\psi}=\frac{f''}{f}=\frac{2+\beta^2x^2-4\beta x}{x^2}=\frac{2}{x^2}-\frac{4\beta}{x}+\beta^2.$

$$-\gamma=-\frac{\hbar^2}{2m}\frac{\psi''}{\psi}+V(x)\Rightarrow V(x)=-\gamma+\frac{\hbar^2}{2m}\left(\frac{2}{x^2}-\frac{4\beta}{x}+\beta^2\right)$$

$$V(x)=-\gamma+\frac{\hbar^2}{2m}\left(\frac{2}{x^2}-\frac{4\beta}{x}+\beta^2\right)$$

Q28. Two non-relativistic particles with masses m_1 and m_2 move with momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, respectively, in an inertial frame S. In another inertial frame S', moving with a constant speed with respect to S, the same particles are observed to have momenta $\boldsymbol{p}_1'$ and $\boldsymbol{p}_2'$, respectively. Galilean invariance implies that

(A) $m_2p_1'-m_1p_2'=m_2p_1-m_1p_2$ (B) $m_2p_1'+m_1p_2'=m_2p_1+m_1p_2$

(C) $m_1p_1'-m_2p_2'=m_1p_1-m_2p_2$ (D) $m_1p_1'+m_2p_2'=m_1p_1+m_2p_2$

Topic: Mechanics and properties of matter

Subtopic: Conservation of momentum

Ans.: (a)

Solution: Let us assume speed of S' with respect to S is v

We know that $v_{1,S}=v_{1s'}+v_{s',s}$ so $p_1=p_1'+m_1v$ ……A

$v_{2,S}=v_{2s'}+v_{s',s}$ so $p_2=p_2'+m_2v$ ……B

m_2A-m_1B will give rise to $m_1p_1-m_2p_2=m_1p_1'-m_2p_2'$

Q29. The binding energy $B(A,Z)$ of an atomic nucleus of mass number A, atomic number Z, and number of neutrons $N=A-Z$, can be expressed as

$$B(A,Z)=a_1A-a_2A^{\frac{2}{3}}-a_3\frac{Z^2}{A^{\frac{1}{3}}}-a_4\frac{(A-2Z)^2}{A},$$

where a_1,a_2,a_3, and a_4 are constants of appropriate dimensions. Let $B(A,Z')$ be the binding energy of a mirror nucleus (which has the same A, but the number of protons and neutrons are interchanged).

Then, at constant A, $[B(A,Z)-B(A,Z')]$ is

(A) proportional to Z^2

(B) Proportional to (Z^2-N^2)

(C) Proportional to N^2

(D) Constant

Topic: Modern physics

Subtopic: Nuclear physics (Binding energy)

Ans.: (b)

Solution: $B(A,Z)=a_1A-a_2A^{2/3}-a_3\frac{Z^2}{A^{1/3}}-a_4\frac{(A-2Z)^2}{A}$.

Mirror nucleus has same A but with $Z'=N=A-Z$.

Compute the difference at fixed A :

$$\Delta B\equiv B(A,Z)-B(A,Z'=A-Z).$$

The first two terms are independent of $Z\rightarrow$ cancel.

Asymmetry term:

$$-\frac{a_4}{A}(A-2Z)^2-\left[-\frac{a_4}{A}(A-2(A-Z))^2\right]=-\frac{a_4}{A}(A-2Z)^2+\frac{a_4}{A}(A-2Z)^2=0.$$

Coulomb term:

$$-\frac{a_3}{A^{1/3}}Z^2-\left[-\frac{a_3}{A^{1/3}}(A-Z)^2\right]=-\frac{a_3}{A^{1/3}}[Z^2-(A-Z)^2]=-\frac{a_3}{A^{1/3}}[2AZ-A^2].$$

So

$$\Delta B=-a_3A^{2/3}(2Z-A)\propto(2Z-A)=Z-(A-Z)=Z-N.$$

At fixed $A, Z^2-N^2=(Z-N)(Z+N)=A(Z-N)$, hence

$$\Delta B\propto(Z^2-N^2).$$

Q30. A magnetic field is given by $B = \nabla \times A$ where A is the magnetic vector potential. If $A = (ax^2 + by^2)\hat{\imath}$, the corresponding current density J is (a and b are non-zero constants)

(A) $-\frac{1}{\mu_0}(2a + 2b)\hat{\imath}$ (B) $\frac{1}{\mu_0}(2a + 2b)\hat{\imath}$ (C) $-\frac{1}{\mu_0}(2a)\hat{\imath}$ (D) $-\frac{1}{\mu_0}(2b)\hat{\imath}$

Topic –Electromagnetic theory

Subtopic – Magnetostatics vector Potential

Ans.: (d)

Solution: $B = \nabla \times A$

$\nabla \times B = \mu_0 J$, $\nabla \times \nabla \times A = \mu_0 J$, $\nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 J$

$-\nabla^2 A = \mu_0 J$, $J = -\frac{1}{\mu_0}\nabla^2 A$

$$= -\frac{1}{\mu_0}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right](ax^2 + by^2)\hat{L} = -\frac{1}{\mu_0}[2a + 2b]\hat{L}$$

Section B: Q. 31 - Q. 40 Carry TWO marks each.

