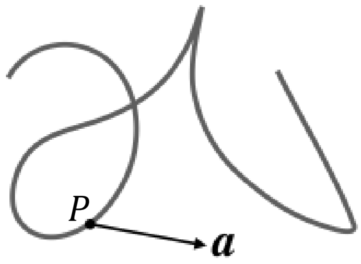
Section A

A1

(1) Is it possible for the trajectory of a particle in a two-dimensional plane with a continuous velocity and acceleration to have the shape shown in the figure?

(2) Is it possible for its acceleration vector at P on the trajectory to point in the direction, $\boldsymbol{a}$, as shown?



(a) (1) Yes (2) No

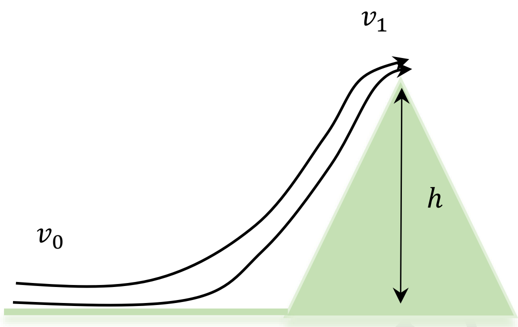
(b) (1) Yes (2) Yes

(c) (1) No (2) No

(d) (1) No (2) Yes

A2

A steady, incompressible air stream of density $\rho = 1.2$ kg/m^3 blows horizontally at $v_0 = 15$ m/s towards a hill of height $h = 50$ m. Because the streamlines constrict over the hilltop, the air speed there increases to $v_1 = 19.5$ m/s. The difference between the pressure at the base and the hilltop is closest to:



(a) 681 N/m^2

(b) 495 N/m^2

(c) 588 N/m^2

(d) 93 N/m^2

A3

A student performs an experiment to measure the acceleration due to gravity, g, using a simple pendulum. The student measures $L = 1.00 \pm 0.01$ m and the time for 50 oscillations $t_{50} = 100.0 \pm 1.0$ s. Based on this experiment, what is the calculated value of g and its uncertainty, assuming that the errors on the length and time are statistically independent?

(a) 9.87 ± 0.22 m/s^2

(b) 9.87 ± 0.11 m/s^2

(c) 9.87 ± 0.44 m/s^2

(d) 9.87 ± 0.05 m/s^2

A4 Consider a 2×2 matrix defined as:

$$A = [a_0 + i\,\vec{\sigma} \cdot \vec{a}][a_0 - i\,\vec{\sigma} \cdot \vec{a}]^{-1}$$

where a_0 is a non-zero real number, $\vec{a}$ is a three-dimensional non-zero real vector, and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli spin matrices. The matrix A:

(a) Is Unitary but not Hermitian

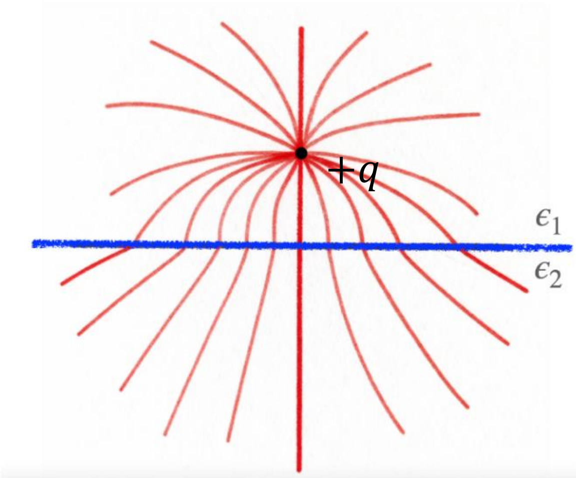
(b) Is Hermitian but not Unitary

(c) Is Unitary and Hermitian

(d) Is neither Unitary nor Hermitian

A5

The figure on the right shows a positive point charge ($+q$) placed above an interface (horizontal line in the figure) between two dielectrics with permittivities, ϵ_1 and ϵ_2. The curves shown are field lines of the electric displacement, $\vec{D}$. Which of the following statements is true?



