

Section A: Q.1 – Q.10 Carry ONE mark each.

Q1. Light of wavelength 632 nm is passing through an optically active medium of thickness 20 cm . The optical rotation exhibited by the medium is 18° .

Which of the following options correctly states the magnitude of the difference in refractive indices corresponding to the left and the right circularly polarized light?

- (a) 1.81×10^5 (b) 3.16×10^7 (c) 3.62×10^5 (d) 6.32×10^7

Q2. Consider an n -type silicon in which the fully ionized dopant concentration is 10^{17} cm^{-3} . The intrinsic electron density is $1.5 \times 10^{10} \text{ cm}^{-3}$. Which of the following options correctly states the equilibrium hole concentration in cm^{-3} ?

- (a) 2.25×10^3 (b) 1.55×10^3 (c) 3.01×10^3 (d) 4.52×10^3

Q3. Consider the superposition of two electromagnetic waves with their electric field vectors given by $\vec{E}_1(z, t) = \hat{i}A_1 \cos(kz - \omega t)$ and $\vec{E}_2(z, t) = \hat{j}A_2 \sin(kz - \omega t + \phi)$, where A_1 and A_2 are the amplitudes, k is the wavenumber, ω is the angular frequency, and ϕ is the relative phase. Which of the following options represents a resultant elliptically polarized wave with its semi-major axis either along \hat{i} or \hat{j} ?

- (a) $\phi = 0$ and $A_1 \neq A_2$ (b) $\phi = \frac{\pi}{2}$ and $A_1 = A_2$
(c) $\phi = \frac{\pi}{2}$ and $A_1 \neq A_2$ (d) $\phi = 0$ and $A_1 = A_2$

Q4. Which of the following options represents the simplified form of the Boolean equation $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$?

- (a) $A\bar{B}$ (b) $\bar{A}B\bar{C}$ (c) $\bar{B}C$ (d) \bar{C}

Q5. A quantum particle is confined in a one-dimensional space of $0 \leq x \leq 2$.

Consider a normalized wavefunction of the particle as

$$\psi(x) = \sqrt{p/5}[1 + \cos(\pi x/2)]\sin(\pi x/2).$$

Which of the following options gives the correct value of p ?

- (a) 2 (b) 3 (c) 4 (d) 1

Q6. Consider the normalized superposed state $\psi = c_0\phi_0 + c_1\phi_1$, where ϕ_0 and ϕ_1 are the ground and first excited states of a simple harmonic oscillator, respectively. c_0 and c_1 are imaginary superposition coefficients. Which of the following options is correct for the expectation value of $(\langle x \rangle + i\langle p \rangle)$?

- (a) Imaginary
- (b) Real
- (c) Zero
- (d) Complex with non-zero real and imaginary parts

Q7. The potential of a quantum harmonic oscillator is modified from $\frac{1}{2}kx^2$ to $\frac{1}{2}kx^2 + 3ax$, where k and a are constants and x is the position variable. When, the values of $a = 2$ and $k = 1$, which of the following options gives the change in the ground state energy?

(a and k are in appropriate units)

- (a) -3
- (b) -6
- (c) -12
- (d) -18

Q8. Consider the following linear second order differential equation

$$\frac{d^2y}{dt^2} + \omega^2y = 0,$$

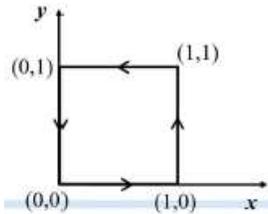
where ω is a positive constant. The boundary conditions are $\left. \frac{dy}{dt} \right|_{t=0} = 1$, and $y(t = 0) = \frac{1}{2}$.

Which of the following options gives the value of $y\left(t = \frac{\pi}{2\omega}\right)$?

- (a) $\frac{1}{\omega}$
- (b) $\frac{2\pi}{\omega}$
- (c) 0
- (d) 1

Q9. Consider the electrostatic potential in two dimensions $V(x, y) = x^8y^9$. What is the line integral of the corresponding electric field along the path shown in the figure?