Q31. In the logic circuit shown below, for which of the following combination(s) of inputs P and Q, the output Y will be 0 ?

(A) $P = 0, Q = 0$ (B) $P = 0, Q = 1$ (C) $P = 1, Q = 0$ (D) $P = 1, Q = 1$

Topic: Solid state and electronics

Subtopic: Digital electronics

Ans.: (d)

Solution: Evaluate for all input pairs:

1. $P = 0, Q = 0$

$A = \overline{0 \cdot 0} = 1, B = \overline{1 \cdot 0} = 1, Y = \overline{1 \cdot 0} = 1$

2. $P = 0, Q = 1$

$A = \overline{0 \cdot 1} = 1, B = \overline{1 \cdot 1} = 0, Y = \overline{0 \cdot 1} = 1$

3. $P = 1, Q = 0$

$A = \overline{1 \cdot 0} = 1, B = \overline{1 \cdot 0} = 1, Y = \overline{1 \cdot 0} = 1.$

4. $P = 1, Q = 1$

$A = \overline{1 \cdot 1} = 0, B = \overline{0 \cdot 1} = 1, Y = \overline{1 \cdot 1} = 0$

Thus the output $Y = 0$ only when $P = 1$ and $Q = 1$.

Q32. Two particles of masses m_1 and m_2, interacting via gravity, rotate in circular orbits about their common center of mass with the same angular velocity ω.

For masses m_1 and m_2, respectively,

- r_1 and r_2 are the constant distances from the center of mass,
- L_1 and L_2 are the magnitudes of the angular momenta about the center of mass, and
- K_1 and K_2 are the kinetic energies.

Which of the following is(are) correct?

(G is the universal gravitational constant)

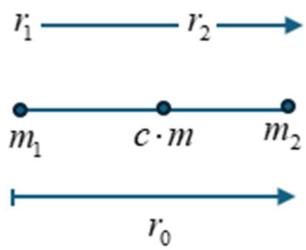
(A) $\frac{L_1}{L_2} = \frac{m_2}{m_1}$ (B) $\frac{K_1}{K_2} = \frac{m_2}{m_1}$ (C) $\omega = \sqrt{\frac{G(m_1+m_2)}{(r_1+r_2)^3}}$ (d) $m_2 r_1 = m_1 r_2$

Topic – Mechanics and properties of matter

Sub topic – two body problem

Ans. : (a),(b) and (c)

Solution:



$m_1 r_1 = m_2 r_2 \quad r_0 = r_1 + r_2$ solving these two equations we get $r_1 = \dfrac{m_2 r_0}{m_1 + m_2}$ and $r_2 = \dfrac{m_1 r_0}{m_1 + m_2}$

Option D is wrong

And we know that two body will behave like one body on Centre of mass reference frame .

Angular momentum of system is given by $L = \left(m_1 r_1^2 + m_2 r_2^2\right)\omega$ when we put value of r_1 and r_2

and using relation $m_1 r_1 = m_2 r_2$ we can get $L = \mu r_0^2 \omega$ where μ is reduce mass $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$

Option A Now $\dfrac{L_1}{L_2} = \dfrac{m_1 r_1^2 \omega}{m_2 r_2^2 \omega}$ put value $r_1 = \dfrac{m_2 r_0}{m_1 + m_2}$ and $r_2 = \dfrac{m_1 r_0}{m_1 + m_2}$ we get $\dfrac{L_1}{L_2} = \dfrac{m_2}{m_1}$ is correct

Option B Now $\dfrac{T_1}{T_2} = \dfrac{\frac{1}{2} m_1 r_1^2 \omega^2}{\frac{1}{2} m_2 r_2^2 \omega^2}$ put value $r_1 = \dfrac{m_2 r_0}{m_1 + m_2}$ and $r_2 = \dfrac{m_1 r_0}{m_1 + m_2}$ we get $\dfrac{T_1}{T_2} = \dfrac{m_2}{m_1}$ is correct

Condition of circular orbit $\dfrac{G m_1 m_2}{r_0^2} = \mu \omega^2 r_0 \Rightarrow \dfrac{G m_1 m_2}{r_0^2} = \dfrac{m_1 m_2}{m_1 + m_2}\omega^2 r_0 \Rightarrow \omega = \sqrt{G \dfrac{m_1 + m_2}{r_0^3}}$ so option

Q33. Which of these cubic lattice plane pairs is(are) perpendicular to each other?

(A) (100), (010) (B) (220), (001) (C) (110), (010) (D) (112), (220)

Topic: Solid state and electronics

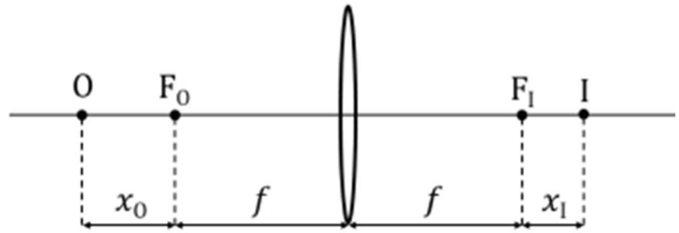
Subtopic: crystallography

Ans.: (a) and (b)

