PREVIOUS YEAR'S SOLUTION

IIT-JAM 2026

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IIT-JAM Physics Course

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IIT-JAM 2026

Section A: Q1 – Q10 Carry ONE mark each.

Q1. Light of wavelength 632 *nm* is passing through an optically active medium of thickness 20 *cm*. The optical rotation exhibited by the medium is 18.

Which of the following options correctly states the magnitude of the difference in refractive indices corresponding to the left and the right circularly polarized light?

(a) 1.81×10^5 (b) 3.16×10^7 (c) 3.62×10^5 (d) 6.32×10^7

Ans.: (b)

Solution: $\Delta n = \frac{\theta_{rad}\lambda}{\pi d}$

$\lambda = 632$ nm, $d = 0.20$ m, $\theta = 18^\circ = \pi/10$ rad.

$$\Delta n = \frac{(\pi/10) \times 632 \times 10^{-9}}{\pi \times 0.20} = \frac{632 \times 10^{-9}}{2.0} = 3.16 \times 10^{-7}$$

$$\boxed{\Delta n = 3.16 \times 10^{-7}}$$

Q2. Consider an *n*-type silicon in which the fully ionized dopant concentration is 10^{17} cm^{-3}. The intrinsic electron density is 1.5×10^{10} cm^{-3}. Which of the following options correctly states the equilibrium hole concentration in cm^{-3}?

(a) 2.25×10^3 (b) 1.55×10^3 (c) 3.01×10^3 (d) 4.52×10^3

Ans.: (a)

Solution: Mass-action law: $p_0 = n_i^2/n_0 \approx n_i^2/N_D$

$$p_0 = \frac{(1.5 \times 10^{10})^2}{10^{17}} = \frac{2.25 \times 10^{20}}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

$$\boxed{p_0 = 2.25 \times 10^3 \text{ cm}^{-3}}$$

Q3. Consider the superposition of two electromagnetic waves with their electric field vectors given by $\vec{E}_1(z,t) = \hat{\imath}A_1\cos(kz - \omega t)$ and $\vec{E}_2(z,t) = \hat{\jmath}A_2\sin(kz - \omega t + \phi)$, where A_1 and A_2 are the amplitudes, k is the wavenumber, ω is the angular frequency, and ϕ is the relative phase. Which of the following options represents a resultant elliptically polarized wave with its semi-major axis either along $\hat{\imath}$ or $\hat{\jmath}$?

(a) $\phi = 0$ and $A_1 \neq A_2$ (b) $\phi = \frac{\pi}{2}$ and $A_1 = A_2$

(c) $\phi = \frac{\pi}{2}$ and $A_1 \neq A_2$ (c) $\phi = 0$ and $A_1 = A_2$

Ans.: (a)

Solution: $\vec{E}_1 = \hat{\imath}A_1\cos(kz-\omega t)$, $\vec{E}_2 = \hat{\jmath}A_2\sin(kz-\omega t+\phi)$.

For $\phi = 0$: $E_x = A_1\cos\xi$, $E_y = A_2\sin\xi$, giving $\frac{E_x^2}{A_1^2}+\frac{E_y^2}{A_2^2} = 1$ – an axis-aligned ellipse when $A_1 \neq A_2$.

$$\boxed{\phi = 0,\ A_1 \neq A_2}$$

Q4. Which of the following options represents the simplified form of the Boolean equation $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$?

(a) $A\bar{B}$ (b) $\bar{A}B\bar{C}$ (c) $\bar{B}C$ (d) $\bar{C}$

Ans.: (d)

Solution: $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$. Factor $\bar{C}$:

$$Y = \bar{C}\underbrace{(\bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB)}_{=1} = \bar{C}$$

$$\boxed{Y = \bar{C}}$$

Q5. A quantum particle is confined in a one-dimensional space of $0 \leq x \leq 2$.

Consider a normalized wavefunction of the particle as

$$\psi(x) = \sqrt{p/5}[1+\cos(\pi x/2)]\sin(\pi x/2).$$

Which of the following options gives the correct value of p ?

(a) 2 (b) 3 (c) 4 (d) 1

Ans.: (c)

Solution: The energy eigenstates of the infinite square well $(0 \leq x \leq 2)$ are:

$$|\phi_n\rangle = \sqrt{\frac{2}{2}}\sin\left(\frac{n\pi x}{2}\right)$$

The given wavefunction is:

$$\psi(x) = \sqrt{\frac{p}{5}}\left(1+\cos\frac{\pi x}{2}\right)\sin\left(\frac{\pi x}{2}\right)$$

Using the identity $2\cos\theta\sin\theta = \sin 2\theta$, expand:

$$\sqrt{\frac{p}{5}}\left(\sqrt{\frac{2}{2}}\sin\left(\frac{\pi x}{2}\right)+\sqrt{\frac{2}{2}}\cdot 2\cos\frac{\pi x}{2}\sin\frac{\pi x}{2}\right) = \sqrt{\frac{p}{5}}\left(|\phi_1\rangle + \frac{1}{2}|\phi_2\rangle\right)$$

For the normalisation condition $\langle\psi \mid \psi\rangle = 1$:

$$\frac{p}{5}\left(1^2+\left(\frac{1}{2}\right)^2\right) = \frac{p}{5}\cdot\frac{4+1}{4} = \frac{p}{5}\cdot\frac{5}{4} = \frac{p}{4} = 1 \Longrightarrow p = 4$$

Q6. Consider the normalized superposed state $\psi = c_0\phi_0 + c_1\phi_1$, where ϕ_0 and ϕ_1 are the ground and first excited states of a simple harmonic oscillator, respectively. c_0 and c_1 are imaginary superposition coefficients. Which of the following options is correct for the expectation value of $(\langle x\rangle + i\langle p\rangle)$?

(a) Imaginary

(b) Real

(c) Zero

(d) Complex with non-zero real and imaginary parts

Ans.: (b)

Solution: $\psi = c_0\phi_0 + c_1\phi_1$, $c_0 = i\alpha$, $c_1 = i\beta$ (purely imaginary, $\alpha, \beta \in \mathbb{R}$).

$c_0^*c_1 = (-i\alpha)(i\beta) = \alpha\beta \in \mathbb{R}$, so $\langle x\rangle = 2\alpha\beta\sqrt{\hbar/2m\omega} \in \mathbb{R}$.

$\langle p\rangle = c_0^*c_1\langle\phi_0|\hat{p}|\phi_1\rangle +$ c.c. $= \alpha\beta\cdot\left(i\sqrt{m\omega\hbar/2}\right) + \alpha\beta\cdot\left(-i\sqrt{m\omega\hbar/2}\right) = 0.$

$$\boxed{\langle x\rangle + i\langle p\rangle \in \mathbb{R}}$$

Q7. The potential of a quantum harmonic oscillator is modified from $\frac{1}{2}kx^2$ to $\frac{1}{2}kx^2 + 3ax$, where k and a are constants and x is the position variable. When, the values of $a = 2$ and $k = 1$, which of the following options gives the change in the ground state energy?

(a and k are in appropriate units)

(a) -3 (b) -6 (c) -12 (d) -18

Ans.: (d)

Solution: $V = 1/2\,kx^2 + 3ax$. Complete the square:

$$V = \frac{k}{2}\left(x + \frac{3a}{k}\right)^2 - \frac{9a^2}{2k}$$

Energy shift: $\Delta E_0 = -9a^2/(2k)$. With $a = 2$, $k = 1$:

$$\Delta E_0 = -\frac{9\times 4}{2} = -18$$

$$\boxed{\Delta E_0 = -18}$$

Q8. Consider the following linear second order differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \omega^2 y = 0,$$

where ω is a positive constant. The boundary conditions are $\left.\frac{dy}{dt}\right|_{t=0} = 1$, and $y(t=0) = \frac{1}{2}$.

Which of the following options gives the value of $y\left(t = \frac{\pi}{2\omega}\right)$?

(a) $\frac{1}{\omega}$ (b) $\frac{2\pi}{\omega}$ (c) 0 (d) 1

Ans.: (a)

Solution: $\ddot{y} + \omega^2 y = 0$, $y(0) = 1/2$, $\dot{y}(0) = 1$.