- (a) 0
- (b) 1
- (c) 2
- (d) $\frac{1}{2}$



Q10. Consider the matrix

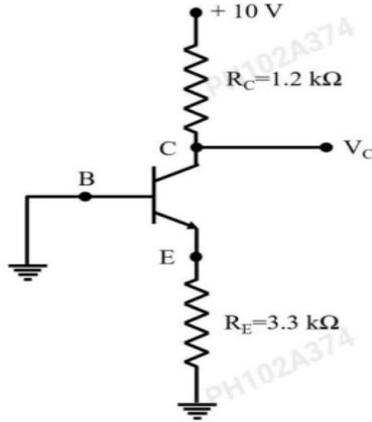
$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

The value of $\det(A^{-1})$ is:

- (a) 4
- (b) 1
- (c) $\frac{1}{4}$
- (d) 0

Section A: Q.11 – Q.30 Carry TWO marks each.

Q11. Consider the circuit of the figure, the voltage V_c , in Volts, is:



- (a) 0 (b) 1.25 (c) 9.8 (d) 10

Q12. A particle of mass m moves in a potential given by

$$V(x, y, z) = -k \frac{y}{(x^2 + y^2 + z^2)}$$

where k is a constant.

Which of the following statements is correct?

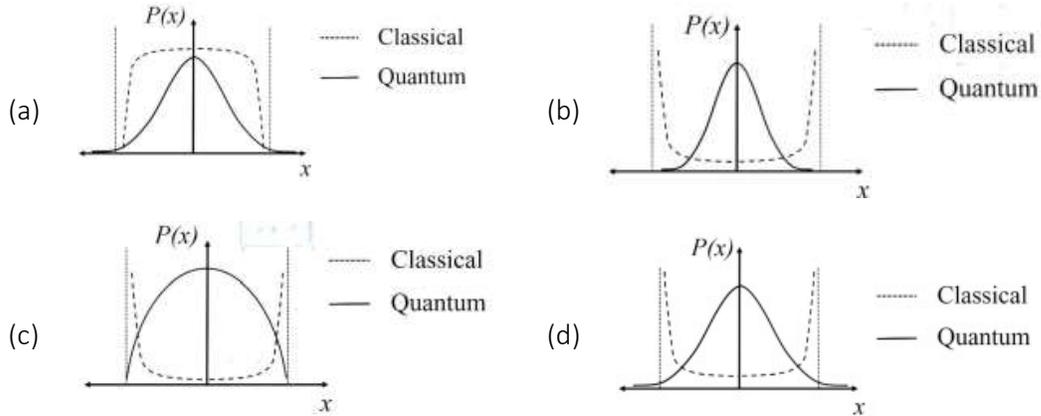
- (a) The force corresponding to the potential is central
- (b) Angular momentum of the system is not conserved
- (c) Linear momentum along the y -direction is conserved
- (d) Energy of the system is not conserved

Q13. (\vec{E}, \vec{B}) are the electric and magnetic fields in a rest frame. (\vec{E}', \vec{B}') represent the corresponding quantities in a reference frame moving with a constant velocity \vec{v}_0 . Using the invariance of Lorentz force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ under Galilean transformations, identify the correct relation.

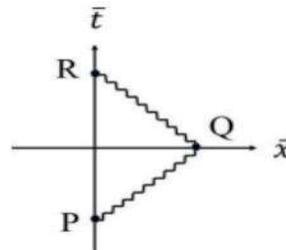
- (a) $\vec{E}' = \vec{E}$ and $\vec{B}' = \vec{B}$
- (b) $\vec{E}' = \vec{E} + \vec{v}_0 \times \vec{B}$ and $\vec{B}' = \vec{B}$
- (c) $\vec{E}' = \vec{E}$ and $\vec{v} \times \vec{B}' = (\vec{v} + \vec{v}_0) \times \vec{B}$
- (d) $\vec{E}' = \vec{v}_0 \times \vec{B}$ and $\vec{v} \times \vec{B}' = \vec{E} + \vec{v} \times \vec{B}$

Q14. Which ONE of the following figures correctly represents the probability density, $P(x)$, of a particle undergoing simple harmonic oscillation as a function of position x ?