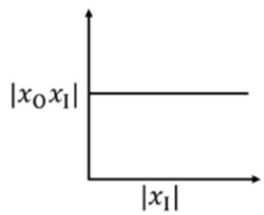
Solution: For cubic crystals, planes $(h_1 k_1 l_1)$ and $(h_2 k_2 l_2)$ are perpendicular if their normals $[h_1 k_1 l_1]$ and $[h_2 k_2 l_2]$ satisfy

$$h_1 h_2 + k_1 k_2 + l_1 l_2 = 0$$

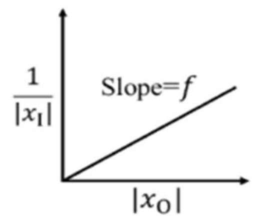
Q34. For a thin convex lens of focal length f, the image of an object at O is formed at I, as shown in the figure below. The distances of object and image from the two focal points ($\mathrm{F_O}$ and $\mathrm{F_I}$) are x_O and x_I, respectively. Which of the following graphs correctly represent(s) the variation of the quantities shown in the figure?



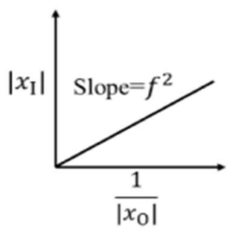
(a)



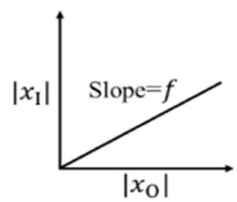
(b)



(c)



(d)



Topic: Wave oscillation and optics

Subtopic: Optics

Ans.: (a) and (c)

Solution: Let the object and image distances from the lens be

$$s_o = f + x_0, s_i = f + x_1.$$

For a thin convex lens,

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}.$$

Substitute:

$$\frac{1}{f + x_0} + \frac{1}{f + x_1} = \frac{1}{f}.$$

Simplify:

$$\frac{2f + x_0 + x_1}{(f + x_0)(f + x_1)} = \frac{1}{f} \Rightarrow f(2f + x_0 + x_1) = (f + x_0)(f + x_1).$$

Expanding and cancelling gives

$$f^2 = x_0 x_1.$$

Q35. Identify which of the following wave functions describe (s) travelling wave (s).

(A_0, B_0, a and b are positive constants of appropriate dimensions)

(A) $\psi(x,t) = A_0(x+t)^2$

(B) $\psi(x,t) = A_0 \sin(ax^2 + bt^2)$

(C) $\psi(x,t) = \frac{A_0}{B_0(x-t)^2+1}$

(D) $\psi(x,t) = A_0 e^{(ax+bt)^2}$

Topic: Wave oscillation and optics

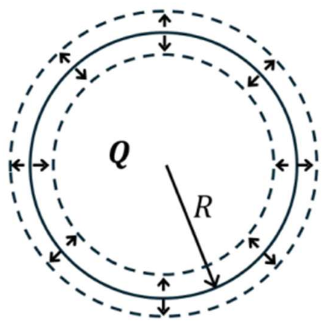
Subtopic: Wave

Ans.: (a) and (c)

Solution: Travelling wave $\Leftrightarrow$ dependence on $x \pm vt$.

Only (A) and (C) have arguments of that linear form.

Q36. A spherical ball having a uniformly distributed charge Q and radius R pulsates with frequency ω such that the radius changes by $\pm 10\%$, as shown in the figure below. Which of the following is(are) correct?



(A) The net outward electric flux across a spherical surface of radius $r > 1.5R$ pulsates with a frequency ω

(B) The net outward electric flux across a spherical surface of radius $r = 2R$ is $\frac{Q}{\varepsilon_0}$

(C) The potential fluctuates with frequency ω at $r = 2R$

(D) The electric field inside the sphere at $r = 0.5R$ will not be time dependent

Topic: Electromagnetic theory

Subtopic: Electrostatics

Ans : (b)

Solution: Use Gauss's Law to see the charge enclosed within a fluctuating radius.

Q37. Which of the following relations is(are) valid for linear dielectrics?

[E = Electric field, P = Polarization, D = Electric displacement, ε_0 = Permittivity of free space, ε = Dielectric permittivity, χ_e = Electric susceptibility, ρ_f = Free charge density, ρ_b = Bound charge density]

(A) $P = \varepsilon_0 \chi_e E$ (B) $\varepsilon = \varepsilon_0(1+\chi_e)$ (C) $D = \varepsilon_0 E + P$ (D) $\nabla \cdot D = \rho_f + \rho_b$

Topic: Electromagnetic theory

Subtopic: Electrostatics

Ans.: (a),(b) and (c)

Solution: $\nabla \cdot D = \rho_f$

Q38. Three gaseous systems, G_1, G_2, and G_3 with pressure and volume $(P_1, V_1), (P_2, V_2)$, and (P_3, V_3), respectively, are such that

(I) when G_1 and G_2 are in thermal equilibrium, $P_1V_1 - P_2V_2 + \alpha P_2 = 0$, is satisfied, and

(II) when G_1 and G_3 are in thermal equilibrium, $P_3V_3 - P_1V_1 + \frac{\beta P_1 V_1}{V_3} = 0$, is satisfied.