General solution: $y = A\cos\omega t + B\sin\omega t$.

Applying BCs: $A = 1/2$, $B\omega = 1 \Rightarrow B = 1/\omega$.

$$y\left(\frac{\pi}{2\omega}\right) = \frac{1}{2}\cos\frac{\pi}{2} + \frac{1}{\omega}\sin\frac{\pi}{2} = 0 + \frac{1}{\omega}$$

$$\boxed{y(\pi/2\omega) = 1/\omega}$$

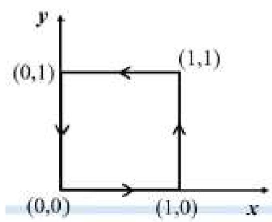
Q9. Consider the electrostatic potential in two dimensions $V(x,y) = x^8 y^9$. What is the line integral of the corresponding electric field along the path shown in the figure?

(a) 0

(b) 1

(c) 2

(d) $\frac{1}{2}$



Ans.: (a)

Solution: $V(x,y) = x^8 y^9$. The electric field $\vec{E} = -\nabla V$ is conservative.

$$\oint_C \vec{E} \cdot d\vec{l} = V(\text{start}) - V(\text{end}) = 0$$

(Path starts and ends at same potential since the endpoints shown give $V = 0$.)

$$\boxed{0}$$

Q10. Consider the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

The value of $\det(A^{-1})$ is:

(a) 4 (b) 1 (c) $\frac{1}{4}$ (d) 0

Ans.: (b)

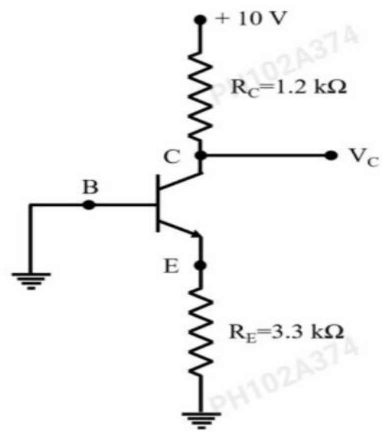
Solution: $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Expanding along column 1: $\det(A) = 1 \cdot (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$.

$$\det(A^{-1}) = \frac{1}{\det(A)} = 1$$

$$\boxed{\det(A^{-1}) = 1}$$

Section A: Q.11 – Q.30 Carry TWO marks each.

Q11. Consider the circuit of the figure, the voltage V_c, in Volts, is:



(a) 0 (b) 1.25 (c) 9.8 (d) 10

Ans.: (d)

Solution: Base tied to ground: $V_B = 0$. For transistor ON, $V_E = V_B - 0.7 = -0.7$ V – impossible with emitter connected to ground through R_E. Transistor is cut off: $I_C = 0$.

$$V_C = V_{CC} - I_C R_C = 10 - 0 = 10 \text{ V}$$

$$\boxed{V_C = 10 \text{ V}}$$

Q12. A particle of mass m moves in a potential given by

$$V(x, y, z) = -k \frac{y}{(x^2 + y^2 + z^2)}$$

where k is a constant.

Which of the following statements is correct?

(a) The force corresponding to the potential is central

(b) Angular momentum of the system is not conserved

(c) Linear momentum along the y-direction is conserved

(d) Energy of the system is not conserved

Ans.: (b)

Solution: $V(x, y, z) = -k\, y/(x^2 + y^2 + z^2)$. This is not $V(r)$ alone (depends on direction), so force is not central. Since not central, angular momentum is not conserved ($\partial V/\, \partial y \neq 0$, E is conserved since V is time-independent).

Q13. ($\vec{E}, \vec{B}$) are the electric and magnetic fields in a rest frame. ($\vec{E}', \vec{B}'$) represent the corresponding quantities in a reference frame moving with a constant velocity $\vec{v}_0$. Using the invariance of Lorentz force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ under Galilean transformations, identify the correct relation.

(a) $\vec{E}' = \vec{E}$ and $\vec{B}' = \vec{B}$

(b) $\vec{E}' = \vec{E} + \vec{v}_0 \times \vec{B}$ and $\vec{B}' = \vec{B}$

(c) $\vec{E}' = \vec{E}$ and $\vec{v} \times \vec{B}' = (\vec{v} + \vec{v}_0) \times \vec{B}$

(d) $\vec{E}' = \vec{v}_0 \times \vec{B}$ and $\vec{v} \times \vec{B}' = \vec{E} + \vec{v} \times \vec{B}$

Ans.: (b)

Solution: Invariance of $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ under $\vec{v} = \vec{v}' + \vec{v}_0$:

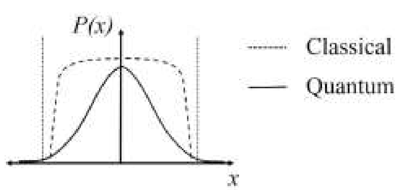
$$q\vec{E}' + q\vec{v}' \times \vec{B}' = q\vec{E} + q(\vec{v}' + \vec{v}_0) \times \vec{B}$$

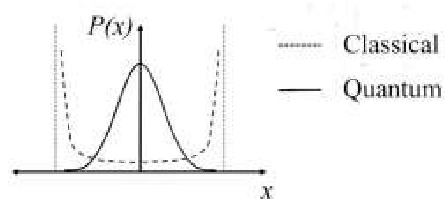
Matching terms: $\vec{E}' = \vec{E} + \vec{v}_0 \times \vec{B}$, $\vec{B}' = \vec{B}$.

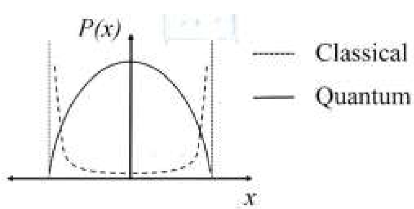
$$\boxed{\vec{E}' = \vec{E} + \vec{v}_0 \times \vec{B}, \quad \vec{B}' = \vec{B}}$$

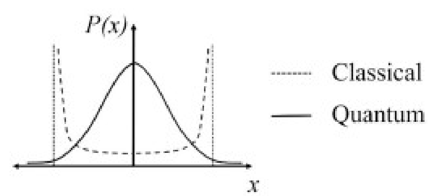
Q14. Which ONE of the following figures correctly represents the probability density, $P(x)$, of a particle undergoing simple harmonic oscillation as a function of position x ?

The dashed line is for the classical case. The solid line is for the quantum case, where the system is in its ground state. The dotted vertical lines on the x axis denote the classical turning points.

(a)


(b)


(c)


(d)


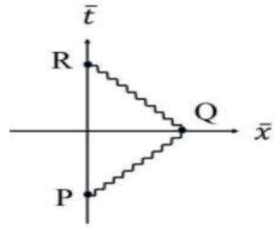
Ans.: (d)

Solution: Classical: $P_{cl}(x) \propto 1/\sqrt{A^2 - x^2}$ – U-shaped, peaks at turning points.

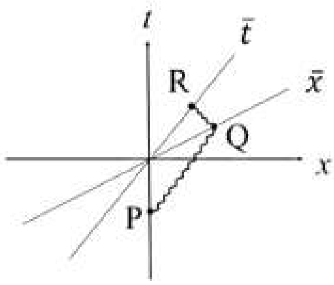
Quantum ground state: $P_{QM}(x) \propto e^{-m\omega x^2/\hbar}$ – Gaussian, peaked at $x = 0$, non-zero beyond turning points.

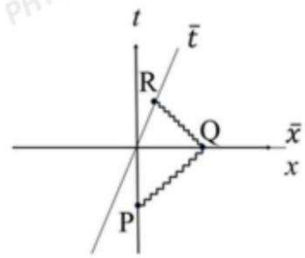
Figure (D): Gaussian quantum curve centred at origin; classical U-shaped curve.

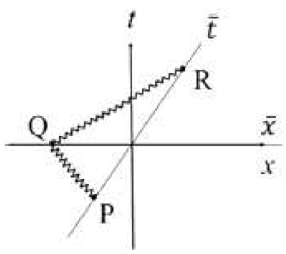
Q15. An observer $\bar{O}(\bar{t},\bar{x})$ moves with a constant velocity in the positive x-direction relative to an observer $O(t,x)$ at rest. In the frame of reference of $\bar{O}(\bar{t},\bar{x})$, a light-ray emitted at a point P at some time reaches the $\bar{x}$-axis at the point Q. Then, on reflection it arrives at point R, as shown in the figure.