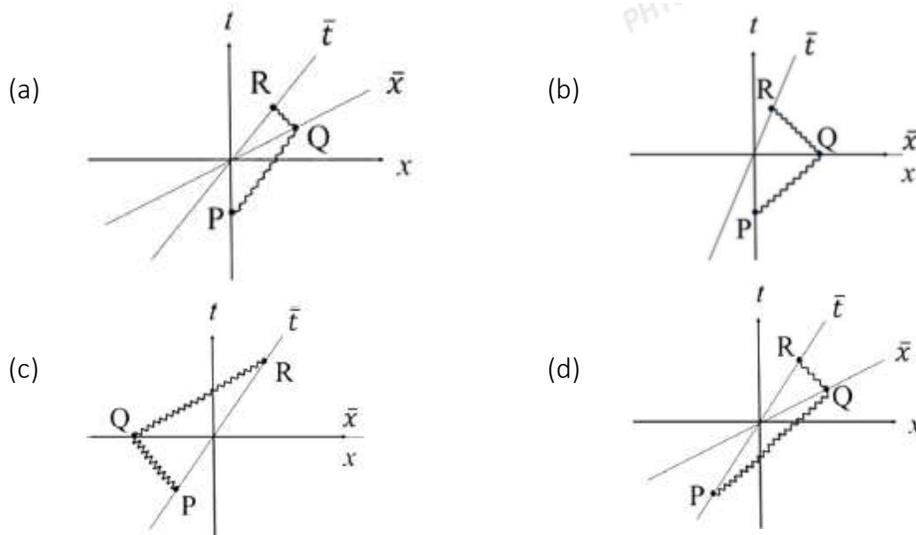
The dashed line is for the classical case. The solid line is for the quantum case, where the system is in its ground state. The dotted vertical lines on the x axis denote the classical turning points.



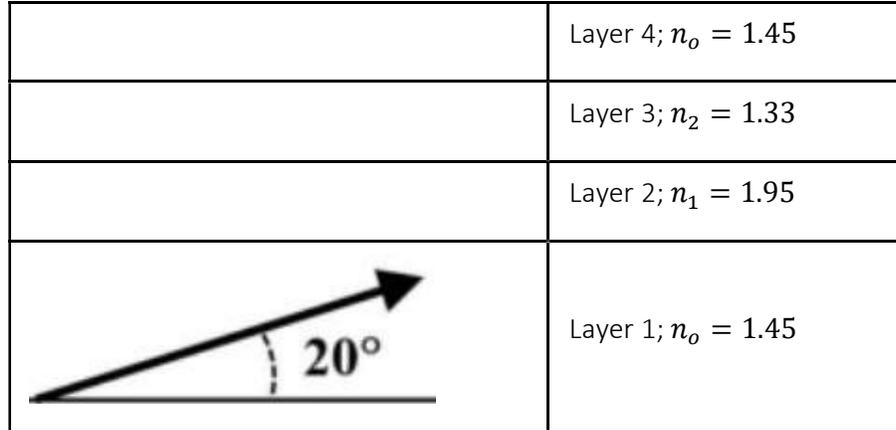
Q15. An observer $\bar{O}(\bar{t}, \bar{x})$ moves with a constant velocity in the positive x -direction relative to an observer $O(t, x)$ at rest. In the frame of reference of $\bar{O}(\bar{t}, \bar{x})$, a light-ray emitted at a point P at some time reaches the \bar{x} -axis at the point Q. Then, on reflection it arrives at point R, as shown in the figure.



Which of the following options represents these events as observed by $O(t, x)$?



Q16. Consider a multilayered structure composed of thin films of refractive indices $n_0, n_1,$ and n_2 as shown in the figure. A ray traveling in the first layer hits the interface at an angle of 20° with the horizontal. Which of the following options is correct?

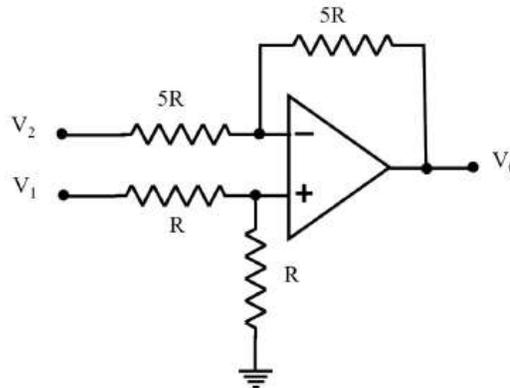


- (a) The ray emerges at an angle of 20° with the horizontal in the 4th layer
- (b) The ray emerges at an angle of 56° with the horizontal in the 4th layer
- (c) The ray emerges at an angle of 44° with the horizontal in the 4th layer
- (d) The ray would not enter the 3rd layer

Q17. Consider a BCC lattice with one atom per lattice point. The maximum packing fraction is close to:

- (a) 53 %
- (b) 68 %
- (c) 74 %
- (d) 81%

Q18. The output voltage V_0 for the circuit shown in the figure is:



- (a) $(V_1 - V_2)$
- (b) $(V_1 - 2V_2)$
- (c) $(V_2 - V_1)$
- (d) $2(V_1 - V_2)$

Q19. A quantum particle of mass 10^{-20} kg is confined within a length of 1 nm in one-dimension. The minimum uncertainty in the measurement of velocity of the particle, in units of $\mu\text{m/s}$, rounded off to the nearest integer is:

[Assume the minimum uncertainty product $\Delta x \Delta p_x \approx \frac{\hbar}{2}$, use Planck's

Constant $h = 6.64 \times 10^{-34}$ Js]

- (a) 2 (b) 5 (c) 10 (d) 1

Q20. A thin ring of mass m and radius R has a total charge Q distributed uniformly. The ring is rotating with a constant angular velocity ω about an axis passing through its center and perpendicular to its plane.

Which of the following options is correct?

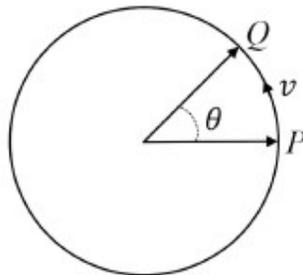
- (a) The magnetic moment of the ring is $\frac{Q\omega R^2}{4}$
 (b) The ratio of the magnetic moment to angular momentum is $\frac{Q}{2m}$
 (c) The magnetic moment of the ring is $\frac{Q\omega R^2}{6}$
 (d) The ratio of the magnetic moment to angular momentum is $\frac{Q}{m}$

Q21. A mass attached to the bottom end of a vertical massless spring stretches the spring by Δx . The system executes oscillation with a time period $T = 0.2\text{s}$. The value of Δx , in cm , rounded off to the nearest integer is:

[Assume the acceleration due to gravity $g = 9.8 \text{ m/s}^2$]

- (a) 2 (b) 3 (c) 4 (d) 1

Q22. A particle is rotating along a circular path with uniform speed v , as shown in the figure. While moving from the point P to Q subtending an angle θ , the magnitude of the change in its velocity is:



- (a) Zero (b) $v \cos \theta$ (c) $2v \sin \frac{\theta}{2}$ (d) $v \cos \frac{\theta}{2}$

Q23. A series LCR circuit contains $L = 175mH$, $C = 62.5\mu F$, and $R = 40\Omega$ and is connected to a source of voltage amplitude $E_0 = 50 V$ and angular frequency $\omega = 400rad/s$.

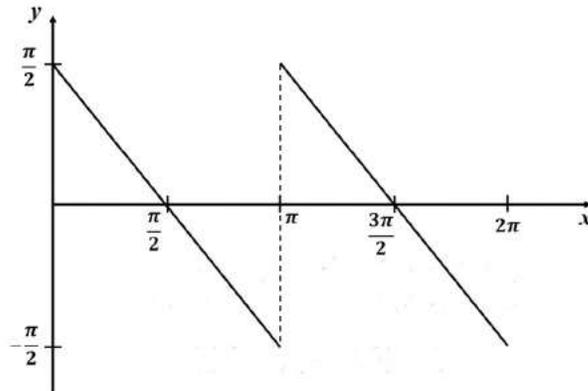
Which of the following statements is correct?

- (a) The magnitude of the maximum voltage across the inductor is less than 50 V
- (b) The magnitude of the maximum voltage across the capacitor is more than 60 V
- (c) The magnitude of the maximum voltage across the resistor is 50 V
- (d) The magnitude of the maximum voltage across the capacitor equals the maximum voltage across the resistor

Q24. A piecewise regular function $f(x)$ is shown in the figure. It is expanded in Fourier series given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx),$$

where a_0, a_n, b_n 's are the Fourier coefficients.



Which of the following options is correct?

- (a) $a_0 = 3\pi$
- (b) All a_n are zero
- (c) All b_n are zero
- (d) $a_0 = \frac{3\pi}{2}$

Q25. Using Taylor series, expand $f(x) = x^3 - \frac{1}{8}$ around $x_0 = 1$ up to second order in x . The coefficient of x is:

- (a) -2
- (b) 3
- (c) 2
- (d) -3

Q26. Consider a 2×2 matrix A . The determinant of A is -1 and the $\text{trace}(A) = 1$. Which of the following options gives the eigenvalues of A ?