The relation(s) valid at thermal equilibrium is(are)

(α and β are constants of appropriate dimensions)

(A) $P_3V_3 - (P_2V_2 - \alpha P_2)\left(1 - \frac{\beta}{V_3}\right) = 0$ (B) $P_3V_3 + (P_2V_2 + \alpha P_2)\left(1 + \frac{\beta}{V_3}\right) = 0$

(C) $P_1V_1 = P_2V_2 = P_3V_3$ (D) $P_3V_3 + P_1V_1\left(\frac{\beta}{V_3} - 1\right) = 0$

Topic – Kinetic theory of gases and thermodynamics

Subtopic –kinetic theory of gases

Ans.: (a) and (d)

Solution: $G_1 \longrightarrow G_2 \quad P_1v_1 - P_2v_2 + \alpha P_2 = 0$

$G_1 \longrightarrow G_3 \quad P_3v_3 - P_1v_1 + \beta\frac{P_1v_1}{v_3} = 0$

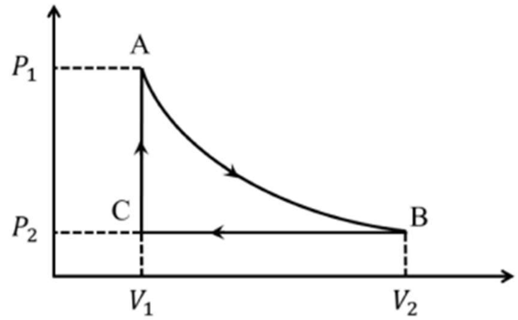
$$P_1v_1 = P_2v_2 - \alpha P_2$$

$$P_3v_3 + \left(\frac{\beta}{v_3}(B_3 - 1)P_1v_1 = 0\right.$$

$$P_3v_3 + \left(\frac{\beta}{v_3} - 1\right)(P_2v_2 - \alpha P_2) = 0\ (A)^v$$

$$P_3v_3 + P_1v_1\left[\frac{\beta}{v_3} - 1\right] = 0$$

Q39. An ideal mono-atomic gas is expanded adiabatically from A to B . It is then compressed in an isobaric process from B to C. Finally, the pressure is increased in an isochoric process from C to A . The cyclic process is shown in the figure below. For this system, which of the following is(are) correct?



(A) Work done along the path AB is $(P_1V_1 - P_2V_2)$

(B) Total work done during the entire process is $\frac{3}{2}(P_1V_1 - P_2V_2) + P_2(V_1 - V_2)$

(C) Total heat absorbed during the entire process is $\frac{3}{2}(P_1 - P_2)V_1$

(D) Total change in internal energy during the entire process is $\frac{5}{2}P_2(V_2 - V_1)$

Topic: Kinetic theory of gases and thermodynamics

Subtopic: First law of thermodynamics

Ans.: (b), (c) and (d)

Solution: $A \to B$ (Adiabatic)

$$dQ = dU + dW = 0$$

$$= \frac{R}{Y-1}[P, v, -P, v_1] - \left(\frac{Y}{3}\right) = -\frac{3}{2}R[R_2v_2 - P_1v_1], r = \frac{5}{3}$$

$$W = \frac{-3}{2}R[P_2v_2 - P_1v_1] + C_2[v_1 - v_2] + 0$$

$$\theta = \theta_{ML} + \theta_{KC} + \theta_{CA} = 0 + C_p dT + C_v dT, \frac{P_1V_1}{T_A} = \frac{P_2V_2}{T_B} = \frac{P_2V_1}{T_C} = nR$$

$$= C_p[T_c - T_B] + C_v[T_A - T_c] = C_p\left[\frac{p_2V_1}{h_R} - \frac{p_1V_2}{h_R}\right] + C_V\left[\frac{p_1V_1}{h_R} - \frac{p_2V_1}{h_R}\right]$$

$$= \frac{P}{r-1}\left[\frac{P_2V_1}{r} - \frac{P_2V_2}{r}\right] + \frac{rP_1}{r-1}\left[\frac{P_1V_1 - P_2V_1}{r}\right] = \frac{3}{2}[V_1 - V_2]P_2 + \frac{SA}{2/8}[P_1V_1 - P_2V_1]_1$$

$$= \frac{3}{2}V_1\rho_2 - \frac{3}{2}V_2\rho_2 + \frac{5}{2}\rho_1V_1 - \frac{5}{2}\rho_2V_1$$

Q40. For a body-cantered cubic (bcc) system, the X-ray diffraction peaks are observed for the following $h^2+k^2+l^2$ value (s) [h, k, and l are Miller indices]

(A) 3 (B) 4 (C) 5 (D) 7

Topic: Solid state and electronics

Sub topic: Crystallography

Ans.: (b)

Solution: Only $h^2+k^2+l^2=4$ gives a bcc reflection.

Section C: Q. 41 - Q. 50 Carry ONE mark each.