(a) $\epsilon_2 > \epsilon_1$ and the sign of the net surface charge at the interface is negative

(b) $\epsilon_1 > \epsilon_2$ and the sign of the net surface charge at the interface is positive

(c) $\epsilon_2 > \epsilon_1$ and the sign of the net surface charge at the interface is positive

(d) $\epsilon_1 > \epsilon_2$ and the sign of the net surface charge at the interface is negative

A6

A thin film of oil (refractive index, $n_{oil} = 1.40$) floats on a puddle of water (refractive index, $n_{water} = 1.33$) on a road. The film is illuminated at normal incidence from air by white light. In the light reflected vertically, green colour (with wavelength in air $\lambda_0 = 560$ nm) is enhanced. The smallest possible thickness of the oil film is approximately:

(a) 100 nm

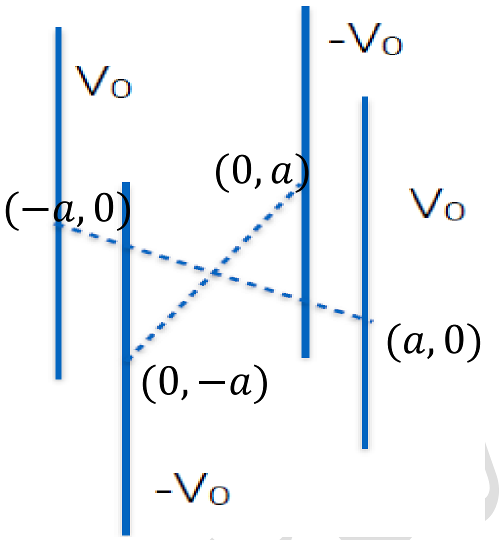
(b) 2000 nm

(c) 200 nm

(d) 360 nm

A7

Four infinitely long electrodes of negligible thickness are placed parallel to the z axis and pass through the points at the edges of a square in the (x, y) plane at $(\pm a, 0)$ and $(0, \pm a)$. The potential at $(0,0)$ is 0. If the electrodes at $(\pm a, 0)$ are maintained at a potential V_0, while the electrodes at $(0, \pm a)$ are maintained at a potential $-V_0$, the electric field in the vicinity of the electrodes is given by:



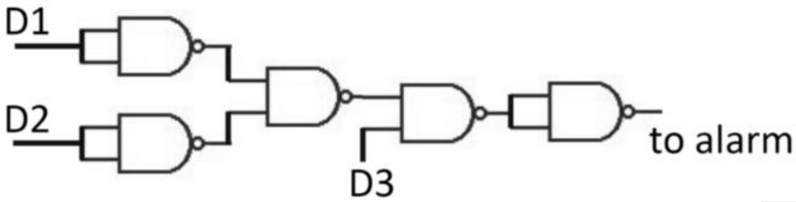
(a) $$\vec{E} = -\frac{2V_0}{a^2}(x\hat{x} - y\hat{y})$$

(b) $$\vec{E} = -\frac{V_0}{a}(\hat{x} - \hat{y})$$

(c) $$\vec{E} = \frac{2\pi V_0}{a^2}(\sin(\frac{2\pi x}{a})\,\hat{x} - \sin(\frac{2\pi y}{a})\,\hat{y})$$

(d) $$\vec{E} = -\frac{V_0}{x^2 + y^2}(\hat{x} - \hat{y})$$

A8 Three nearby laboratory rooms **1**, **2**, **3** have one smoke detector each labelled **D1**, **D2**, **D3** respectively, for fire safety. In case smoke is detected, the detector output goes to a logic state *True*. However, to prevent false alarm, these outputs are connected to the logic circuit below whose output activates a single alarm.



In which of the following cases will the alarm definitely ring:

(a) Smoke is detected in rooms 2 and 3

(b) Smoke is detected in rooms 1 and 2

(c) Smoke is detected in rooms 1 or 2

(d) Smoke is detected in any two of three rooms

A9

Find:

$$\lim_{x \to +\infty} x \log\left(\frac{x+1}{x-1}\right)$$

(a) 2

(b) 1

(c) 0

(d) The limit does not exist

A10

Scientists are conducting an underground experiment where two interactions, A and B can occur. In general, interaction A is twice as likely to occur compared to interaction B. Interaction A would give a signal in the detector with probability $1/8$, while interaction B would give a signal with probability $1/2$. If the detector registers a signal, what is the probability that the signal was due to interaction A?

(a) $\frac{1}{3}$

(b) $\frac{1}{5}$

(c) $\frac{1}{9}$

(d) $\frac{2}{3}$

A11

The equation of state of a gas is given by:

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT,$$

where v is the volume per mole and R, a, b are constants. The internal energy per mole of the gas is given by:

$$U(T, v) = \frac{3}{2}RT - \frac{a}{v}$$

Which of the following gives the adiabatic equation of state at fixed particle number for the gas?