- (a) 1,0
- (b) $\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$
- (c) $\frac{1+\sqrt{5}}{4}, \frac{1-\sqrt{5}}{4}$
- (d) $\frac{\sqrt{5}}{2}, 1 - \frac{\sqrt{5}}{2}$

Section B: Q.31 – Q.40 Carry TWO marks each.

Q31. The displacement of a vibrating string of finite length stretched along the x -axis is given by

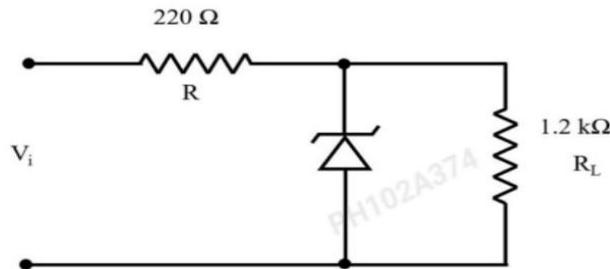
$$y(x, t) = 2A \cos(kx) \sin(\omega t)$$

where, A is the amplitude, $k = 2\pi/\lambda$ is the wavenumber and ω is the angular frequency.

Which of the following statements is/are correct for the standing wave?

- (a) The nodes are at $x = n\frac{\lambda}{2}$, where n is an integer
- (b) The antinodes are at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$, where n is an integer
- (c) The nodes are at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$, where n is an integer
- (d) The antinodes are at $x = n\frac{\lambda}{2}$, where n is an integer

Q32. In the circuit shown in the figure, the Zener voltage V_z is 20 V and the maximum Zener current I_{zM} is 60 mA .



For what value(s) of the input voltage V_i , the Zener diode is in ON state?

- (a) 20 V
- (b) 25 V
- (c) 35 V
- (d) 40 V

Q33. Consider a particle of mass m in a rotating frame. The force acting on the particle is expressed as $\vec{F} = (F_1 + F_2)\hat{e}_r + (F_3 + F_4)\hat{e}_\theta$, where \hat{e}_r and \hat{e}_θ are the radial and angular unit vectors, respectively, and $F_1 = m\ddot{r}$, $F_2 = -mr\dot{\theta}^2$, $F_3 = mr\ddot{\theta}$, $F_4 = 2m\dot{r}\dot{\theta}$.

Which of the following statements is/are correct?

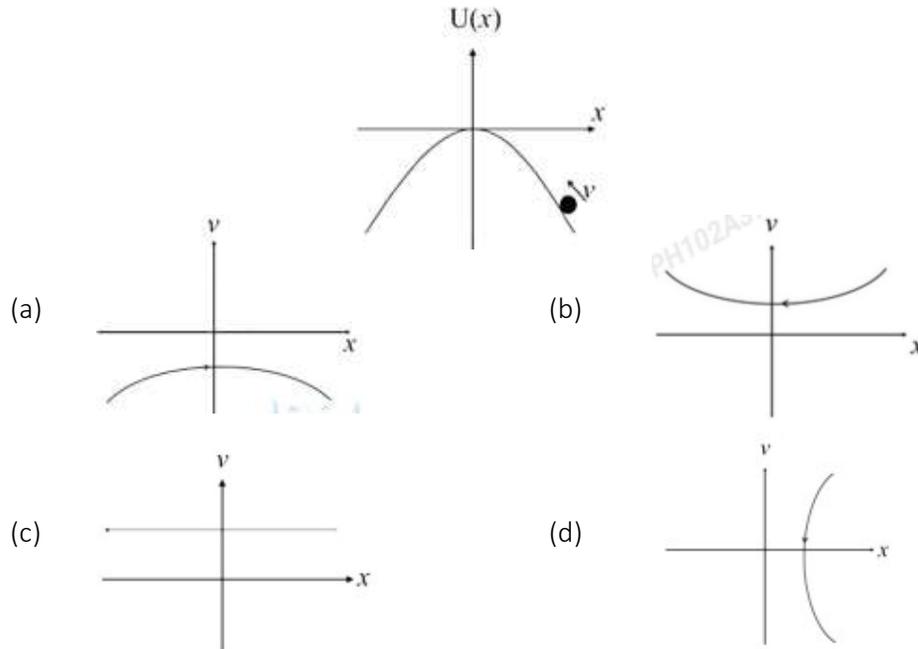
- (a) \hat{e}_r and \hat{e}_θ are not constant unit vectors
- (b) F_1 and F_3 are fictitious forces
- (c) F_2 and F_4 are fictitious forces
- (d) F_1 and F_2 are zero in uniform circular motion

Q34. The events are represented by coordinates (ct, x, y, z) in some frame of reference. Which of the following statements is/are correct?