Q41. Two solid cylinders of the same density are found to have the same moment of inertia about their respective principal axes. The length of the second cylinder is 16 times that of the first cylinder. If the radius of the first cylinder is 4 cm , the radius of the second cylinder is _______cm . (in integer)

Topic – Mechanics and properties of mechanics

Sub topic –Moment of inertia

Ans.: 2

Solution: $\frac{M_1R_1^2}{2}=\frac{M_2R_2^2}{2}\Rightarrow\frac{\pi R_1^2 l_1\rho R_1^2}{2}=\frac{\pi R_2^2 l_2\rho R_2^2}{2}$

$$\Rightarrow\frac{R_1}{R_2}=\left(\frac{l_2}{l_1}\right)^{1/4}\Rightarrow\frac{R_1}{R_2}=(16)^{1/4}=2\Rightarrow R_2=\frac{R_1}{2}=\frac{4}{2}=2$$

Q42. The shortest distance between an object and its real image formed by a thin convex lens of focal length 20 cm is______cm. (in integer)

Topic –Wave oscillation and optics

Sub topic – Ray optics (lens)

Ans.: 80

Solution: The shortest distance between an object and its real image in a convex lens occurs when both the object and the image are at 2 f , i.e. both at the center of curvature.

Hence, $D_{\min}=4f$.

Q43. Consider two media 1 and 2 having permittivities ε_0 and $\varepsilon_1(=2\varepsilon_0)$, respectively. The interface between the two media aligns with the $x-y$ plane. An electric field $\boldsymbol{E_1}=4\hat{\imath}-5\hat{\jmath}-\hat{k}$ exists in medium 1. The magnitude of the displacement vector $\boldsymbol{D_2}$ in medium 2 is_______ε_0. (up to two decimal places)

Topic: Electromagnetic theory

Subtopic: Electrostatics (Boundary condition)

Ans.: 12.65 to 13.05

Solution: $P_{1a}=D_{1b}$

$(-1)=2t_0'\epsilon_2$

$\epsilon_r=-1/2$

$D_2=2\epsilon_0\left[4\hat{\imath}-5\hat{\jmath}+\frac{1}{2}\hat{k}\right], D=\varepsilon_0\cdot[8\hat{\imath}-10+1\hat{k}]_2$

$|D_2|=\sqrt{64+100+1}=\sqrt{165}=12.84\mathrm{e}$

Q44. G1 and G2 are two ideal gases at temperatures T_1 and T_2, respectively. The molecular weight of the constituents of G1 is half that of G2 If the average speeds of the molecules of both gases are equal, then assuming Maxwell-Boltzmann distributions for the molecular speeds, the ratio $\frac{T_2}{T_1}$ is________(in integer)

Topic: Kinetics theory and thermodynamics

Subtopic: Kinetic theory of gases

Ans.: 2

Solution: $M_{a_1}=\frac{1}{2}M_{a_2}, \langle V_{a_1}\rangle=\langle V_{a_2}\rangle, \sqrt{\frac{8RT_1}{KM_{a_1}}}=\sqrt{\frac{8RT_2}{KM\ \ _2}}$

$\frac{T_1}{M_{a_1}}=\frac{T_2}{M_{a_2}}, \frac{T_2}{T_1}=\frac{Ma_2}{Ma_1}=2$

Q45. An ideal p-n junction diode (ideality factor $\eta=1$) is operating in forward bias at room temperature (thermal energy $=26\mathrm{meV}$). If the diode current is 26 mA for an applied bias of 1.0 V , the dynamic resistance (r_{ac}) is______Ω. (up to two decimal places)

Topic: Solid state and electronics

Subtopic: Diode

Ans.: 0.95 to 1.05

Solution: $V_T=26\mathrm{mV}$

$$V_{ac}=\frac{\eta V_T}{I}=\frac{1\times 26\mathrm{mV}}{26\ \mathrm{mA}}=1\Omega$$

Q46. In a two-level atomic system, the excited state is 0.2 eV above the ground state. Considering the Maxwell-Boltzmann distribution, the temperature at which 2% of the atoms will be in the excited state is__________K. (up to two decimal places)

(Boltzmann constant $k_B = 8.62 \times 10^{-5} \text{eV/K}$)

Topic: Kinetics theory of gases and thermodynamics

Subtopic: identical particle

Ans.: 591.00 to 597.00

Solution:

$$n = n_0 e^{-\beta 0.2eV}$$

$$\frac{1}{50}\frac{2}{n} = \frac{n}{n_0} = \frac{0.2eV}{k_B T}$$

$$-\ln 1 - \ln 50 = -\frac{0.2 \text{ eV}}{k_B T}$$

$$T = \frac{0.2\times 1.6\times 10^{-1}}{1.38\times 10^{-2}\ \times \ln 50} = \frac{0.32}{1.38\times 150} \times 10^{23-19} = 0.0593,\ T = 593 \text{ K}$$

Q47. Neutrons of energy 8 MeV are incident on a potential step of height 48 MeV . As they penetrate the classically forbidden region, the distance at which the probability density of finding neutrons decreases by a factor of 100 is______________________fm. (up to two decimal places) (Take $\hbar c = 200\text{MeVfm}$, and the rest mass energy of neutron $= 1\text{GeV}$.)