(a) $(v - b)^2 T^3 = \text{constant}$

(b) $(v - b)^2 T^{-3} = \text{constant}$

(c) $(v - b)^2 T^{\frac{3}{2}} = \text{constant}$

(d) $(v - b)^2 T^{-\frac{3}{2}} = \text{constant}$

A12

Electrons in a metal are accelerated under a constant electric field $\vec{E}$ and experience a drag $-\eta\vec{v}$ due to the surrounding medium. If the density of electrons is n and the current is constant, what is the conductivity σ of the metal?

(a) $$\sigma = \frac{ne^2}{\eta}$$

(b) $$\sigma = \frac{ne^2E}{\eta}$$

(c) $$\sigma = \frac{ne^2}{m\eta}$$

(d) $$\sigma = \frac{neE}{m\eta}$$

A13

A thermistor measures the temperature by measuring the change of electric resistivity of a material. Which of the following types of materials are most suited for making thermistors to work in the temperature range -100 degree C to 300 degree C?

(a) Semiconductors

(b) Metals

(c) Superconductors

(d) Gas of atoms

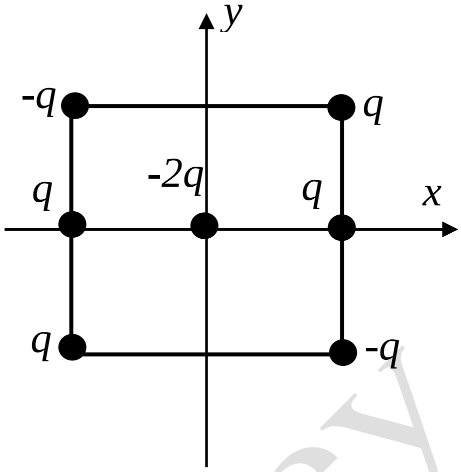
A14

An electron beam is accelerated over a potential V and strikes a crystal with lattice constant 5Å. What is the minimum V needed to resolve the crystal?

(a)	5 V
(b)	30 V
(c)	300 V
(d)	0.5 V

A15

The figure on the right, shows point charges arranged in the x, y plane on the centre, vertices, and some edges of a square with vertices at $(\pm1, \pm1, 0)$ in standard units. Given the charge distribution in the figure, the electric field far away from the charges ($r = \sqrt{x^2 + y^2 + z^2}$ denotes the distance from the origin) falls off as:



(a) $\frac{1}{r^4}$

(b) $\frac{1}{r^2}$

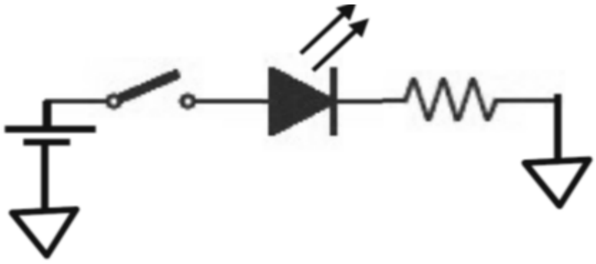
(c) $\frac{1}{r^3}$

(d) $\frac{1}{r^5}$

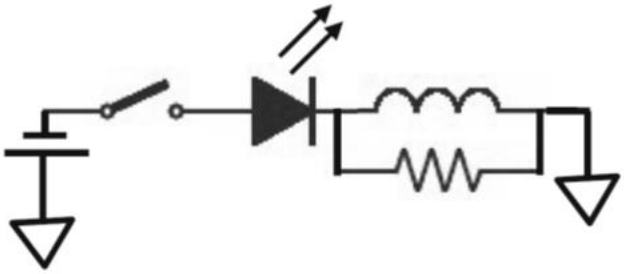
A16

Which of these circuits made with resistor, capacitor and inductor will be the best to get light out of the LED using a 9V battery, after the switch is closed?

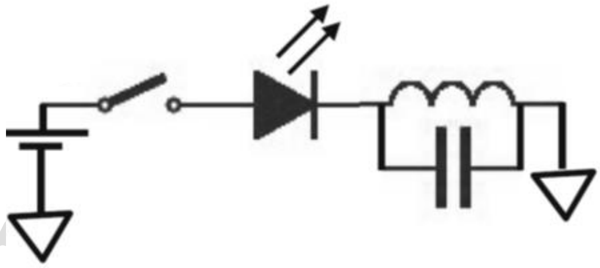
(a)



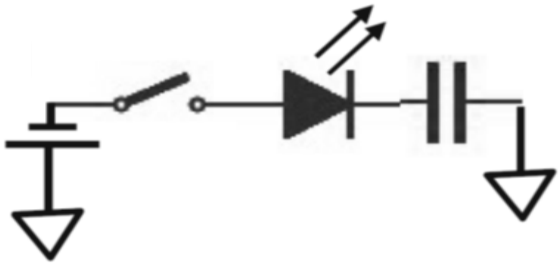
(b)



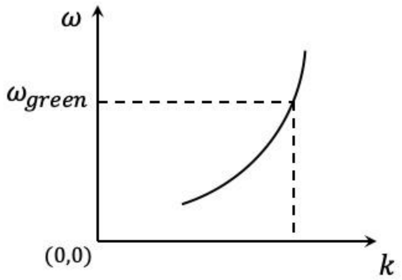
(c)



(d)



A17 For a medium, the dispersion relation for the propagation of light is shown below on a linear scale:



Here, ω_{green} is the frequency of the green light. Which of the following statements is correct?

