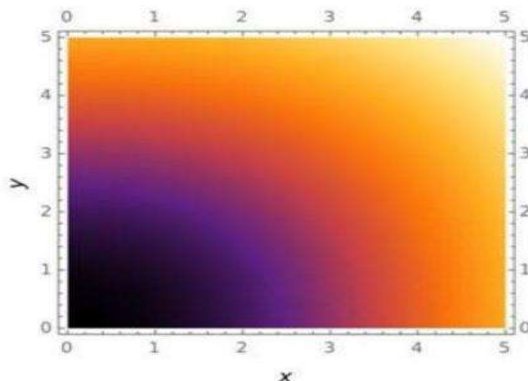


NET December 2025

PART-A

- Q1. Five students graduated from a college, not all in the same year, after each has studied for four years. If batchmates Jiten and Anwar were between Ramesh and Prakash but senior to Sam while Ramesh had left the college before Jiten took admission, then it is certain that
1. Anwar was the most senior
 2. Ramesh was the most senior
 3. Sam was the most junior
 4. Prakash was the most junior
- Q2. Suppose $a_1, a_2, \dots, a_{300}$ are integers such that $a_{i-1} + a_i + a_{i+1} = 2025$ for all $i = 2, 3, \dots, 299$. If $a_7 = -5, a_9 = 37$, then the value of a_{106} is
1. 1993
 2. 37
 3. -5
 4. 2030
- Q3. The geometric mean of 100 observations is 25. If each observation is multiplied by 4, what will be the new geometric mean?
1. 100
 2. 50
 3. 25
 4. $(25 \times 4)^{1/2}$
- Q4. Some, but not all, faces of a six-faced cubical fair die are painted red (R) and the remaining green (G); and the die is thrown until red faces come up on top 4 times. Consider the following sequences of colours listed left to right as they appear on the top.
- A: GRRRR
- B: GRGRRR
- Which one of the following is true?
1. A is more probable than B
 2. B is more probable than A
 3. Both have the same probability
 4. Whether A or B is more probable depends upon how many faces are painted green

- Q5. The following plot shows temperature as a function of x and y . Along which path is the temperature change minimum?