Topic –Modern physics

Subtopic –Quantum mechanics (step potential)

Ans.: 1.55 to 1.70

Solution: $T = \exp -2\left(\sqrt{\frac{2m(V_0 - E)}{\hbar^2}}\right)x \Rightarrow T = \exp -2\left(\sqrt{\frac{2mc^2(V_0 - E)}{\hbar^2 c^2}}\right)x$

$$\frac{1}{100} = \exp -2\left(\sqrt{\frac{2\times 1\times 1000\times 40}{200\times 200}}\right)x \Rightarrow 0.01 = \exp -2\sqrt{2}x \Rightarrow x = 2.07$$

Q48. At a particular temperature T, Planck's energy density of black body radiation in terms of frequency is $\rho_T(v) = 8 \times 10^{-18} \frac{\text{J/m}^3}{\text{Hz}}$ at $v = 3 \times 10^{14}$ Hz. Then Planck's energy density $\rho_T(\lambda)$ at the corresponding wavelength (λ) has the value_______ $\times 10^2 \frac{\text{J/m}^3}{\text{m}}$. (in integer)

[Speed of light $c = 3 \times 10^8$ m/s]

Topic: Modern physics

Subtopic: Black body radiation

Ans.: 24

Solution: Given:

$$\rho_T(\nu) = 8\times 10^{-18}\ \mathrm{J/m^3/Hz}\ \text{at}\ \nu = 3\times 10^{14}\ \mathrm{Hz}$$

Relation:

$$\rho_T(\lambda) = \rho_T(\nu)\frac{c}{\lambda^2}, \lambda = \frac{c}{\nu}$$

So, $\lambda = \frac{3\times10^8}{3\times10^{14}} = 10^{-6}$ m

$$\rho_T(\lambda) = 8\times10^{-18}\times\frac{3\times10^8}{(10^{-6})^2} = 8\times10^{-18}\times3\times10^{20} = 24\times10^2 b$$

Q49. The ratio of the density of atoms between the (111) and (110) planes in a simple cubic (sc) lattice is__________. (up to two decimal places)

Topic –Solid state and electro

Sub topic – black body radiation

Ans.: 0.80 to 0.84

Solution: For cubic lattice,

$d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$, and planar density $\propto \frac{1}{d_{hk}}$.

So, $\frac{\rho_{111}}{\rho_{110}} = \frac{d_{110}}{d_{111}} = \frac{a/\sqrt{2}}{a/\sqrt{3}} = \sqrt{\frac{3}{2}} = 1.22.$

Q50. The packing fraction for a two-dimensional hexagonal lattice having sides $2r$ with atoms of radii r placed at each vertex and at the center is________. (up to two decimal places)

Topic –Solid state and electronics

Sub topic – black body radiation

Ans.: 0.89 to 0.93

Solution: For a 2D hexagonal lattice of side $2r$, total atoms per unit cell $= 6\times\frac{1}{6}+1 = 2.$

Area of unit cell $A_{\text{cell}} = \frac{3\sqrt{3}}{2}(2r)^2 = 6\sqrt{3}r^2.$

Area occupied by atoms $A_{\text{atoms}} = 2\pi r^2.$

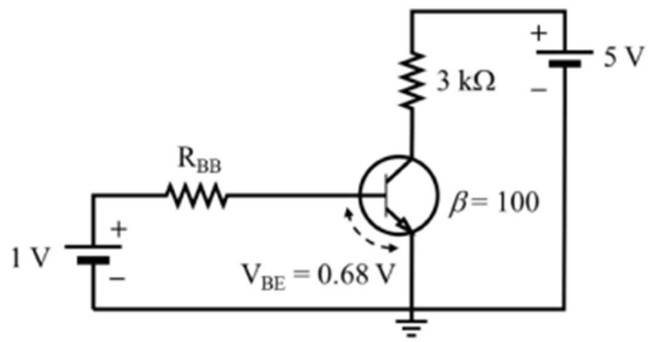
Hence, packing fraction

$$\eta = \frac{A_{\text{atoms}}}{A_{\text{cell}}} = \frac{2\pi r^2}{6\sqrt{3}r^2} = \frac{\pi}{3\sqrt{3}} = 0.60.$$

Packing fraction $= 0.60$

Section C: Q. 51 - Q. 60 Carry TWO marks each.

Q51. A NPN bipolar junction transistor (BJT) is connected in common emitter (CE) configuration as shown in the circuit diagram below. The amplifier is operating in the saturation regime. The collector-emitter saturation voltage (V_{CE}^{sat}) is 0.2 V . The current gain $\beta = 100$. The maximum value of base resistance R_{BB} is ________ $k\Omega$. (in integer)



Topic –Solid state and electronics

Sub topic – Transistor

Ans.: 20

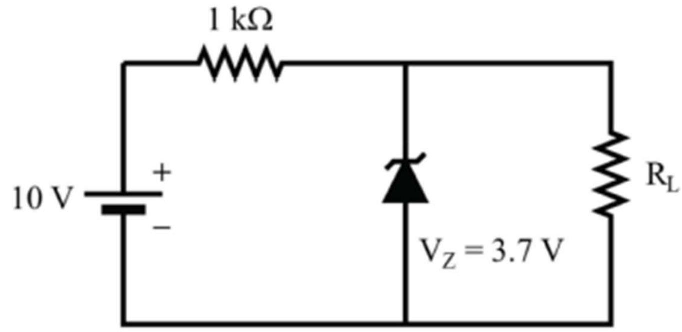
Solution: $-1 + I_B R_{BB} + 0.68 = 0$

$3I_C + 5 + V_{EC} = 0$

$3I_C + 5 = V_{CE} = 0.2$

$I_C = \frac{0.2-5}{3} = \frac{-4.8}{3}$ = -1.6, $\frac{I_c}{I_B} = 100$, $R_{BB} =$ (1-0.68)/ $I_B = 20$

Q52. For a Zener diode as shown in the circuit diagram below, the Zener voltage V_Z is 3.7 V. For a load resistance (R_L) of $1k\Omega$, a current I_1 flows through the load. If R_L is decreased to 500Ω, the current changes to I_2.