(a) The phase velocity for the green light is smaller than the group velocity at that frequency

(b) The refractive index of the medium for the red light is smaller than that for the blue light

(c) The phase velocity of the green light is larger than the group velocity at that frequency

(d) The speed of light in this medium for green light is same as that in vacuum

A18

Hydrogen atom has a decay rate from $n = 2$ state to $n = 1$ state as about 6×10^8 decays per second. If the energy level expression for the hydrogen atom is

$$E_n = \frac{13.6}{n^2}\,\text{eV}$$

the uncertainty in the wavelength of this emission would be closest to

(a) 30 fm

(b) 0.1 fm

(c) 122 nm

(d) 19.4 nm

A19

A quantum particle of mass m in three-dimensions is governed by the Hamiltonian:

$$\hat{H} = \frac{\hat{\vec{p}}^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2 + z^2) + \lambda(x^6 + y^6)$$

with $\lambda > 0$. If $\psi_3(\vec{r})$ is the third excited state of the system, which of the following is correct?

(a) $|\psi_3(\vec{r})|^2 = |\psi_3(-\vec{r})|^2$

(b) $\psi_3(\vec{r})$ is an eigenstate of L^2

(c) $\psi_3(\vec{r})$ is an eigenstate of L_z

(d) $\psi_3(\vec{r})$ is an eigenstate of p_z

A20

Consider the series:

$$S = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

Which of the following statements is correct?

(a) S is convergent and is greater than 2

(b) S is convergent and is less than 2

(c) S is convergent and is equal to 2

(d) S is not convergent

A21 A relativistic particle of mass m moving under the central force of gravity with angular momentum L around a massive body of mass M experiences the following potential:

$$V(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{m\,c^2r^3}$$

where the last term is the relativistic correction to the Newtonian formula. For sufficiently large L, the particle has:

(a) Two circular orbits. The smaller one is unstable and the larger is stable

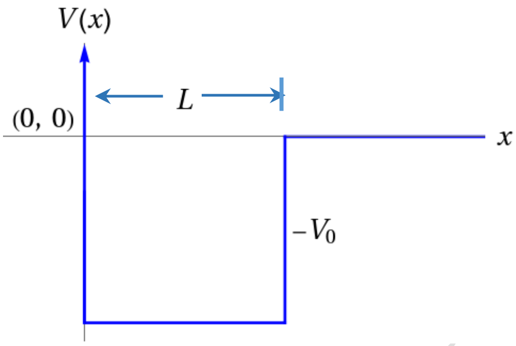
(b) Two circular orbits. The smaller one is stable and the larger is unstable

(c) Three circular orbits. The middle one is unstable and the others stable

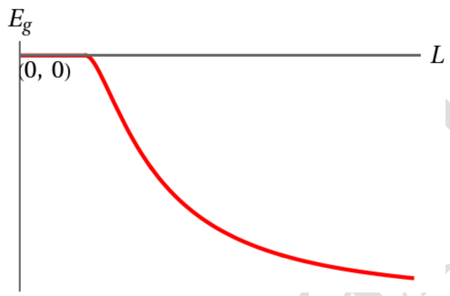
(d) Three circular orbits. The middle one is stable and the others unstable

A22

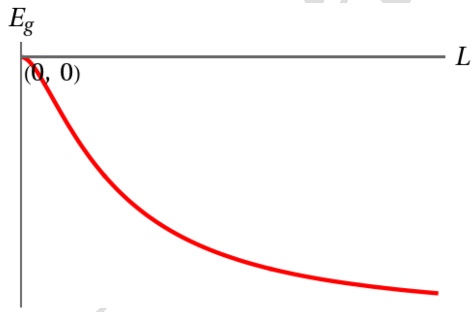
Consider a quantum particle in a one-dimensional potential of the shape shown, with the well of width L, and $V(x<0) \to \infty$. The potential in the well is $-V_0$ and the potential for $x \in (L, \infty)$ is 0. Which plot describes the change in the ground state energy (E_g) as L is changed keeping V_0 constant?