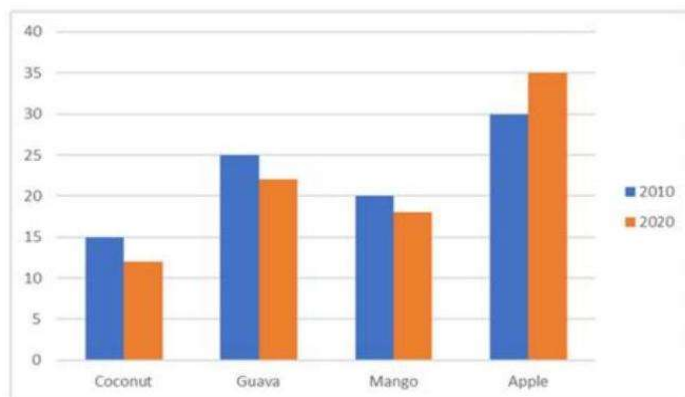


1. $x = \text{constant}$ or $y = \text{constant}$
 2. $\frac{y}{x^2} = \text{constant}$
 3. $y^2 + x^2 = \text{constant}$
 4. $yx = \text{constant}$
- Q6. Which among the following cities can be said most appropriately to bear the same relation to Tamil Nadu that Pune bears to Maharashtra; Surat to Gujarat and Asansol to West Bengal?
1. Tirupati
 2. Mysore
 3. Chennai
 4. Coimbatore
- Q7. A recent survey suggests that the total fertility rate in a country has fallen below 2.1, the population replacement ratio. This necessarily implies that the
1. Infant mortality rate has increased reducing the net fertility ratio.
 2. Total population will decline.
 3. Population of young people is going to increase with a faster rate in the long run if the same status continues.
 4. proportion of elderly people is going to decrease in the long run if the same status continues.
- Q8. The value of $1 + \left(\frac{1}{2^1} + \frac{1}{3}\right) + \left(\frac{1}{2^2} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}\right) + \dots + \left(\frac{1}{2^9} + \dots + \frac{1}{1023}\right)$ lies between
1. 2 and 10
 2. 11 and 20
 3. 21 and 30
 4. 31 and 40
- Q9. A lady bought some apples, each costing Rs. 25, and some bananas each costing Rs 6, for a total of Rs. 378. In how many ways could she have chosen the numbers of apples and bananas?
1. 1
 2. 2
 3. 3
 4. 4
- Q10. In an exam, questions of three difficulty levels hard, medium, and easy fetch respectively 7,3, and 2 marks if correct and 0 if incorrect. Three students got 30 marks each but in three different ways, though the total number of questions correctly answered by each student was the same. Then what could be the total number of questions correctly answered by each of these students?
1. 12
 2. 10
 3. 9
 4. 6

Q11. In a class, 40% and 20% students passed in Mathematics and Physics, respectively, and 10% students passed in both subjects. What is the probability of a randomly selected student to have passed in Physics if the student already passed in Mathematics?

1. $\frac{1}{2}$ 3. $\frac{1}{20}$ 4. $\frac{1}{4}$ 4. $\frac{2}{25}$

Q12. The numbers (in millions) of coconut, guava, mango and apple trees in a region in 2010 and 2020 are shown in the following figure.



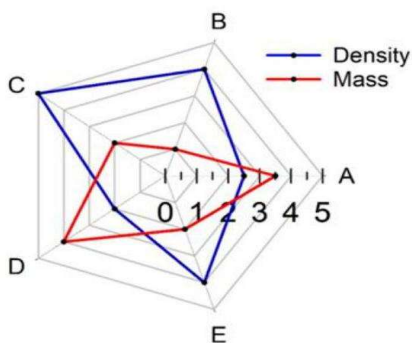
The maximum relative change in numbers was for

1. coconut trees 2. guava trees 3. mango trees 4. apple trees

Q13. How many 5-digit numbers can be formed from the digits 0, 2, 3, 4, 6, 7 and 9, using each at most once, which are divisible by 5 ?

1. 120 2. 240 3. 360 4. 720

Q14. The following figure shows densities and masses of five objects (A to E).



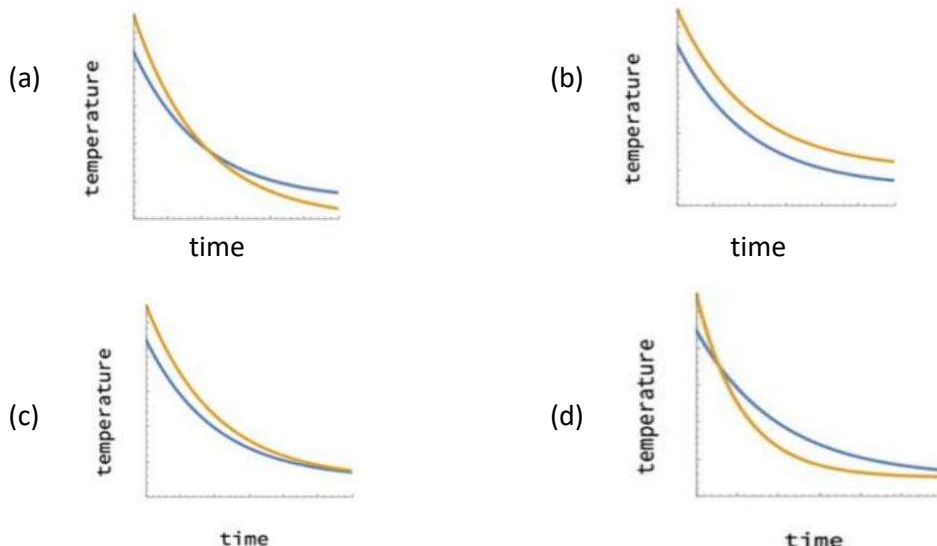
The object with the largest volume is ____ .