[c is the speed of light]

- (a) Events $(1,0,-10,1)$ and $(-1,1,-9,1)$ are space-like separated
- (b) Events $(-1,0,-9,1)$ and $(1,1,-10,1)$ are space-like separated
- (c) Events $(-10,0,1,-1)$ and $(-9,1,-1,-1)$ are light-like separated
- (d) Events $(9,-1,1,-1)$ and $(-10,1,0,-1)$ are time-like separated

Q35. A ball of mass m climbs the potential $U(x) = -\frac{1}{2}kx^2$, as shown in the figure. Assuming that the total energy of the system $E = \frac{1}{2}mv^2 + U(x)$ is conserved, which of the following correctly describe(s) the plot of velocity (v) as a function of position (x) of the system for $E < 0$?



Q36. An LC oscillator circuit contains a capacitor of $4.0\mu\text{ F}$. The maximum potential difference across the capacitor is 2 V and the maximum current through the inductor is 80 mA .

Which of the following statements is/are correct?

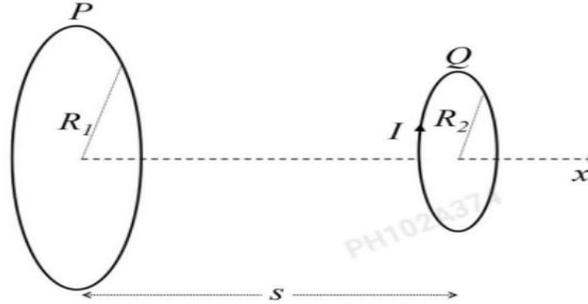
- (a) The value of inductance is 2.5 mH
- (b) The frequency of oscillator is 3.2 kHz
- (c) The time for the charge in the capacitor to rise from zero to the maximum is nearly 0.157 ms
- (d) The maximum potential difference across the inductor is 2.0 V

Q37. Consider an ideal gas of entropy S , molar specific heat C_v , pressure P and volume V .

Which of the following options is/are true?

- (a) Internal energy of the gas depends only on the temperature
- (b) $S \propto \ln V$, assuming C_v is constant
- (c) $S = 0$
- (d) Internal energy of the gas depends both on temperature and pressure

Q38. Two small circular copper loops P and Q of radii R_1 and R_2 , respectively, are coaxially placed along the x -axis as shown in the figure. The loops are at a distance s apart, where $s \gg R_1, R_2$. The loop Q carrying a steady current I is moving away along x -axis with a speed v .



Which of the following statements is/are correct?

- (a) The magnetic field at the center of P is $\propto \frac{1}{s^2}$
- (b) The magnetic flux through the loop P is $\propto \frac{1}{s^2}$
- (c) The emf induced in the loop P is $\propto \frac{1}{s^4}$
- (d) The emf induced in the loop P is $\propto v^2$

Q39. A charge Q is distributed uniformly on the surface of a sphere of radius R . It is placed inside a concentric conducting hollow sphere of radius $2R$. The outer sphere is earthed.

Which of the following statements is/are correct?

- (a) The charge on the inner surface of the outer sphere is $-Q$
- (b) The flux through a closed surface through the material of the outer sphere is $\frac{Q}{\epsilon_0}$
- (c) The charge on the outer surface of the outer sphere is zero
- (d) The potential at a radial distance r between the two spheres, $R < r < 2R$, is $\frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

Q40. Which of the following statements is/are true for a first order phase transition?

[C_p is the molar heat capacity, T_c is the critical temperature and S is the entropy]

- (a) At the transition point $C_p \rightarrow \infty$
- (b) The derivative of the Gibbs function with respect to pressure changes continuously across the phase transition
- (c) The two thermodynamic states between which the transition takes place are distinct
- (d) Entropy changes discontinuously with temperature at T_c

Section C: Q.41 – Q.50 Carry ONE mark each.

Q41. Considering the diameter of the pupil of a human eye to be 2 mm , the angular resolution of the eye at a wavelength of 500 nm , in minute of arc, is _____.