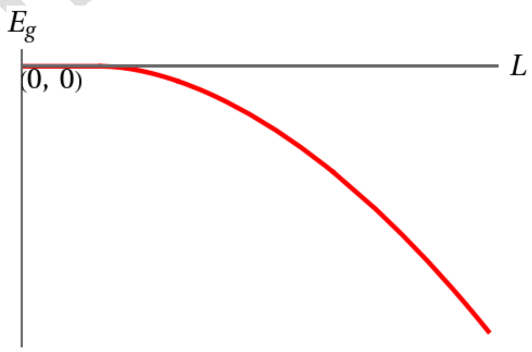
(a)



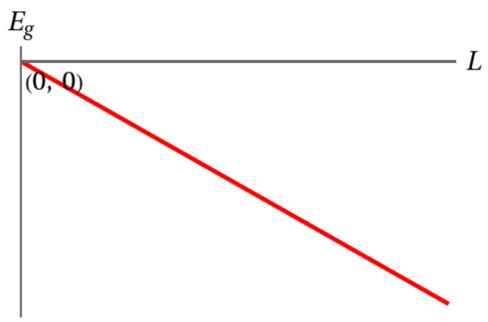
(b)



(c)



(d)



A23

A non-interacting, classical gas is made up of N Nitrogen molecules. The specific heat at constant volume is C_v. $C_v/(Nk_BT)$ is given by:

(Ignore rotations about the axis of symmetry.)

(a) $3/2$ for some very low T, $5/2$ for temperatures around the room temperature, $7/2$ for some high temperatures

(b) $7/2$ for some very low T, $5/2$ for temperatures around the room temperature, $3/2$ for some high temperatures

(c) $3/2$ for some very low T, 2 for temperatures around the room temperature, $5/2$ for some high temperatures

(d) $3/2$ for some very low T, 2 for temperatures around the room temperature, 3 for some high temperatures

A24

The longitudinal disturbance generated by an earthquake, travels through the earth's crust and reaches 1000 km in 3 mins from the epicentre of the earthquake. Assuming the density of the Earth's crust is 2700 kg/m^3, the Bulk's modulus of the crust is closest to:

(Ignore the shear modulus and local variations in the density and the Bulk modulus.)

(a) 8.3×10^{10} N m^{-2}

(b) 2.1×10^{11} N m^{-2}

(c) 8.3×10^{8} N m^{-2}

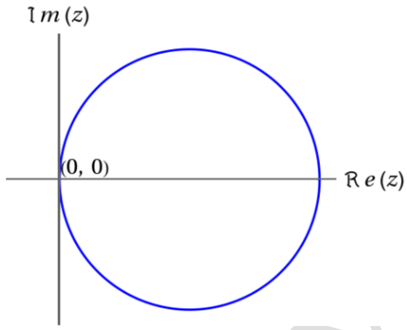
(d) 2.1×10^{9} N m^{-2}

A25

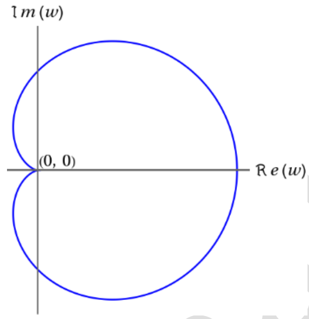
An analytic transformation:

$$w = z^2$$

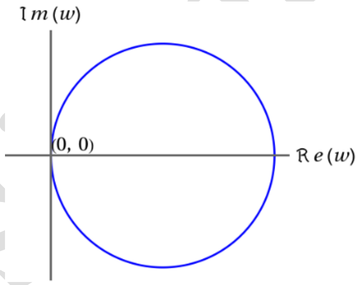
is applied on the complex plane. Consider the circle C in z as shown on the right. Which of the following represents the image of C in the w plane?



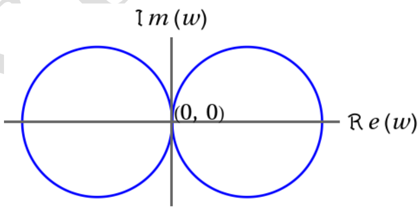
(a)



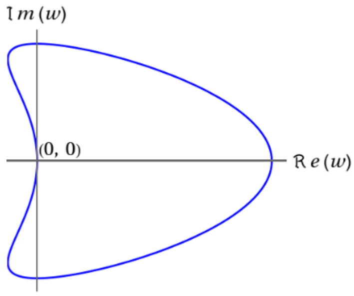
(b)



(c)



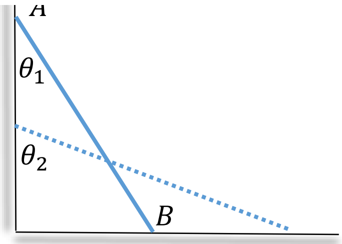
(d)



Section B

B1

Consider a uniform rod with length L and mass m with one end A on the wall and the other end B on the ground. Initially it is at rest at an angle θ_1 with the wall and starts slipping. What is the speed of A when the ladder's angle with the wall is θ_2? Ignore friction.