1. A 2. B 3. D 4. E

Q15. What is the minimum number of pourings required to transfer exactly 6 L of water from a 12 L fully filled container to an 8 L empty container when a 5 L empty from a 12 L fully filled container to an 8 L empty container when a 5 L empty container is also available to use?

1. 4 2. 5 3. 6 4. 7

Q16. Two identical metal bars are heated to different temperatures and allowed to cool in the same surroundings. Which one of the following figures correctly shows their cooling curves?



Q17. In a community, some artists are teachers, no teacher is a painter, all painters are artists, and all teachers are professionals. Then it can be definitely asserted that

1. no painter is a professional
2. all artists are professionals
3. no professionals are teachers
4. some artists are professionals

Q18. Alloy A is formed by mixing iron (Fe) and nickel (Ni) in the ratio 3: 4, while alloy B is formed by mixing Fe and Ni in the ratio 9: 5. If equal quantities of alloys A and B are melted together to form a new alloy C, what will be the ratio of Fe to Ni in the alloy C?

1. 4: 3
2. 5: 3
3. 15: 13
4. 13: 9

Q19. Three periodic events repeat every 24 seconds, 54 seconds, and 56 seconds. If they coincide at 10:20:00, when will they next coincide?

1. 10: 35: 12
2. 10:45:20
3. 10: 45: 12
4. 10: 35: 20

Q20. The minimum height of a plane vertical mirror that will allow a 6-feet tall person to see himself fully in it

1. depends on the distance between the person and the mirror
2. is 3 feet
3. is 4.5 feet
4. is 6 feet

PART B

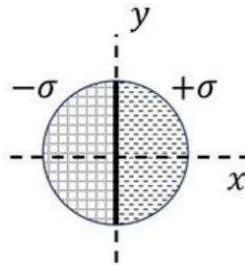
Q21. A 1-dimensional random walker's displacement is always positive and is equally likely to be anywhere in the range $[L, L + b]$. After N statistically independent steps the mean distance covered by the walker is

1. NL 2. $N\sqrt{L^2 + b^2}$ 3. $N\left(L + \frac{b}{2}\right)$ 4. $NL + b\sqrt{N}$

Q22. A classical mono-atomic ideal gas is in thermal equilibrium at temperature T . The velocity of a molecule of this gas, of mass m , is $\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$. The value of the ensemble average $\langle v_x^2 v_y^2 \rangle$ is

1. $\left(\frac{k_B T}{2m}\right)^2$ 2. $\left(\frac{k_B T}{m}\right)^2$ 3. $\left(\frac{3k_B T}{2m}\right)^2$ 4. $\left(\frac{2k_B T}{m}\right)^2$

Q23. A circular disc of radius R is made of 2 halves (as shown in the figure), separated by a dielectric of negligible thickness (along the y axis.)



If the surface charge density on the right half is $+\sigma$ and that on the left half is $-\sigma$, the dipole moment of the disc is

1. $P_x = 0, P_y = \frac{1}{3}\sigma R^3$ 2. $P_x = 0, P_y = \frac{4}{3}\sigma R^3$
 3. $P_x = \frac{1}{3}\sigma R^3, P_y = 0$ 4. $P_x = \frac{4}{3}\sigma R^3, P_y = 0$