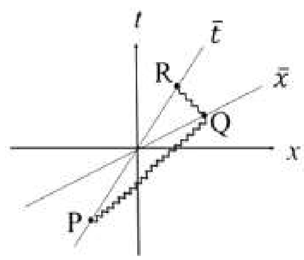


Which of the following options represents these events as observed by $O(t,x)$?

(a)


(b)


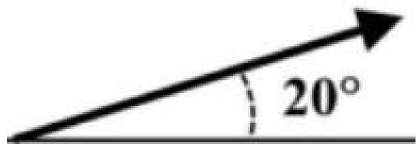
(c)


(d)


Ans.: (d)

Solution: $\bar{O}$ moves in $+x$ direction. In O's frame: $\bar{t}$-axis tilts right, $\bar{x}$-axis tilts up, both symmetrically about the light cone. Light rays remain at $45°$. Events P (lower), Q (on tilted $\bar{x}$-axis), R (upper) are visible with light paths at $45°$.

Q16. Consider a multilayered structure composed of thin films of refractive indices n_o, n_1, and n_2 as shown in the figure. A ray traveling in the first layer hits the interface at an angle of $20°$ with the horizontal. Which of the following options is correct?

	Layer 4; $n_o = 1.45$
	Layer 3; $n_2 = 1.33$
	Layer 2; $n_1 = 1.95$
	Layer 1; $n_o = 1.45$

(a) The ray emerges at an angle of $20°$ with the horizontal in the 4^{th} layer

(b) The ray emerges at an angle of $56°$ with the horizontal in the 4^{th} layer

(c) The ray emerges at an angle of $44°$ with the horizontal in the 4^{th} layer

(d) The ray would not enter the 3^{rd} layer

Ans.: (c)

Solution: Angle with normal at Layer 1/2 interface: $\theta_1 = 70°$.

Snell's law at Layer 1/2 ($n_o = 1.45 \to n_1 = 1.95$):

$$\sin\theta_2 = \frac{1.45\sin 70°}{1.95} = \frac{1.3625}{1.95} = 0.699, \quad \theta_2 = 44.3°$$

Critical angle at Layer 2/3 ($n_1 = 1.95 \to n_2 = 1.33$): $\sin\theta_c = 1.33/1.95 = 0.682, \quad \theta_c = 43.0°$.

Since $\theta_2 = 44.3° > \theta_c = 43.0°$: Total Internal Reflection – ray does not enter Layer 3.

Q17. Consider a BCC lattice with one atom per lattice point. The maximum packing fraction is close to:

(a) 53 % (b) 68 % (c) 74 % (d) 81%

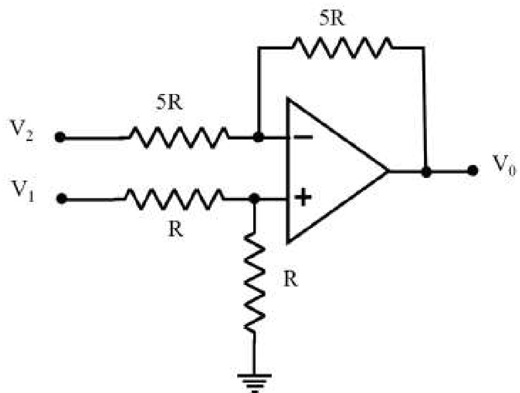
Ans.: (b)

Solution: Atoms per cell $= 2$; touching along body diagonal: $r = \sqrt{3}a/4$.

$$\eta = \frac{2 \times \frac{4}{3}\pi r^3}{a^3} = \frac{\pi\sqrt{3}}{8} \approx 0.6802 \approx 68\%$$

$$\boxed{\eta \approx 68\%}$$

Q18. The output voltage V_0 for the circuit shown in the figure is:



(a) $(V_1 - V_2)$ (b) $(V_1 - 2V_2)$ (c) $(V_2 - V_1)$ (d) $2(V_1 - V_2)$

Ans.: (a)

Solution: $V_+ = V_1/2$ (voltage divider R, R). Virtual short: $V_- = V_1/2$. KCL at inverting node:

$$\frac{V_2 - V_-}{5R} = \frac{V_- - V_0}{5R} \Longrightarrow V_0 = 2V_- - V_2 = V_1 - V_2$$

$$\boxed{V_0 = V_1 - V_2}$$

Q19. A quantum particle of mass 10^{-20} kg is confined within a length of 1 nm in one-dimension. The minimum uncertainty in the measurement of velocity of the particle, in units of $\mu\text{m/s}$, rounded off to the nearest integer is:

[Assume the minimum uncertainty product $\Delta x \Delta p_x \approx \frac{\hbar}{2}$, use Planck's Constant $h = 6.64 \times 10^{-34}$ Js]

(a) 2 (b) 5 (c) 10 (d) 1

Ans.: (b)

Solution: $\Delta v = \hbar/(2m\Delta x)$ with $m = 10^{-2}$ kg, $\Delta x = 10^{-9}$ m:

$$\Delta v = \frac{1.0567 \times 10^{-34}}{2 \times 10^{-20} \times 10^{-9}} = 5.28 \times 10^{-6} \text{ m/s} \approx 5\ \mu\text{m/s}$$

$$\boxed{\Delta v \approx 5\ \mu\text{m/s}}$$

Q20. A thin ring of mass m and radius R has a total charge Q distributed uniformly.

The ring is rotating with a constant angular velocity ω about an axis passing through its center and perpendicular to its plane.

Which of the following options is correct?

(a) The magnetic moment of the ring is $\frac{Q\omega R^2}{4}$

(b) The ratio of the magnetic moment to angular momentum is $\frac{Q}{2m}$

(c) The magnetic moment of the ring is $\frac{Q\omega R^2}{6}$

(d) The ratio of the magnetic moment to angular momentum is $\frac{Q}{m}$

Ans.: (b)

Solution: $I = Q\omega/2\pi$. Magnetic moment: $\mu = I\pi R^2 = Q\omega R^2/2$.

Angular momentum: $L = mR^2\omega$. Ratio: $\mu/L = Q/2m$.

$$\boxed{\mu = Q\omega R^2/2, \quad \mu/L = Q/2m}$$

Q21. A mass attached to the bottom end of a vertical massless spring stretches the spring by Δx. The system executes oscillation with a time period $T = 0.2s$. The value of Δx, in cm, rounded off to the nearest integer is:

[Assume the acceleration due to gravity $g = 9.8 \text{ m/s}^2$]

(a) 2 (b) 3 (c) 4 (d) 1

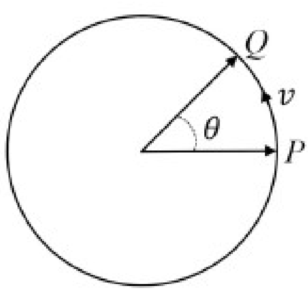
Ans.: (d)

Solution: $T = 2\pi\sqrt{\Delta x/g} \Longrightarrow \Delta x = g(T/2\pi)^2$:

$$\Delta x = 9.8 \times \left(\frac{0.2}{2\pi}\right)^2 = 9.8 \times 1.013 \times 10^{-3} = 9.93 \times 10^{-3} \text{ m} \approx 1 \text{ cm}$$

$$\boxed{\Delta x \approx 1 \text{ cm}}$$

Q22. A particle is rotating along a circular path with uniform speed v, as shown in the figure. While moving from the point P to Q subtending an angle θ, the magnitude of the change in its velocity is:



(a) Zero (b) $v\cos\ \theta$ (c) $2v\sin\ \frac{\theta}{2}$ (d) $v\cos\ \frac{\theta}{2}$

Ans.: (c)

Solution: Velocity vectors $\vec{v}_P$ and $\vec{v}_Q$ both have magnitude v, angle θ apart:

$$|\Delta\vec{v}| = \sqrt{2v^2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}$$

$$\boxed{|\Delta\vec{v}| = 2v\sin(\theta/2)}$$

Q23. A series LCR circuit contains $L = 175mH, C = 62.5\mu F$, and $R = 40\Omega$ and is connected to a source of voltage amplitude $E_0 = 50$ V and angular frequency $\omega = 400\text{rad/s}$.