(FYI: The moment of inertia of a uniform rod about its centre is $I = \frac{1}{12}mL^2$.)

(a) $\sin\theta_2\sqrt{3gL(\cos\theta_1 - \cos\theta_2)}$

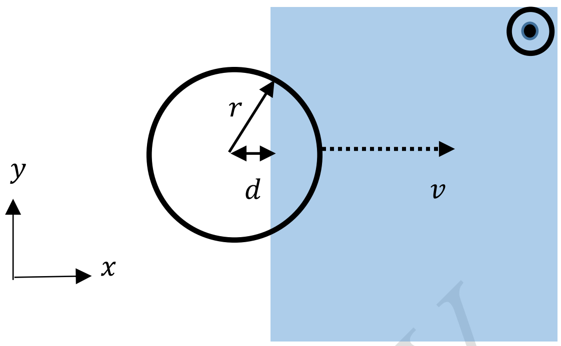
(b) $\sqrt{3gL(\cos\theta_1 - \cos\theta_2)}$

(c) $\sqrt{12gL(\cos\theta_1 - \cos\theta_2)}$

(d) $\sin\theta_2\sqrt{6gL(\cos\theta_1 - \cos\theta_2)}$

B2

A metallic ring of radius r is pushed into a region of uniform magnetic field $B\hat{z}$ (shaded region) with a constant speed v as shown. The magnetic field is 0 outside. The electrical resistance of the loop is R. What is the external force one needs to apply on the loop to maintain its speed, when the distance of the region's edge to the centre is d (>0)?



(a) $$\frac{4r^2B^2v}{R}\left(1-\frac{d^2}{r^2}\right)\hat{x}$$

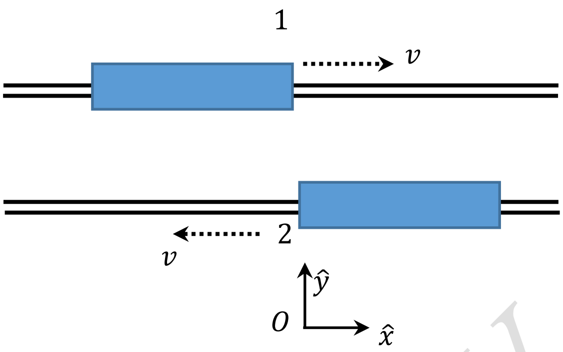
(b) $$\frac{\pi\, r^2B^2v}{R}\left(1+\frac{d^2}{r^2}\right)\hat{x}$$

(c) $$\frac{r^2B^2v}{R}\left(1+\frac{d}{r}\right)\hat{x}$$

(d) $$\frac{\pi\, r^2B^2v}{R}\left(1-\frac{d}{r}\right)\hat{x}$$

B3

Two trains of proper length L each move on neighbouring tracks. An observer at rest with the tracks, O, notes the speed of train 1 to be $v\hat{x}$ and train 2 to be $-v\hat{x}$. O also notes that at $t = 0$ the front ends of the two trains have the same x position, $x = 0$. According to a train engineer sitting at the front of train 1, what is the time interval between the passing by of the front and the back of train 2?



(a) $\frac{L}{2v}(1 - v^2/c^2)$

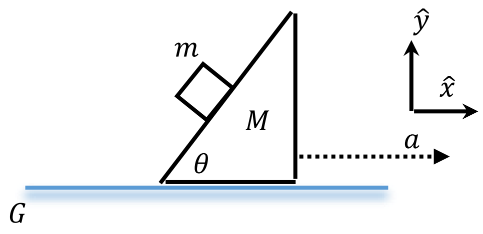
(b) $\frac{L}{2v}\sqrt{1 - v^2/c^2}$

(c) $\frac{L}{2v}\sqrt{\frac{1 - v^2/c^2}{1 + v^2/c^2}}$

(d) $\frac{L}{2v}$

B4

A mass m is placed on a wedge of mass M and angle θ as shown, which in turn is placed on the horizontal ground. Initially, m and M are at rest with respect to the ground and are released from this position. What is the acceleration a of M? Assume that all surfaces are frictionless.