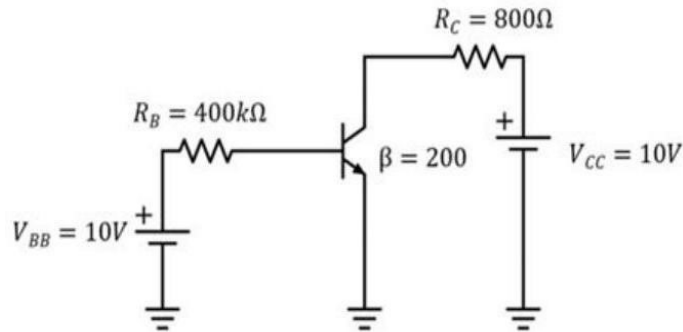
Q24. If C be the unit circle traversed clockwise, then the integral $\oint_C dz |1 + 2z|^2$ equals

1. $-4\pi i$ 2. $-\pi i$ 3. 0 4. $-2\pi i$

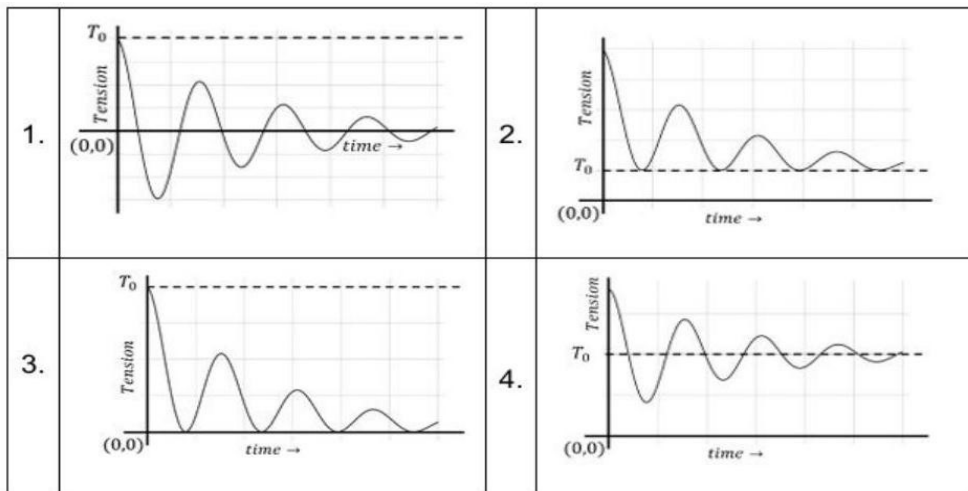
Q25. If $\hat{L}$ is the angular momentum operator for a quantum particle, then $\hat{L} \times \hat{L}$ is

1. $\hbar^2$ 2. $-i\hbar\hat{L}$ 3. 0 4. $i\hbar\hat{L}$

- Q26. In the transistor circuit given below the voltage V_{CC} fluctuates by 5%. Then the fluctuation in V_{CE} would be closest to (take $V_{BE} = 0.7 \text{ V}$)



1. 8% 2. 7% 3. 6% 4. 5%
- Q27. An isolated two-electron quantum state is described by the orbital angular momentum quantum number l and the total spin S . An allowed value of l and S is
1. $S = 1, l = 0$ 2. $S = 0, l = 1$ 3. $S = 1, l = 1$ 4. $S = 1, l = 2$
- Q28. A bow has a taut string of tension T_0 (when it is at rest). The string is pulled and released at time $t = 0$. Which plot best represents the tension in the bow string as a function of time?



- Q29. The residue of $f(z) = \frac{\cos \pi z}{(1-z^2)^3}$ at $z = 1$ is

1. $\frac{\pi^2}{16}$ 2. $\frac{3}{16}$ 3. $\frac{3+\pi^2}{16}$ 4. $\frac{3-\pi^2}{16}$

Q30. Five indistinguishable atoms are sitting on the distinguishable vertices of a pentagon. The atoms can be in one of the two states: g with energy 0, and e with energy E . However neighbouring atoms cannot both be in the e state. The partition function of this system at temperature T , is