Which of the following statements is correct?

(a) The magnitude of the maximum voltage across the inductor is less than 50 V

(b) The magnitude of the maximum voltage across the capacitor is more than 60 V

(c) The magnitude of the maximum voltage across the resistor is 50 V

(d) The magnitude of the maximum voltage across the capacitor equals the maximum voltage across the resistor

Ans.: (d)

Solution: $X_L = \omega L = 400 \times 0.175 = 70\ \Omega$, $X_C = 1/\omega C = 40\ \Omega$, $R = 40\ \Omega$.

$Z = \sqrt{40^2 + 30^2} = 50\ \Omega$, $I_0 = 50/50 = 1$ A.

$V_R = 40$ V, $V_L = 70$ V, $V_C = 40$ V.

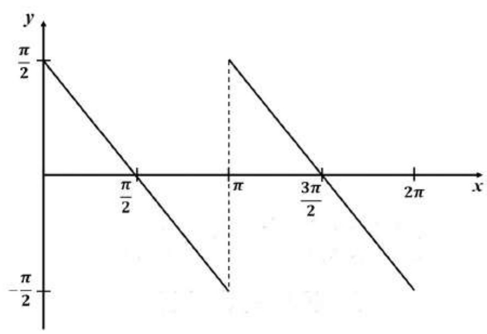
Since $X_C = R$: $V_C = V_R$.

Maximum voltage across capacitor equals maximum voltage across resistor ($= 40$ V).

Q24. A piecewise regular function $f(x)$ is shown in the figure. It is expanded in Fourier series given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + \sum_{n=1}^{\infty} b_n\sin(nx),$$

where a_0, a_n, b_n 's are the Fourier coefficients.



Which of the following options is correct?

(a) $a_0 = 3\pi$ (b) All a_n are zero (c) All b_n are zero (d) $a_0 = \frac{3\pi}{2}$

Ans.: (b)

Solution: The function $f(x)$ is odd (antisymmetric, zero mean) over the period. All cosine coefficients vanish for an odd function:

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)\,dx = 0 \quad \forall n \geq 0$$

All a_n are zero.

Q25. Using Taylor series, expand $f(x) = x^3 - \frac{1}{8}$ around $x_0 = 1$ up to second order in x. The coefficient of x is:

(a) -2 (b) 3 (c) 2 (d) -3

Ans.: (d)

Solution: $f(x) = x^3 - 1/8$ about $x_0 = 1$: $f(1) = 7/8$, $f'(1) = 3$, $f''(1) = 6$.

$$f(x) \quad = 7/8 + 3(x-1) + 3(x-1)^2 = 3x^2 - 3x + 7/8$$

Coefficient of x: -3.

$$\boxed{-3}$$

Q26. Consider a 2×2 matrix A. The determinant of A is -1 and the $\text{trace}(A) = 1$. Which of the following options gives the eigenvalues of A ?

(a) 1,0 (b) $\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$ (c) $\frac{1+\sqrt{5}}{4}, \frac{1-\sqrt{5}}{4}$ (d) $\frac{\sqrt{5}}{2}, 1 - \frac{\sqrt{5}}{2}$

Ans.: (b)

Solution: $\det A = -1$, $\text{tr}\, A = 1$. Characteristic equation: $\lambda^2 - \lambda - 1 = 0$.

$$\lambda = \frac{1 \pm \sqrt{5}}{2}$$

$$\boxed{\lambda_{1,2} = (1 \pm \sqrt{5})/2}$$

Q27. Consider the following vector $\vec{V} = xyz\hat{k}$.

Which of the following statements is correct for the resultant vector $\vec{\nabla} \times \vec{\nabla} \times \vec{V}$?

(a) It lies in xy – plane (b) It lies in yz – plane

(c) It lies in xz – plane (d) It is along the x direction

Ans.: (a)

Solution: $\vec{V} = xyz\hat{k}$. Using $\nabla \times (\nabla \times \vec{V}) = \nabla(\nabla \cdot \vec{V}) - \nabla^2\vec{V}$:

$\nabla \cdot \vec{V} = xy$; $\nabla(xy) = y\hat{\imath} + x\hat{\jmath}$; $\nabla^2\vec{V} = \vec{0}$.

$$\nabla \times \nabla \times \vec{V} = y\hat{\imath} + x\hat{\jmath} \Longrightarrow \text{lies in } xy\text{-plane}$$

Lies in xy-plane.

Q28. Two coherent plane waves having wavelength λ and wavevectors $\vec{k}_1 = \frac{2\pi}{\lambda}\left(\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right)$ and $\vec{k}_2 = \frac{2\pi}{\lambda}\left(-\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right)$, are incident on a screen placed on the xz-plane. The overlap of the waves on the plane of the screen produces interference fringes. The fringe width (center-to-center spacing of bright fringes) will be:

(a) $\frac{\lambda}{2}$ (b) 2λ (c) $\frac{\sqrt{3}\lambda}{2}$ (d) λ

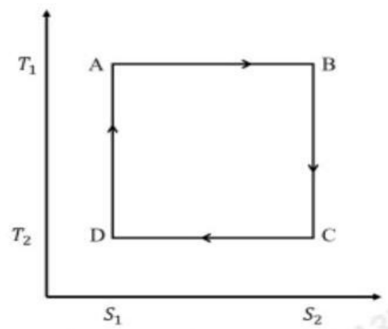
Ans.: (d)

Solution: $\vec{k}_{1,2} = \frac{2\pi}{\lambda}\left(\pm\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right)$. Phase difference on screen ($y = 0$):

$$\Delta\phi = \frac{2\pi x}{\lambda} \Longrightarrow \text{bright fringes at } x = m\lambda \Longrightarrow \beta = \lambda$$

$$\boxed{\beta = \lambda}$$

Q29. Consider a Carnot cycle as shown in the figure. The ideal gas expanded from volume V_1 to V_2 along the path AB . During this process, the temperature T_1 is constant and the entropy changes from S_1 to $S_2 \cdot T_2$ is the temperature of the cold bath.



Which of the following statements is correct?

(a) Work done along the path AB is $W_{AB} = -NRT_1 \ln \frac{V_2}{V_1}$

(b) Work done along the path BC is $W_{BC} = 0$

(c) Work done along the path CD is $W_{CD} = -NRT_1 \ln \frac{V_2}{V_1}$

(d) Work done along the path DA is $W_{DA} = 0$

Ans.: (a)

Solution: Path AB: isothermal expansion at T_1. Work done on gas (sign convention $dW = PdV$):

$$W_{AB} = NRT_1 \ln \frac{V_2}{V_1}$$

(BC: adiabatic, $W \neq 0$; CD: isothermal at $T_2 \neq T_1$; DA: adiabatic, $W \neq 0$.)

$$\boxed{W_{AB} = NRT_1 \ln(V_2/V_1)}$$

Q30. The speed of an object of mass 20 kg increases from $2\ \mathrm{m/s}$ to $6\ \mathrm{m/s}$ in 10 s.

The power required, in Watts, is:

(a) 64 (b) 54 (c) 32 (d) 72

Ans.: (c)

Solution: $W = \Delta KE = \frac{1}{2} \times 20 \times (6^2 - 2^2) = 10 \times 32 = 320\ \mathrm{J}$.

$$P = W/t = 320/10 = 32\ \mathrm{W}$$

$$\boxed{P = 32\ \mathrm{W}}$$

Section B: Q.31 – Q.40 Carry TWO marks each.

Q31. The displacement of a vibrating string of finite length stretched along the x-axis is given by

$$y(x,t) = 2A\cos(kx)\sin(\omega t)$$

where, A is the amplitude, $k = 2\pi/\lambda$ is the wavenumber and ω is the angular frequency.

Which of the following statements is/are correct for the standing wave?