(Rounded off to two decimal places)

Q42. Consider a 10 mW laser beam focused using a biconvex lens to a circular spot of area 10^{-10} m^2 . The magnitude of the electric field in the focal plane of the lens, in kV/m , is _____ . (Rounded off to one decimal place)

[Use permittivity of free space $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/(\text{Nm}^2)$, and speed of light $c = 3 \times 10^8 \text{ m/s}$]

Q43. An OP-AMP has differential gain of $A_d = 4000$, two input voltages $V_{i1} = 120\mu V$ and $V_{i2} = 80\mu V$, and $\text{CMRR} = 100$. The output voltage, in mV , is _____
(Answer in integer)

Q44. A particle of mass 10^{-2} kg is moving along a circular orbit of radius 1 nm . The speed of the particle corresponds to the average thermal energy at temperature 10^{-6} K . Assuming the Bohr's angular momentum quantization condition, the quantum number of the circular path of the particle is _____. (Answer in integer)

[Use $h = 6.64 \times 10^{-34} \text{ Js}$ and $k_B = 1.38 \times 10^{-23} \text{ J/K}$]

Q45. One mole of an ideal gas undergoes a reversible isothermal expansion from $V_i = 1.5 \times 10^{-5} \text{ m}^3$ to $V_f = 1.6 \times 10^{-5} \text{ m}^3$ at a temperature 273 K . The amount of heat transfer during the process is αR , where R is the gas constant.

The value of α is _____. (Rounded off to one decimal place)

Q46. The value of $(1 - i\sqrt{3})^3$ is ____ . (Answer in integer)

Q47. Two thermodynamic systems separated by diathermic wall have the equations of state $U_1 = \frac{3}{2}RN_1T_1$ and $U_2 = \frac{5}{2}RN_2T_2$, where R is the gas constant. N_1, N_2 and T_1, T_2 are the mole numbers and the temperature of the two systems, respectively. The composite system in equilibrium has the total energy $1.5 \times 10^3 \text{ Joule}$. If $N_1 = 3$ and $N_2 = 2$, then the internal energy U_1 of the system one is ____ . (Answer in integer)

Q48. Light of wavelength 500 nm is incident on the surface of Na metal for photoelectric emission. The corresponding threshold wavelength is 600 nm . The maximum kinetic energy of the emitted electron, in eV , is _____(Rounded off to two decimal places)

[Use Planck's constant $h = 6.625 \times 10^{-34}$ J s, speed of light $c = 3 \times 10^8$ m/s, charge of electron $e = 1.6 \times 10^{-19}$ C]

[Assume refractive index is independent of wavelength]

Q49. The first order Bragg peak for (100) plane of a material with simple cubic structure is measured using an X-ray of wavelength 1\AA . If the lattice constant is 5\AA then the Bragg peak is observed at an angle, in degrees, ____ .

(Rounded off to two decimal places)

Q50. Consider an ensemble of hydrogen gas. The temperature, in K , at which the rms speed of the hydrogen molecule is twice the rms speed of the molecule at 300 K is _____. (Answer in integer)

Section C: Q.51 – Q.60 Carry TWO marks each.

Q51. A particle of mass m undergoes periodic motion in one-dimension with its total energy given as $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{4}kx^4$, where k is a positive constant and $\dot{x} = \frac{dx}{dt}$. Assuming that E is conserved, the time period T has the relation $T \propto E^{-1/n}$. The value of n is ____ . (Answer in integer)

Q52. A spacecraft is placed 200 km above Earth in a circular orbit. The minimum change in the speed required to place the spacecraft in a parabolic orbit, in km/s, is ____ .

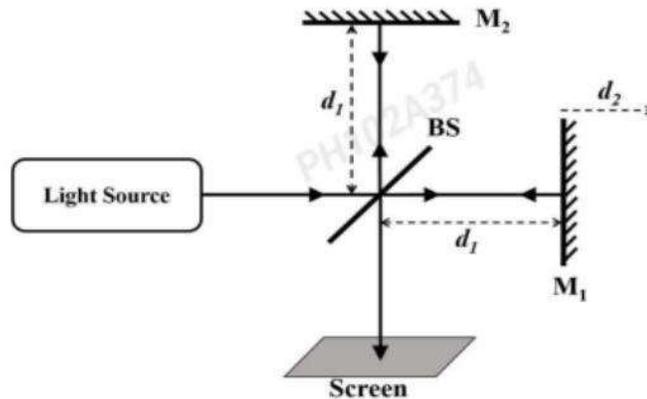
(Rounded off to one decimal place)

[Use $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$, mass of Earth = $6 \times 10^{24} \text{ kg}$, radius of Earth = 6400 km]

Q53. Consider a light source having a spectral linewidth of 10^{10} Hz , used in a Michelson interferometer. The mirrors M_1 and M_2 are equidistant from the beam-splitter of negligible thickness as shown in the figure. The minimum distance d_2 that the mirror M_1 is to be moved for the interference pattern to completely disappear, in cm, is ____ .