The ratio $\frac{I_2}{I_1}$ is__________ . (up to two decimal places)

Topic –Solid state and electronics

Sub topic – Zener diode

Ans.: 1.78 to 1.82

Solution: $R_L = 1\text{k}\Omega$

Check Zener regulation

If Zener is ON, output voltage:

$$V_o = V_Z = 3.7 \text{ V}$$

Load current, $I_1 = \frac{V_o}{R_L} = \frac{3.7}{1000} = 3.7 \text{ mA}$

Current through series resistor

$$I_S = \frac{10 - 3.7}{1000} = 6.3 \text{ mA}$$

Zener current:

$$I_Z = I_S - I_1 = 6.3 - 3.7 = 2.6 \text{ mA} > 0$$

Zener remains ON, so

$$I_1 = 3.7 \text{ mA}$$

$R_L = 500\Omega$

If Zener were ON:

$$I_L = \frac{3.7}{500} = 7.4 \text{ mA}$$

But maximum available current from source:

$$I_S = 6.3 \text{ mA}$$

Since $I_L > I_S$, Zener cannot conduct $\Rightarrow$ Zener turns OFF.

Circuit becomes a simple voltage divider

Total resistance:

$$R_{\text{total}} = 1000 + 500 = 1500\Omega$$

Current through load:

$I_2 = \frac{10}{1500} = 6.67 \text{ mA}, \frac{I_2}{I_1} = \frac{6.67}{3.7} \approx 1.80, \frac{I_2}{I_1} = 1.80$

Q53. One kg of water at 27°C is brought in contact with a heat reservoir kept at 37°C. Upon reaching thermal equilibrium, this mass of water is brought in contact with another heat reservoir kept at 47°C. The final temperature of water is 47°C. The change in entropy of the whole system in this entire process is_______cal/K. (up to two decimal places)

[Take specific heat at constant pressure of water as $1\text{cal}/(\text{gK})$]

Topic: Kinetic theory of gases and thermodynamics

Subtopic: Second law of thermodynamics

Ans.: 0.90 to 1.10

Solution: $\Delta S_{\text{total}} = 1000\ln\frac{310}{300} - \frac{10000}{310} + 1000\ln\frac{320}{310} - \frac{10000}{320} = 1.03\text{cal/K}$

Q54. Consider a vector $\boldsymbol{F} = \frac{1}{\pi}[-\sin\ y\hat{\imath} + x(1-\cos\ y)\hat{\jmath}]$. The value of the integral $\oint \mathbf{F}\cdot d\mathbf{r}$ over a circle $x^2 + y^2 = 1$ evaluated in the anti-clockwise direction is________. (in integer)

Topic: Mathematical physics

Subtopic: Vector

Ans.: 1

Solution: $\mathrm{F} = \frac{1}{\pi}(-\sin\ y\hat{\imath} + x(1-\cos\ y)\hat{\jmath} + 0\hat{k})$

$$\oint_C \mathrm{F}\cdot d\mathrm{r} = \iint_S (\nabla\times\mathrm{F})\cdot\hat{n}dS$$

$$\nabla\times\mathrm{F} = \left(0,0,\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) = \left(0,0,\frac{1}{\pi}(1-\cos y) - \frac{1}{\pi}(-\cos y)\right) = \left(0,0,\frac{1}{\pi}\right)$$

For anticlockwise C in xy-plane, $n = \hat{k}$, and S is the unit disk:

$$\oint_C \mathrm{F}\cdot d\mathrm{r} = \iint_S \frac{1}{\pi}dA = \frac{1}{\pi}(\pi) = 1$$

Q55. A particle is moving with a constant angular velocity 2rad/s in an orbit on a plane. The radial distance of the particle from the origin at time t is given by $r = r_0 e^{2\beta}$ where r_0 and β are positive constants. The radial component of the acceleration vanishes for $\beta =$ _____rad/s. (in integer)

Topic- Mechanics and general properties of matter

Subtopic – Newton's law in polar coordinates

Ans. : 1

Solution: $\dot{\theta} = 2, r = r_0\exp 2\beta t$

$$a_r = \ddot{r} - r\dot{\theta}^2 = r_0\exp 2\beta t(2\beta)^2 - r.4 = r(2\beta)^2 - r.4$$

$$a_r = 0 \Rightarrow \beta = 1$$

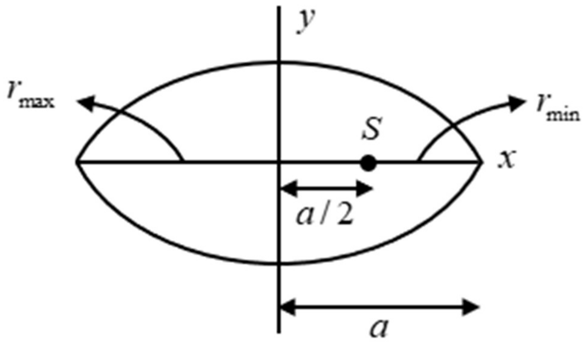
Q56. A planet rotates in an elliptical orbit with a star situated at one of the foci. The distance from the center of the ellipse to any foci is half of the semi-major axis. The ratio of the speed of the planet when it is nearest (perihelion) to the star to that at the farthest (aphelion) is_______. (in integer)