(a) The nodes are at $x = n\frac{\lambda}{2}$, where n is an integer

(b) The antinodes are at $x = \left(n+\frac{1}{2}\right)\frac{\lambda}{2}$, where n is an integer

(c) The nodes are at $x = \left(n+\frac{1}{2}\right)\frac{\lambda}{2}$, where n is an integer

(d) The antinodes are at $x = n\frac{\lambda}{2}$, where n is an integer

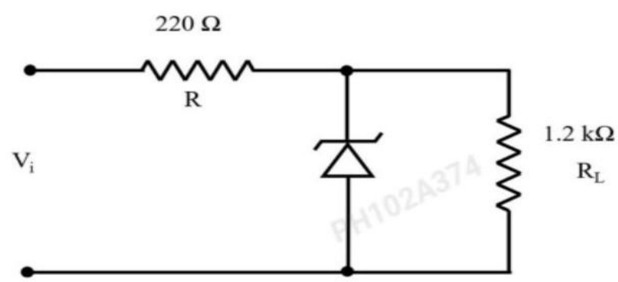
Ans.: (c) and (d)

Solution: $y(x,t) = 2A\cos(kx)\sin(\omega t)$, $k = 2\pi/\lambda$.

Nodes ($\cos(kx) = 0$): $x = (n + 1/2)\lambda/2$

Antinodes ($|\cos(kx)| = 1$): $x = n\lambda/2$

Q32. In the circuit shown in the figure, the Zener voltage V_Z is 20 V and the maximum Zener current I_{ZM} is 60 mA .



For what value(s) of the input voltage V_i, the Zener diode is in ON state?

(a) 20 V (b) 25 V (c) 35 V (d) 40 V

Ans.: (b) and (c)

Solution: $V_Z = 20$ V, $I_{ZM} = 60$ mA, $R = 220\ \Omega$, $R_L = 1.2$ kΩ.

$I_L = 20/1200 = 16.67$ mA.

Minimum V_i (Zener just ON, $I_Z \approx 0$): $V_{i,\min} = 20 + 16.67 \times 10^{-3} \times 220 = 23.7$ V.

Maximum V_i ($I_Z = I_{ZM}$): $V_{i,\max} = 20 + (16.67 + 60) \times 10^{-3} \times 220 = 36.9$ V.

Check: (A) 20 V $<$ 23.7 (OFF); (B) 25 V (ON); (C) 35 V (ON); (D) 40 V $>$ 36.9 (exceeds I_{ZM}).

Q33. Consider a particle of mass m in a rotating frame. The force acting on the particle is expressed as $\vec{F} = (F_1 + F_2)\hat{e}_r + (F_3 + F_4)\hat{e}_\theta$, where $\hat{e}_r$ and $\hat{e}_\theta$ are the radial and angular unit vectors, respectively, and $F_1 = m\ddot{r}, F_2 = -mr\dot{\theta}^2, F_3 = mr\ddot{\theta}, F_4 = 2m\dot{r}\dot{\theta}$.

Which of the following statements is/are correct?

(a) $\hat{e}_r$ and $\hat{e}_\theta$ are not constant unit vectors (b) F_1 and F_3 are fictitious forces

(c) F_2 and F_4 are fictitious forces (d) F_1 and F_2 are zero in uniform circular motion

Ans.: (a) and (c)

Solution: $\vec{F} = (F_1 + F_2)\hat{e}_r + (F_3 + F_4)\hat{e}_\theta$ where $F_1 = m\ddot{r}$, $F_2 = -mr\dot{\theta}^2$, $F_3 = mr\ddot{\theta}$, $F_4 = 2m\dot{r}\dot{\theta}$.

(A) $\hat{e}_r, \hat{e}_\theta$ rotate with the particle: True.

(B) F_1, F_3 are real (inertial) terms: False.

(C) $F_2 = -mr\dot{\theta}^2$ (centrifugal) and $F_4 = 2m\dot{r}\dot{\theta}$ (Coriolis) are fictitious: True.

(D) $F_2 = -mr\omega^2 \neq 0$ in uniform circular motion: False.

Q34. The events are represented by coordinates (ct, x, y, z) in some frame of reference. Which of the following statements is/are correct?

[c is the speed of light]

(a) Events (1,0,-10,1) and (-1,1,-9,1) are space-like separated

(b) Events ($-1,0,-9,1$) and ($1,1,-10,1$) are space-like separated

(c) Events ($-10,0,1,-1$) and ($-9,1,-1,-1$) are light-like separated

(d) Events $(9,-1,1,-1)$ and $(-10,1,0,-1)$ are time-like separated

Ans.: (d)

Solution: Interval: $\Delta s^2 = -(c\Delta t)^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$.

(A) $\Delta s^2 = -4 + 1 + 1 = -2 < 0$ (time-like, not space-like): False.

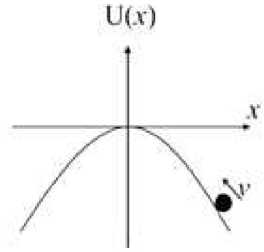
(B) $\Delta s^2 = -4 + 1 + 1 = -2 < 0$ (time-like, not space-like): False.

(C) $\Delta s^2 = -1 + 1 + 4 = 4 > 0$ (space-like, not light-like): False.

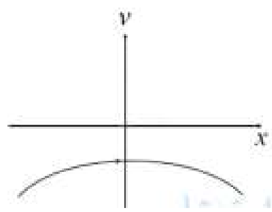
(D) $\Delta s^2 = -361 + 4 + 1 = -356 < 0$ (time-like): True.

Correct: (D) only

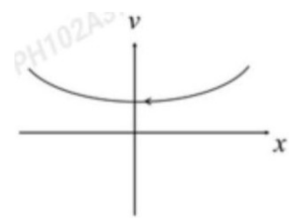
Q35. A ball of mass m climbs the potential $U(x) = -\frac{1}{2}kx^2$, as shown in the figure. Assuming that the total energy of the system $E = \frac{1}{2}mv^2 + U(x)$ is conserved, which of the following correctly describe(s) the plot of velocity (v) as a function of position (x) of the system for $E < 0$?



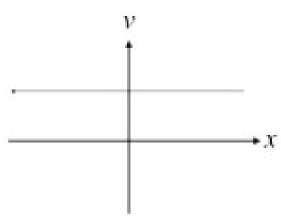
(a)



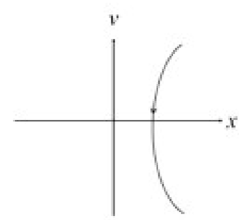
(b)



(c)



(d)



Ans.: (d)

Solution: $U(x) = -1/2\,kx^2$, $E < 0$. Energy conservation:

$$v^2 = \frac{k}{m}(x^2 - x_0^2), \quad x_0 = \sqrt{-2E/k}$$

This is a hyperbola in the v-x plane, existing only for $|x| \geq x_0$ (particle escapes outward from turning points). (Hyperbola opening left and right.)

Q36. An LC oscillator circuit contains a capacitor of 4.0μ F. The maximum potential difference across the capacitor is $2V$ and the maximum current through the inductor is 80 mA .

Which of the following statements is/are correct?

(a) The value of inductance is 2.5 mH

(b) The frequency of oscillator is 3.2 kHz

(c) The time for the charge in the capacitor to rise from zero to the maximum is nearly 0.157 ms

(d) The maximum potential difference across the inductor is 2.0 V

Ans.: (a), (c) and (d)

Solution: $C = 4\ \mu\text{F}$, $V_{\max} = 2$ V, $I_{\max} = 80$ mA.

(A) $L = C(V_{\max}/I_{\max})^2 = 4\times10^{-6}\times(25)^2 = 2.5\text{ mH}$: True.

(B) $f = 1/(2\pi\sqrt{LC}) = 1/(2\pi\times10^{-4}) \approx 1.59\text{ kHz} \neq 3.2\text{ kHz}$: False.

(C) $t_{T/4} = \pi\sqrt{LC}/2 = \pi\times10^{-4}/2 = 1.57\times10^{-4}\text{ s} \approx 0.157\text{ ms}$: True.

(D) $V_{L,\max} = \sqrt{L/C}\cdot I_{\max} = \sqrt{625}\times0.08 = 2.0\text{ V}$: True.

Q37. Consider an ideal gas of entropy S, molar specific heat C_v, pressure P and volume V. Which of the following options is/are true?