(Rounded off to one decimal place)

[Use speed of light to be $3 \times 10^8 \text{ m/s}$]



Q54. Muons are unstable relativistic particles created at high altitudes above the Earth, having a lifetime of $2.2 \times 10^{-6} \text{ s}$ in their rest frame. As measured by an observer on the ground, the minimum velocity the muon requires to travel a distance of 6000m is v . The value of v/c is _____ (Rounded off to three decimal places)

[Speed of light $c = 3 \times 10^8 \text{ m/s}$]

Q55. On the surface of a thin water film of refractive index 1.33, two light beams of wavelength $\lambda_1 = 0.64\mu\text{m}$ and $\lambda_2 = 0.40\mu\text{m}$ are incident at an angle of 30° . The light of wavelength λ_1 exhibits maximum reflection, but that of wavelength λ_2 is not reflected at all. The minimum thickness of the water film, in μm , is _____. (Rounded off to two decimal places)

[Assume refractive index is independent of wavelength]

Q56. An electron is confined in a one-dimensional box of width $L = 10\text{\AA}$. The electron in the first excited state de-excites to the ground state. The wavelength of the emitted radiation, in μm , is ____.

(Rounded off to one decimal place)

[Use the mass of the electron $m_e = 9.1 \times 10^{-31}\text{ kg}$, Planck's constant

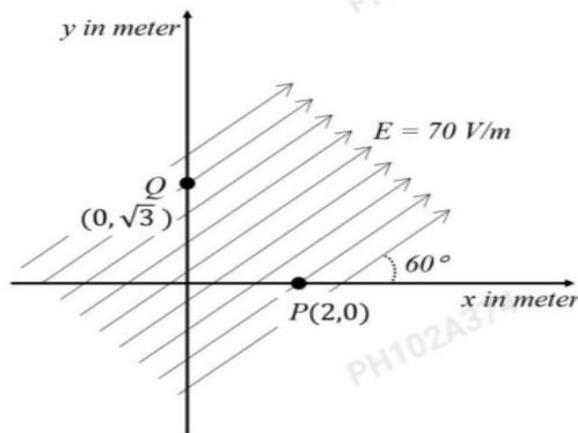
$h = 6.625 \times 10^{-34}\text{ Js}$, $c = 3 \times 10^8\text{ m/s}$]

Q57. An electron is accelerated through a potential of 200 V and then it passes through a slit of width 1.0 nm held normal to the path of the electron. Assuming the uncertainty relation $\Delta x \Delta p_x \approx \hbar/2$, maximum scattering angle of the electron after the slit is $\alpha \times 10^{-3}$ radian.

The value of α is _____. (Rounded off to nearest integer)

[Given $\hbar = 1.054 \times 10^{-34}\text{ Js}$]

Q58. A uniform electric field of 70 V/m makes an angle of 60° with the positive x axis, as shown in the figure. The potential difference between the points P and Q which are 2m and $\sqrt{3}\text{m}$ away from the origin, in Volts, is _____. (Rounded off to one decimal place)



Q59. Consider a simple pendulum of length l and time period T . In a laboratory experiment, the time for 100 oscillations is measured to be 80s using a stopwatch with least count 1 s. The gravitational constant is known with a percentage error of 2.5%.

The percentage error in the measured length of the pendulum, in %, is_____.

(Answer in integer)

Q60. A particle of mass m in a potential $V(x) = \frac{1}{2}kx^2$ is described by normalized wavefunction $\sum_{n=0}^{\infty} (\sqrt{2})^{-(n+1)} \phi_n(x)$, where $\{\phi_n\}$ are the eigenstates of the particle. The

energy corresponding to the wavefunction, in units of $\frac{h}{\pi} \sqrt{\frac{k}{m}}$, is

_____. (Rounded off to two decimal places)

[Given: $\sum_{n=0}^{\infty} (a)^{-n} = \frac{a}{a-1}, a > 1$]