- | | |
|--|---|
| 1. $1 + 5e^{-\frac{E}{k_B T}} + 2e^{-\frac{2E}{k_B T}}$ | 2. $1 + 5e^{-\frac{E}{k_B T}} + 3e^{-\frac{2E}{k_B T}}$ |
| 3. $1 + 5e^{-\frac{E}{k_B T}} + 10e^{-\frac{2E}{k_B T}}$ | 4. $1 + 5e^{-\frac{E}{k_B T}} + 5e^{-\frac{2E}{k_B T}}$ |

Q31. A spin- $\frac{1}{2}$ particle is in a magnetic field $\vec{B} = B_x \hat{x} + B_y \hat{y}$ for which the spin-independent Hamiltonian is $\hat{H} = -A \hat{S} \cdot \vec{B}$ (A is a positive constant and $\hat{S}$ is the spin-operator). The eigenvalues of the Hamiltonian are

- | | | | |
|--|---|--|------|
| 1. $\pm A \frac{\hbar}{2} (B_x + B_y)$ | 2. $\pm A \frac{\hbar}{2} \sqrt{B_x B_y}$ | 3. $\pm A \frac{\hbar}{2} (B_x^2 + B_y^2)^{\frac{1}{2}}$ | 4. 0 |
|--|---|--|------|

Q32. B, C and F are three systems which have particles of same mass and same number density kept at the same low temperature T . Here C is a classical ideal gas, F is a free Fermi gas and B is a free Bose gas, with pressures P_C, P_F and P_B respectively. Then

- | | | | |
|----------------------|----------------------|----------------------|----------------------|
| 1. $P_B > P_C > P_F$ | 2. $P_F > P_C > P_B$ | 3. $P_C > P_F > P_B$ | 4. $P_C > P_B > P_F$ |
|----------------------|----------------------|----------------------|----------------------|

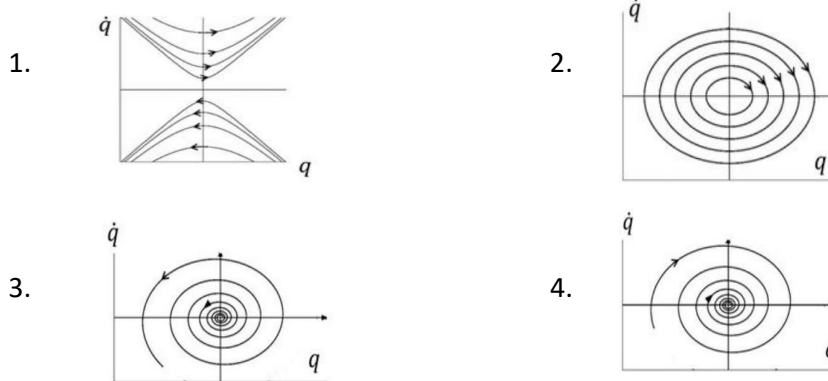
Q33. A fraction $\frac{2}{3}$ of the volume of a parallel plate capacitor is filled with dielectric of relative permittivity $\kappa = 1.5$ (as shown in the figure).

$$\kappa = 1.5$$

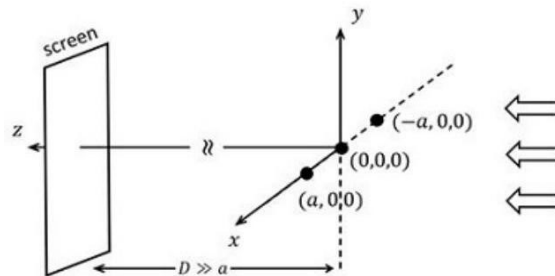
When the filled volume is reduced to $\frac{1}{3}$ of the total volume, the capacitance is smaller by a factor of

- | | | | |
|------------------|------------------|------------------|------------------|
| 1. $\frac{7}{8}$ | 2. $\frac{5}{6}$ | 3. $\frac{3}{4}$ | 4. $\frac{2}{3}$ |
|------------------|------------------|------------------|------------------|