(a) Internal energy of the gas depends only on the temperature

(b) $S \propto \ln V$, assuming C_v is constant

(c) $S = 0$

(d) Internal energy of the gas depends both on temperature and pressure

Ans.: (a) and (b)

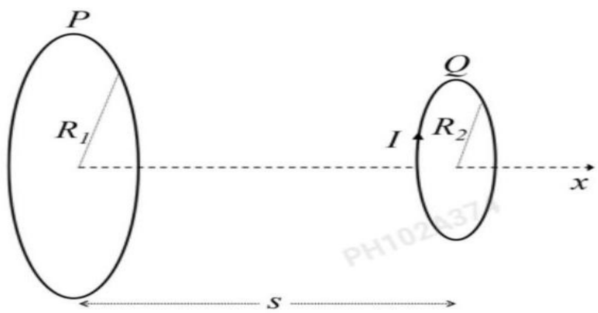
Solution: (A) $U = nC_vT$ – depends only on temperature: True.

(B) From $TdS = dU + PdV$: at constant T, $dS = (nR/V)dV \Rightarrow S \propto \ln V$: True.

(C) $S = 0$ only at $T = 0$: False.

(D) U independent of pressure for ideal gas: False.

Q38. Two small circular copper loops P and Q of radii R_I and R_2, respectively, are coaxially placed along the x-axis as shown in the figure. The loops are at a distance s apart, where $s \gg R_1, R_2$. The loop Q carrying a steady current I is moving away along x-axis with a speed v.



Which of the following statements is/are correct?

(a) The magnetic field at the center of P is $\propto \frac{1}{s^2}$

(b) The magnetic flux through the loop P is $\propto \frac{1}{s^2}$

(c) The emf induced in the loop P is $\propto \frac{1}{s^4}$

(d) The emf induced in the loop P is $\propto v^2$

Ans.: (c)

Solution: For $s \gg R_1, R_2$: dipole field $B \propto 1/s^3$, flux $\Phi \propto 1/s^3$.

EMF: $\mathcal{E} = -d\Phi/dt = -(d\Phi/ds)\dot{s} \propto v/s^4$.

(A) $B \propto 1/s^2$: False. (B) $\Phi \propto 1/s^2$: False. (C) $\mathcal{E} \propto 1/s^4$: True. (D) $\mathcal{E} \propto v^2$: False.

Correct: (C) only

Q39. A charge Q is distributed uniformly on the surface of a sphere of radius R. It is placed inside a concentric conducting hollow sphere of radius $2R$. The outer sphere is earthed.

Which of the following statements is/are correct?

(a) The charge on the inner surface of the outer sphere is $-Q$

(b) The flux through a closed surface through the material of the outer sphere is $\frac{Q}{\epsilon_0}$

(c) The charge on the outer surface of the outer sphere is zero

(d) The potential at a radial distance r between the two spheres, $R < r < 2R$, is $\frac{1}{4\pi\epsilon_0}\frac{Q}{r}$

Ans.: (a) and (c)

Solution: (A) Gauss's law inside conductor: $Q_{inner} = -Q$: True.

(B) $\vec{E} = 0$ inside conductor, so flux $= 0 \neq Q/\epsilon_0$: False.

(C) Grounded outer shell has $V = 0$; outer surface charge $= 0$: True.

(D) $V(r) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{2R}\right) \neq \frac{Q}{4\pi\epsilon_0 r}$: False.

Q40. Which of the following statements is/are true for a first order phase transition?

[C_p is the molar heat capacity, T_c is the critical temperature and S is the entropy]

(a) At the transition point $C_p \rightarrow \infty$

(b) The derivative of the Gibbs function with respect to pressure changes continuously across the phase transition

(c) The two thermodynamic states between which the transition takes place are distinct

(d) Entropy changes discontinuously with temperature at T_c

Ans.: (a),(c) and (d)

Solution: (A) Latent heat at constant T: $C_p = dQ/dT \rightarrow \infty$: True.

(B) $V = (\partial G/\partial P)_T$ is *discontinuous* (volume jump): False.

(C) Two distinct coexisting phases at the transition: True.

(D) $\Delta S = L/T_c \neq 0$ – entropy jumps discontinuously: True.

Section C: Q.41 – Q.50 Carry ONE mark each.

Q41. Considering the diameter of the pupil of a human eye to be 2 mm , the angular resolution of the eye at a wavelength of 500 nm , in minute of arc, is ____________.

(Rounded off to two decimal places)

Ans.: 1.01 To 1.11

Solution: Rayleigh criterion: $\theta = 1.22\lambda/D$.

$$\theta = \frac{1.22 \times 500 \times 10^{-9}}{2 \times 10^{-3}} = 3.05 \times 10^{-4} \text{ rad} = 3.05 \times 10^{-4} \times \frac{180 \times 60}{\pi} = 1.05'$$

$$\boxed{1.05 \text{ arcmin}}$$

Q42. Consider a 10 mW laser beam focused using a biconvex lens to a circular spot of area $10^{-1}\ \text{m}^2$. The magnitude of the electric field in the focal plane of the lens, in kV/m, is __________. (Rounded off to one decimal place)

[Use permittivity of free space $\epsilon_o = 8.854 \times 10^{-12} \text{C}^2/(\text{Nm}^2)$, and speed of light $c = 3 \times 10^8 \text{ m/s}$]

Ans.: 273.0 To 276.0

Solution: $I = P/A = 10 \times 10^{-3}/10^{-1} = 10^8 \text{ W/m}^2$. $E_0 = \sqrt{2I/\epsilon_0 c}$:

$$E_0 = \sqrt{\frac{2 \times 10^8}{8.854 \times 10^{-12} \times 3 \times 10^8}} = \sqrt{7.529 \times 10^{10}} = 274.4 \text{ kV/m}$$

$$\boxed{E_0 \approx 274.4 \text{ kV/m}}$$

Q43. An OP-AMP has differential gain of $A_d = 4000$, two input voltages $V_{i1} = 120\mu V$ and $V_{i2} = 80\mu V$, and CMRR $= 100$. The output voltage, in mV, is___________ (Answer in integer)

Ans.: 160 To 170

Solution: $V_d = 40\ \mu\text{V}$, $V_c = 100\ \mu\text{V}$, $A_c = A_d/\text{CMRR} = 40$.

$$V_o = 4000 \times 40 \times 10^{-6} + 40 \times 100 \times 10^{-6} = 160 + 4 = 164 \text{ mV}$$

$$\boxed{164 \text{ mV}}$$

Q44. A particle of mass 10^{-20} kg is moving along a circular orbit of radius 1 nm . The speed of the particle corresponds to the average thermal energy at temperature 10^{-6} K. Assuming the Bohr's angular momentum quantization condition, the quantum number of the circular path of the particle is_________. (Answer in integer)

[Use $h = 6.64 \times 10^{-34} Js$ and $k_B = 1.38 \times 10^{-23} J/K$]

Ans.: 6 To 6

Solution: $v=\sqrt{3k_BT/m}=\sqrt{3\times 1.38\times 10^{-23}\times 10^{-6}/10^{-20}}=6.434\times 10^{-5}$ m/s.

$L=mvr=10^{-20}\times 6.434\times 10^{-5}\times 10^{-9}=6.434\times 10^{-34}$ J · s.

$n=L/\hbar=6.434\times 10^{-34}/1.057\times 10^{-3}\ \approx 6.$

$$\boxed{n=6}$$

Q45. One mole of an ideal gas undergoes a reversible isothermal expansion from $V_i=1.5\times 10^{-5}\ \mathrm{m}^3$ to $V_f=1.6\times 10^{-5}\ \mathrm{m}^3$ at a temperature 273 K . The amount of heat transfer during the process is αR, where R is the gas constant.