(a) $$a = \frac{mg\cos\theta\ \sin\theta}{M + m\sin^2\theta}$$

(b) $$a = \frac{mg\cos\theta}{M + m}$$

(c) $$a = \frac{mg\sin\theta}{M + m}$$

(d) $$a = 0$$

B5 Scientists have discovered a new type of particle, which can occupy a quantum state with 0, 1 or 2 particles. If a quantum state has energy ω, the average occupation of this state in a system at temperature T and chemical potential μ is given by

(a) $$\frac{2 + e^{\frac{(\omega-\mu)}{T}}}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}}$$

(b) $$\frac{2}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}}$$

(c) $$\frac{1 + 2e^{\frac{(\omega-\mu)}{T}}}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}}$$

(d) $$\frac{1}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}}$$

B6 A cycle for an ideal gas consists of the following three reversible processes:

i. Isothermal expansion at T_H from V_1 to V_2.

ii. Isochoric (constant volume) cooling from T_H to T_C.

iii. Adiabatic compression from T_C and V_2 back to the initial state (T_H, V_1).

If C_V is the constant volume heat capacity of the gas, the efficiency η of this cycle is:

(a)
$$\eta = 1 - \frac{C_V\,(T_H - T_C)}{RT_H\,\log(V_2/V_1)}$$

(b)
$$\eta = 1 - \frac{T_C}{T_H}$$

(c)
$$\eta = 1 - \frac{C_V(T_H - T_C)}{C_P T_H}$$

(d)
$$\eta = 1 - \frac{C_V\,T_C}{R\,T_H\,\log(V_2/V_1)}$$

B7

Consider the function:

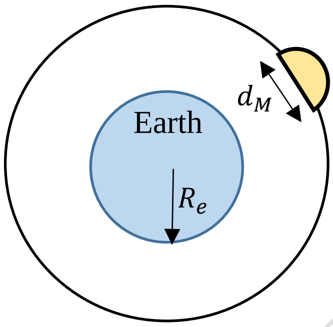
$$f(x) = \frac{4x - 11x^3}{(x^2 + 1)^2}$$

In the domain $x \in [1,4]$. What is the difference between the maximum value of the function and its minimum value? (Select the closest answer.)

(a)	1.45
(b)	3.2
(c)	3.96
(d)	0.82

B8

An earth imaging satellite is in a near earth circular orbit (see figure) with an orbital period of 100 minutes and observes earth with a light of wavelength 550 nm. If it can resolve the ground with 0.5 m, the minimum diameter of the mirror on the satellite should be closest to:



(Assume the system is diffraction limited. The mass and radius of Earth are 5.97×10^{24} kg and 6.4×10^{3} km, respectively.)

(Figure not to scale)

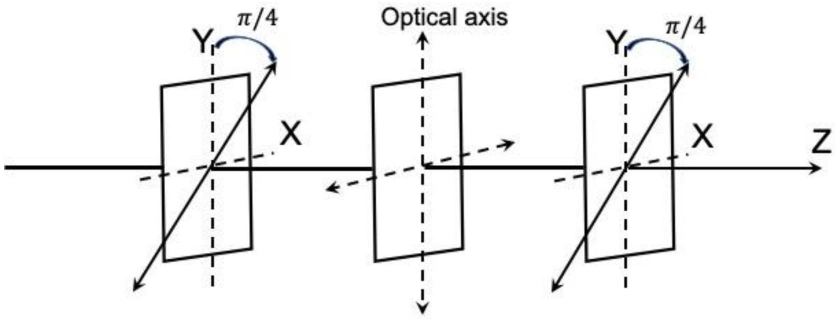
(a) 1 m

(b) 5 m

(c) 0.5 m

(d) 10 m

B9 In the experimental arrangement below, a plane polarised light of intensity I and wavelength 500 nm is propagating along the z-axis. It is polarised along the y-axis and is incident on the first polarizer which has the transmission axis at an angle of $\pi/4$ wrt the y-axis, as shown.



Subsequently, the beam passes through a flat birefringent crystal of thickness 950 μm and optical axis along the y-axis. The crystal is kept normal to the z-axis. If the refractive indices of the crystal are $n_o = 1.514$ and $n_e = 1.519$, the beam intensity measured after the second polarizer whose transmission axis is parallel to that of the first one would be:

(a) 0

(b) $I/4$

(c) $I/2$

(d) I

B10

The curve that solves $\frac{dy}{dx} = \frac{2y^3 - x^3}{3xy^2}$ and passes through the point $(1,0)$ is given by:

(a) $$y^3 + x^3 - x^2 = 0$$

(b) $$2y^3 - x^3 + x = 0$$

(c) $$y^3 + x^3 - x^2 - y^2 = 0$$

(d) $$2y^3 - x^3 + x + y = 0$$

B11

The vector field, $E = (2xy^2 + z^3, 2x^2y, 3xz^2)$ V/m, gives the electric field in a region where (x, y, z) are the cartesian coordinates, given in metres. The difference in electric potential between the two points with coordinates $(x, y, z) = (1, 1, 1)$ and $(x, y, z) = (1, 2, 3)$, is given by:

(a) -29 V

(b) -16 V

(c) 0 V

(d) An electric potential cannot be defined for this electric field configuration

B12

Reactions in a closed container convert one molecule of A to one molecule of B at a rate Γ_1 and one molecule B to one molecule of C at a rate Γ_2. At time $t = 0$, there are N_0 number of A molecules and no B or C molecules in the container. At what time is the number of B molecules in the container the maximum?

(a) $$\frac{\log(\Gamma_2/\Gamma_1)}{\Gamma_2 - \Gamma_1}$$

(b) $$\frac{\log(\Gamma_1/\Gamma_2)}{\Gamma_2 - \Gamma_1}$$

(c) $$\frac{\log(\Gamma_2)}{\Gamma_2} + \frac{\log(\Gamma_1)}{\Gamma_1}$$

(d) $$\frac{\log(\Gamma_2)}{\Gamma_2} - \frac{\log(\Gamma_1)}{\Gamma_1}$$

B13

Consider a quantum particle of mass m in one-dimension with a Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \lambda \hat{x}^8$$

with $\lambda > 0$. The ground state energy of the particle scales with λ as:

(a) $\propto \lambda^{1/5}$

(b) $\propto \lambda^{1/4}$

(c) $\propto \lambda^{1/3}$

(d) $\propto \lambda^{1/6}$

B14

A one-dimensional quantum mechanical wave function is given by:

$$\psi(x) = Ne^{-|x-b|/a},$$

where $x \in (-\infty, \infty)$ and N is a normalization factor. What is the expectation value of the kinetic energy in this state?

(a) $$\frac{\hbar^2}{2ma^2}$$

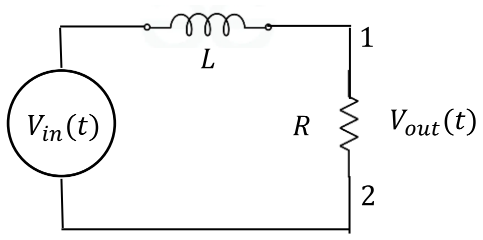
(b) $$\frac{2\hbar^2}{ma^2}$$

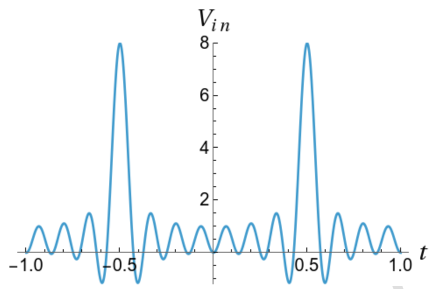
(c) $$\frac{\hbar^2}{2ma^2} + \frac{\hbar^2}{2mb^2}$$

(d) $$\frac{2\hbar^2}{mb^2}$$

B15

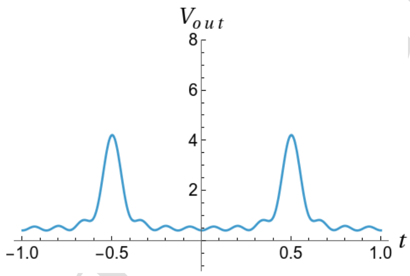
The figure below shows a circuit. The input waveform $V_{in}(t)$ is shown. $R = 5\Omega$ and $L = 0.25$ V.s/A.



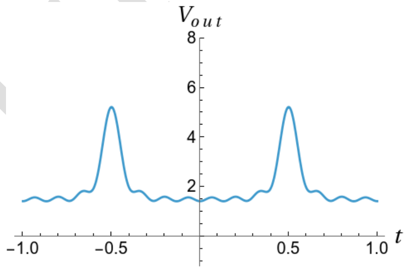


An ideal oscilloscope is connected between points 1 and 2, and shows a signal $V_{out}(t)$. What is the closest representation of $V_{out}(t)$?

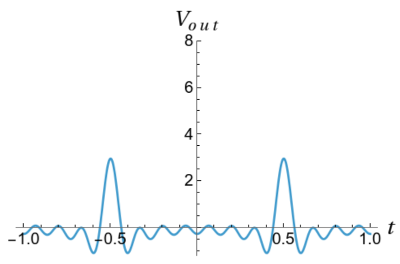
(a)



(b)



(c)



(d)