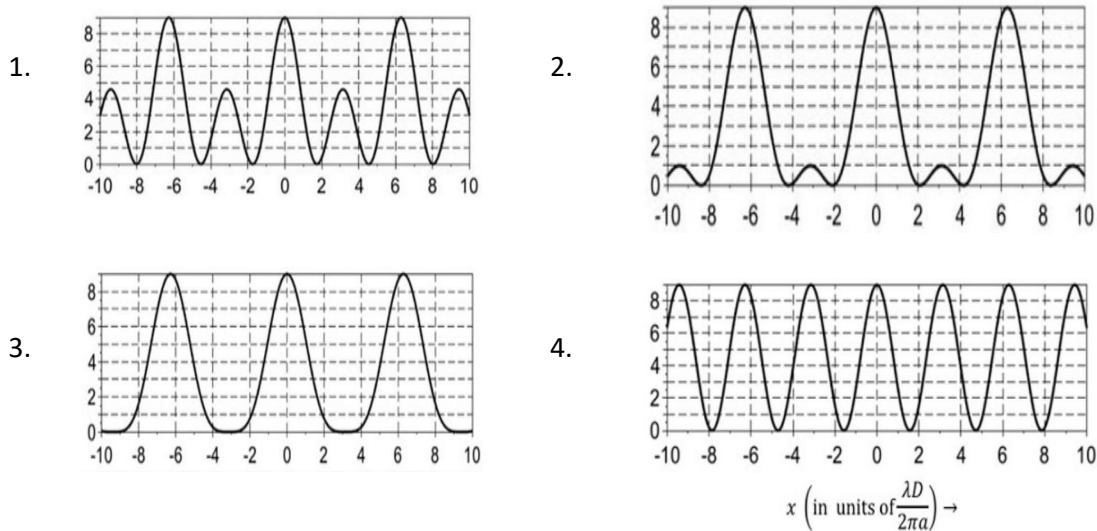
Q34. Which of the following figures best represents the motion of an oscillator described by the differential equation $\ddot{q} + \dot{q} + q = 0$ in $q - \dot{q}$ plane?



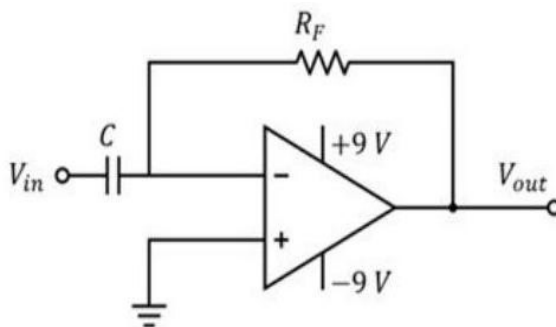
Q35. Three identical pinholes separated by distance a along the x -axis are illuminated by a collimated monochromatic coherent beam of light (wavelength λ) as shown in the figure below.



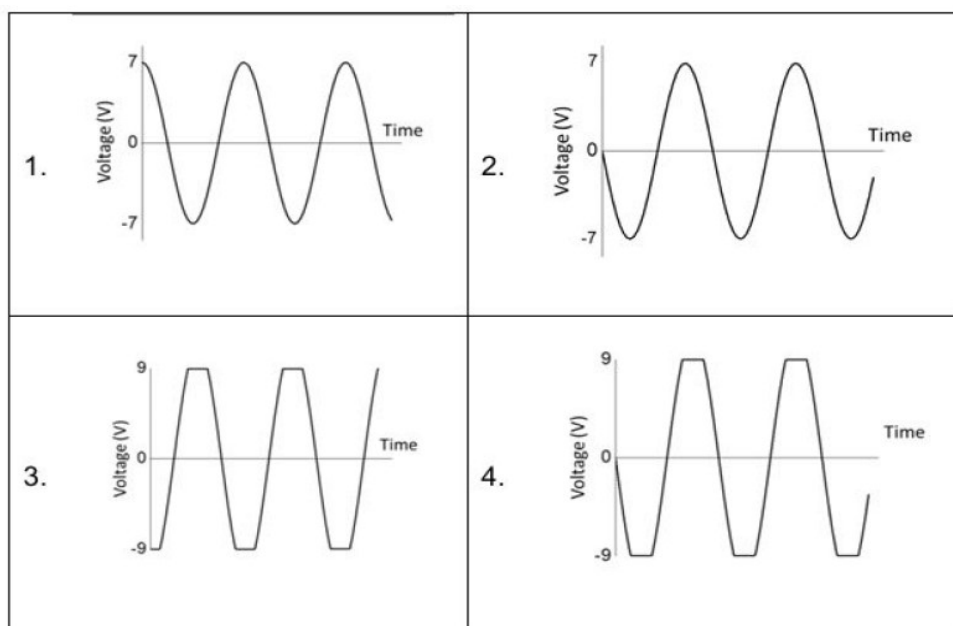
The intensity (in arbitrary units) pattern of fringes obtained on a screen kept at distance D ($D \gg a$) along the z -axis is best represented by