The value of α is __________. (Rounded off to one decimal place)

Ans.: 16.5 To 18.5

Solution: $Q=nRT\ln\left(V_f/V_i\right)=\alpha R.$

$$\alpha=nT\ln\frac{V_f}{V_i}=1\times 273\times \ln\frac{1.6\times 10^{-5}}{1.5\times 10^{-5}}=273\times 0.0645=17.6$$

$$\boxed{\alpha=17.6}$$

Q46. The value of $(1-i\sqrt{3})^3$ is ____ . (*Answer in integer*)

Ans.: -8 To -8

Solution: $z=1-i\sqrt{3}=2e^{-i\pi/3}.$

$$z^3=2^3e^{-i\pi}=8(\cos\pi+i\sin\pi)=-8$$

$$\boxed{-8}$$

Q47. Two thermodynamic systems separated by diathermic wall have the equations of state $U_1=\frac{3}{2}RN_1T_1$ and $U_2=\frac{5}{2}RN_2T_2$, where R is the gas constant. N_1,N_2 and T_1,T_2 are the mole numbers and the temperature of the two systems, respectively. The composite system in equilibrium has the total energy 1.5×10^3 Joule. If $N_1=3$ and $N_2=2$, then the internal energy U_1 of the system one is ____ . (Answer in integer)

Ans.: 709 To 712

Solution: $U_1=\frac{3}{2}RN_1T$, $U_2=\frac{5}{2}RN_2T$. At equilibrium $T_1=T_2=T$:

$$\frac{U_2}{U_1}=\frac{5N_2}{3N_1}=\frac{10}{9}\Longrightarrow U_1\left(1+\frac{10}{9}\right)=1500\Longrightarrow U_1=\frac{1500\times 9}{19}=710.5\ \mathrm{J}$$

$$\boxed{U_1\approx 710\ \mathrm{J}}$$

Q48. Light of wavelength 500 nm is incident on the surface of Na metal for photoelectric emission. The corresponding threshold wavelength is 600 nm . The maximum kinetic energy of the emitted electron, in eV , is________(Rounded off to two decimal places)

[Use Planck's constant $h = 6.625 \times 10^{-34}$ J s, speed of light $c = 3 \times 10^8$ m/s, charge of electron $e = 1.6 \times 10^{-19}$C]

[Assume refractive index is independent of wavelength]

Ans.: 0.38 To 0.44

Solution: $KE_{\max} = hc(1/\lambda - 1/\lambda_0)$:

$$\begin{aligned} KE_{\max} &= 6.625 \times 10^{-34} \times 3 \times 10^8 \times \left(\frac{1}{500 \times 10^{-9}} - \frac{1}{600 \times 10^{-9}}\right) \\ &= 6.625 \times 10^{-20} \text{ J} = 0.41 \text{ eV} \end{aligned}$$

$$\boxed{KE_{\max} = 0.41 \text{ eV}}$$

Q49. The first order Bragg peak for (100) plane of a material with simple cubic structure is measured using an X-ray of wavelength 1Å. If the lattice constant is 5Å then the Bragg peak is observed at an angle, in degrees, ____ .

(Rounded off to two decimal places)

Ans.: 5.55 To 5.95

Solution: $d_{100} = a = 5$ AA. Bragg's law ($n = 1$): $\sin\theta = \lambda/(2d) = 1/10 = 0.1$.

$$\theta = \arcsin(0.1) = 5.74^\circ$$

$$\boxed{\theta = 5.74^\circ}$$

Q50. Consider an ensemble of hydrogen gas. The temperature, in K, at which the rms speed of the hydrogen molecule is twice the rms speed of the molecule at 300 K is_______________.

(Answer in integer)

Ans.: 1200 To 1200

Solution: $v_{rms} \propto \sqrt{T}$. For $v_{rms}(T) = 2v_{rms}(300)$:

$$\sqrt{T/300} = 2 \Longrightarrow T = 4 \times 300 = 1200 \text{ K}$$

$$\boxed{T = 1200 \text{ K}}$$

Section C: Q.51 – Q.60 Carry TWO marks each.

Q51. A particle of mass m undergoes periodic motion in one-dimension with its total energy given as $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{4}kx^4$, where k is a positive constant and $\dot{x} = \frac{dx}{dt}$. Assuming that E is conserved, the time period T has the relation $T \propto E^{-1/n}$. The value of n is ____ . (Answer in integer)

Ans.: 4 To 4

Solution: $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{4}kx^4$. Scaling $x \to \lambda x$, $t \to \mu t$:

PE term: $k\lambda^4 \sim E \Rightarrow \lambda \sim E^{1/4}$.

KE term: $m\lambda^2/\mu^2 \sim E \Rightarrow \mu \sim \lambda E^{-1/2} \sim E^{-1/4}$.

$T \propto \mu \propto E^{-1/4} = E^{-1/n} \Rightarrow n = 4$.

$$\boxed{n = 4}$$

Q52. A spacecraft is placed 200 km above Earth in a circular orbit. The minimum change in the speed required to place the spacecraft in a parabolic orbit, in $\mathrm{km/s}$, is ____ .

(Rounded off to one decimal place)

[Use $G = 6.67 \times 10^{-1}\ \mathrm{Nm^2/kg^2}$, mass of Earth $= 6 \times 10^{24}$ kg, radius of Earth $=$ 6400 km]

Ans.: 3.0 To 3.5

Solution: $r = R_E + h = 6600$ km. $v_c = \sqrt{GM/r} = 7787$ m/s.

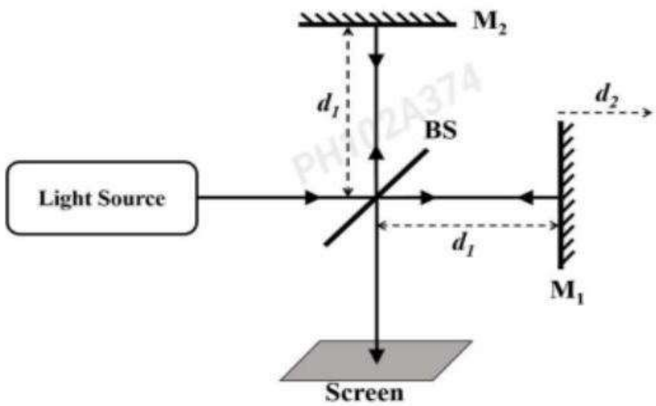
$$\Delta v = (\sqrt{2} - 1)v_c = 0.4142 \times 7787 = 3225 \text{ m/s} = 3.2 \text{ km/s}$$

$$\boxed{\Delta v = 3.2 \text{ km/s}}$$

Q53. Consider a light source having a spectral linewidth of 10^{10} Hz, used in a Michelson interferometer. The mirrors M_1 and M_2 are equidistant from the beam-splitter of negligible thickness as shown in the figure. The minimum distance d_2 that the mirror M_1 is to be moved for the interference pattern to completely disappear, in cm, is ____ .

(Rounded off to one decimal place)

[Use speed of light to be 3×10^8 m/s]



Ans.: 1.5 To 1.5

Solution: Coherence length: $L_c = c/\Delta\nu = 3 \times 10^8/10^{10} = 0.03 \text{ m} = 3 \text{ cm}$.

Optical path difference $= 2d_2 = L_c$:

$$d_2 = \frac{L_c}{2} = 1.5 \text{ cm}$$

$$\boxed{d_2 = 1.5 \text{ cm}}$$

Q54. Muons are unstable relativistic particles created at high altitudes above the Earth, having a lifetime of $2.2 \times 10^{-6} s$ in their rest frame. As measured by an observer on the ground, the minimum velocity the muon requires to travel a distance of $6000m$ is v. The value of v/c is__________(Rounded off to three decimal places)

[Speed of light $c = 3 \times 10^8$ m/s]

Ans.: 0.992 To 0.997

Solution: $v\tau = L \Rightarrow \frac{v\tau_0}{\sqrt{1-\frac{v^2}{c^2}}} = L \Rightarrow \beta\tau_0 = \sqrt{1-\beta^2}\frac{L}{c}$

$$\beta^2\tau_0^2 = (1-\beta^2)\frac{L^2}{c^2} \Rightarrow \beta^2 = \frac{\frac{L^2}{c^2}}{1+\frac{L^2}{c^2}} \Rightarrow \beta = \frac{\frac{L}{c}}{\sqrt{\tau_0^2 + \frac{L^2}{c^2}}} = \frac{\frac{6000}{3 \times 10^8}}{\sqrt{(2.2 \times 10^{-6})^2 + \left(\frac{6000}{3 \times 10^8}\right)^2}}$$

$= 0.994$

Q55. On the surface of a thin water film of refractive index 1.33, two light beams of wavelength $\lambda_1 = 0.64\mu\text{ m}$ and $\lambda_2 = 0.40\mu\text{ m}$ are incident at an angle of $30°$. The light of wavelength λ_1 exhibits maximum reflection, but that of wavelength λ_2 is not reflected at all. The minimum thickness of the water film, in μm, is_________. (Rounded off to two decimal places)

[Assume refractive index is independent of wavelength]

Ans.: 0.61 To 0.71

Solution: $n = 1.33$, $\theta_{inc} = 30°$. Snell: $\sin\theta_r = 0.5/1.33 = 0.376$, $\cos\theta_r = 0.9267$.