Topic: Mechanics and general properties of matter

Subtopic: Central force problem

Ans.: 3

Solution:



Using conservation of angular momentum $mv_P\left(a-\frac{a}{2}\right)=mv_a\left(a+\frac{a}{2}\right)\Rightarrow\frac{v_p}{v_a}=3$

Q57. A light beam given by $\boldsymbol{E}(z,t) = E_{01}\sin(kz-\omega t)\hat{\imath} + E_{02}\sin\left(kz-\omega t+\frac{\pi}{6}\right)\hat{\jmath}$ passes through an ideal linear polarizer whose transmission axis is tilted by 60° from x-axis (in x-y plane). If $E_{01} = 4\ \mathrm{V/m}$ and $E_{02} = 2\ \mathrm{V/m}$, the electric field amplitude of the emerging light beam from the polarizer is _______ V/m. (up to two decimal places)

Topic: Electromagnetic Theory

Subtopic: Polarisation

Ans.: 3.59 to 3.63

Solution: $\hat{u} = \cos\ 60^\circ\hat{\imath} + \sin\ 60^\circ\hat{\jmath} = \frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}$

$$E_u = \mathbf{E}\cdot\hat{\imath} = \left(4\cdot\frac{1}{2}\right)\sin\ \phi + \left(2\cdot\frac{\sqrt{3}}{2}\right)\sin\ \left(\phi+\frac{\pi}{6}\right) = 2\sin\ \phi + \sqrt{3}\sin\ \left(\phi+\frac{\pi}{6}\right)$$

$$E_{\text{amp}} = \sqrt{2^2 + (\sqrt{3})^2 + 2(2)(\sqrt{3})\cos\ \left(\frac{\pi}{6}\right)} = \sqrt{4+3+6} = \sqrt{13} = 3.61\ \mathrm{V/m}$$

Q58. A wedge-shaped thin film is formed using soap-water solution. The refractive index of the film is 1.25 . At near normal incidence, when the film is illuminated by a monochromatic light of wavelength $600\ \mathrm{nm}$, 10 interference fringes per cm are observed. The wedge angle (in radians) is_________ $\times 10^{-5}$. (in integer)

Topic: Wave Oscillation and Optics

Subtopic: Interference

Ans.: 24

Solution: For wedge fringes at near-normal incidence, the spacing between successive fringes is

$$\Delta x = \frac{\lambda}{2n\alpha}$$

so the number of fringes per unit length is

$N=\frac{1}{\Delta x}=\frac{2n\alpha}{\lambda}$.

Given $N=10\ \mathrm{cm}^{-1}=1000\ \mathrm{m}^{-1}, n=1.25, \lambda=600\ \mathrm{nm}=6\times10^{-7}\ \mathrm{m}$,

$$\alpha=\frac{N\lambda}{2n}=\frac{1000\times6\times10^{-7}}{2\times1.25}=\frac{6\times10^{-4}}{2.5}=2.4\times10^{-4}\mathrm{rad}=24\times10^{-5}\mathrm{rad}.$$

Q59. In an orthorhombic crystal, the lattice constants are $3.0\text{Å}, 3.2\text{Å}$, and $4.0\ \text{Å}$. The distance d_{101} between the successive (101) planes is_______Å. (up to one decimal place)

Topic: Solid state physics and electronics

Subtopic: Crystallography

Ans.: 2.3 to 2.5

Solution: For an orthorhombic crystal, the interplanar spacing is given by

$$\frac{1}{d_{hkl}^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}$$

Given $a=3.0\text{Å}, b=3.2\text{Å}, c=4.0\text{Å}$ and plane (101): $h=1, k=0, l=1$

$$\frac{1}{d_{101}^2}=\frac{1^2}{3.0^2}+\frac{0^2}{3.2^2}+\frac{1^2}{4.0^2}=\frac{1}{9}+0+\frac{1}{16}$$

$$\frac{1}{d_{101}^2}=0.1111+0.0625=0.1736$$

$$d_{101}=\frac{1}{\sqrt{0.1736}}=2.40\text{Å}$$

Q60. Consider a chamber at room temperature $(27^\circ\mathrm{C})$ filled with a gas having a molecular diameter of 0.35 nm . The pressure (in Pascal) to which the chamber needs to be evacuated so that the molecules have a mean free path of 1 km is________$\times10^{-5}$ Pa. (up to two decimal places) (Boltzmann constant $k_B=1.38\times10^{-2}\ \ \mathrm{J/K}$)

Topic kinetic theory and thermodynamics

Sub topic – kinetic theory

Ans.: 0.70 to 1.20

Solution: $\lambda=\frac{k_BT}{\sqrt{2}\pi d^2p}$

$$p=\frac{k_BT}{\sqrt{2}\pi d^2\lambda}=\frac{(1.38\times10^{-23})(300)}{\sqrt{2}\pi(0.35\times10^{-9})^2(10^3)}=0.76\times10^{-5}\ \mathrm{Pa}, 0.76$$