Constructive ($\lambda_1 = 0.64\ \mu\text{m}$): $2nt\cos\theta_r = \left(m_1 - \frac{1}{2}\right)\lambda_1$.

Destructive ($\lambda_2 = 0.40\ \mu\text{m}$): $2nt\cos\theta_r = m_2\lambda_2$.

Smallest solution: $m_1 = 3$, $m_2 = 4$; both give $2nt\cos\theta_r = 1.6\ \mu\text{m}$.

$$t = \frac{1.6}{2 \times 1.33 \times 0.9267} = 0.65\ \mu\text{m}$$

$$\boxed{t = 0.65\ \mu\text{m}}$$

Q56. An electron is confined in a one-dimensional box of width $L = 10\text{Å}$. The electron in the first excited state de-excites to the ground state. The wavelength of the emitted radiation, in μm, is ___ .

(Rounded off to one decimal place)

[Use the mass of the electron $m_e = 9.1 \times 10^{-31}\text{ kg}$, Planck's constant $h = 6.625 \times 10^{-34} Js, c = 3 \times 10^8\text{ m/s}$]

Ans.: 0.9 To 1.3

Solution: $\Delta E = 3h^2/(8m_eL^2)$, $L = 10$ AA:

$$\Delta E = \frac{3 \times (6.625 \times 10^{-3}\ )^2}{8 \times 9.1 \times 10^{-31} \times (10^{-9})^2} = 1.809 \times 10^{-19}\text{ J}$$

$\lambda = \frac{hc}{\Delta E} = \frac{1.9875 \times 10^{-25}}{1.809 \times 10^{-19}} = 1.099 \times 10^{-6}\text{ m} = 1.1\ \mu\text{m},$ $\boxed{\lambda = 1.1\ \mu\text{m}}$

Q57. An electron is accelerated through a potential of 200 V and then it passes through a slit of width 1.0 nm held normal to the path of the electron. Assuming the uncertainty relation $\Delta x \Delta p_x \approx \hbar/2$, maximum scattering angle of the electron after the slit is $\alpha \times 10^{-3}$ radian.

The value of α is ___ . (Rounded off to nearest integer)

[Given $\hbar = 1.054 \times 10^{-34}\text{ J}s$]

Ans.: 6.6 To 7.4

Solution: $p=\sqrt{2m_e eV}=\sqrt{2\times 9.109\times 10^{-3}\ \times 1.6\times 10^{-1}\ \times 200}$

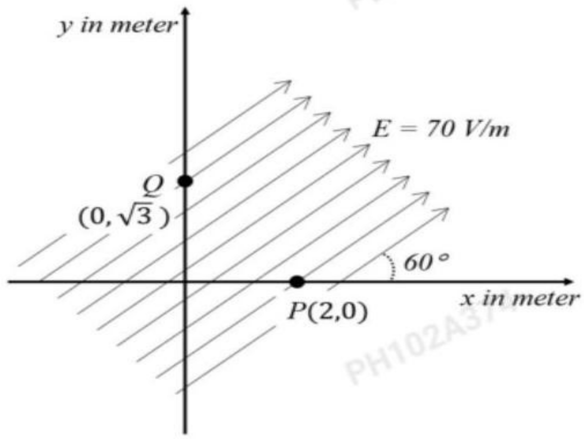
$=7.635\times 10^{-24}$ kg. m/s.

$\Delta p_x=\hbar/(2\Delta x)=1.054\times 10^{-34}/(2\times 10^{-9})=5.27\times 10^{-26}$ kg $\cdot$ m/s.

$$\theta=\Delta p_x/p=5.27\times 10^{-2}\ /7.635\times 10^{-24}=6.90\times 10^{-3}\ \text{rad}\Longrightarrow \alpha\approx 7$$

$$\boxed{\alpha\approx 7\ (6.9\times 10^{-3}\ \text{rad})}$$

Q58. A uniform electric field of $70\ \text{V/m}$ makes an angle of 60° with the positive x axis, as shown in the figure. The potential difference between the points P and Q which are $2m$ and $\sqrt{3}m$ away from the origin, in Volts, is_______ . (Rounded off to one decimal place)



Ans.: 34.5 To 35.5

Solution: $\vec{E}=35\hat{\imath}+35\sqrt{3}\hat{\jmath}\ \text{V/m}$. $P=(2,0)$, $Q=(0,\sqrt{3})$.

$$V_P-V_Q=-\vec{E}\cdot(\vec{r}_P-\vec{r}_Q)=-\left(35\times 2+35\sqrt{3}\times(-\sqrt{3})\right)$$
$$=-(70-105)=35\ \text{V}$$

$$\boxed{V_P-V_Q=35\ \text{V}}$$

Q59. Consider a simple pendulum of length l and time period T. In a laboratory experiment, the time for 100 oscillations is measured to be $80s$ using a stopwatch with least count 1 s. The gravitational constant is known with a percentage error of 2.5%.

The percentage error in the measured length of the pendulum, in %, is_________。

(Answer in integer)

Ans.: 5 To 5

Solution: $l=gT^2/4\pi^2\Rightarrow \%\delta l=\%\delta g+2\%\delta T$.

$\%\ \delta T=(\delta t/t)\times 100=(1/80)\times 100=1.25\%$.

$\%\ \delta l=2.5+2\times 1.25=5\%$

Q60. A particle of mass m in a potential $V(x)=\frac{1}{2}kx^2$ is described by normalized wavefunction $\sum_{n=0}^{\infty}(\sqrt{2})^{-(n+1)}\phi_n(x)$, where $\{\phi_n\}$ are the eigenstates of the particle. The energy corresponding to the wavefunction, in units of $\frac{h}{\pi}\sqrt{\frac{k}{m}}$, is

_____________. (Rounded off to two decimal places)

[Given: $\sum_{n=0}^{\infty}(a)^{-n}=\frac{a}{a-1}, a>1$]

Ans.: 0.75 To 0.75

Solution: $|\psi\rangle=\sum_{n=0}^{\infty}(\sqrt{2})^{-(n+1)}|\phi_n\rangle$ where $\langle\phi_n \mid \phi_m\rangle=\delta_{nm}$

$$\langle\psi \mid \psi\rangle=\sum_n \frac{1}{2^{n+1}}=\frac{1}{2}+\frac{1}{2^2}\ldots=\frac{1}{2}\left(\frac{1}{1-\frac{1}{2}}\right)=1$$

$$\langle E\rangle=\langle\psi|H|\psi\rangle=\sum_n\left(n+\frac{1}{2}\right)\mathrm{h}\omega\frac{1}{2^{n+1}}=\mathrm{h}\omega\left(\sum_n n\cdot\frac{1}{2^{n+1}}+\frac{1}{2}\sum_{n=0}\frac{1}{2^{n+1}}\right)=\mathrm{h}\omega\sum_n n\cdot\frac{1}{2^{n+1}}+\frac{\mathrm{h}\omega}{2}$$

$$\sum_n x^n=\frac{1}{1-x}\Rightarrow\sum_n nx^{n-1}=\frac{1}{1-x}\Rightarrow\sum_n nx^n=\frac{x}{1-x}\Rightarrow\frac{1}{2}\sum_n n\left(\frac{1}{2}\right)^n=\frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}=\frac{2}{2}=1$$

$$\mathrm{h}\omega\sum_n n\cdot\frac{1}{2^{n+1}}+\frac{\mathrm{h}\omega}{2}=\mathrm{h}\omega+\frac{\mathrm{h}\omega}{2}=\frac{3\,\mathrm{h}\omega}{2}$$