- Q36. Two well separated conducting spheres (A and B) of radii 10 cm and 20 cm carry charges +30 C and -20 C respectively. When they are connected by a thin conducting wire, the final charge on A is Q_A and that on B is Q_B . The values of Q_A and Q_B respectively, are closest to
1. 6.7 C and 3.3 C 2. 2.0 C and 8.0 C 3. 3.3 C and 6.7 C 4. 8.0 C and 2.0 C
- Q37. A quantum mechanical particle in a harmonic potential has the wave function $\frac{1}{\sqrt{2}}[\psi_0(x) + \psi_1(x)]$ at $t = 0$, where $\psi_0(x)$ and $\psi_1(x)$ are the wave functions of the ground state and the first excited state respectively. If the frequency of the oscillator is ω , the probability density of finding the particle at x after time $t = \frac{\pi}{\omega}$ is
1. $\frac{1}{2}|\psi_1(x) - \psi_0(x)|^2$ 2. $\frac{1}{2}|\psi_1(x) + \psi_0(x)|^2$
 3. $\frac{1}{2}|\psi_1(x) - i\psi_0(x)|^2$ 4. $\frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_0(x)|^2$
- Q38. In the circuit shown below, the input voltage is $V_{in}(t) = 0.3\sin 50t$ (Volts) and $C = 100\mu F, R_F = 10k\Omega$.



Considering the op-amp to be ideal and neglecting the transients, the best representation of the output voltage $V_{out}(t)$ is



Q39. A fly of mass m rests on the edge of a uniform horizontal disc of radius R and mass M . The disc is free to rotate about the vertical axis through its centre. Initially the disc is stationary. The fly starts to walk around the circumference of the disc with speed v relative to the disc. The speed of the fly for a stationary observer is

1. $\frac{mv}{M+2m}$ 2. $\frac{Mv}{M-2m}$ 3. $\frac{Mv}{M+2m}$ 4. $\frac{mv}{M-2m}$

Q40. Commutator of two matrices A and B is defined as $[A, B] = AB - BA$ and the anti-commutator as $\{A, B\} = AB + BA$. If $\{A, B\} = 0$. Then we can express $[A, BC]$ as

1. $B\{A, C\}$ 2. $-B[A, C]$ 3. $-B\{A, C\}$ 4. $[A, B]C$

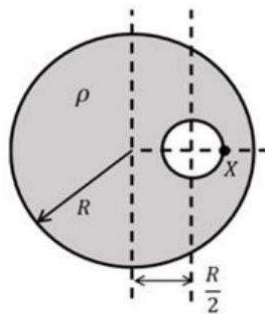
Q41. A quantum particle of mass m is moving in a potential

$$V(x, y) = \frac{m\omega^2}{8} [5(x^2 + y^2) + 8xy].$$

The lowest energy eigenstate with degeneracy has an energy

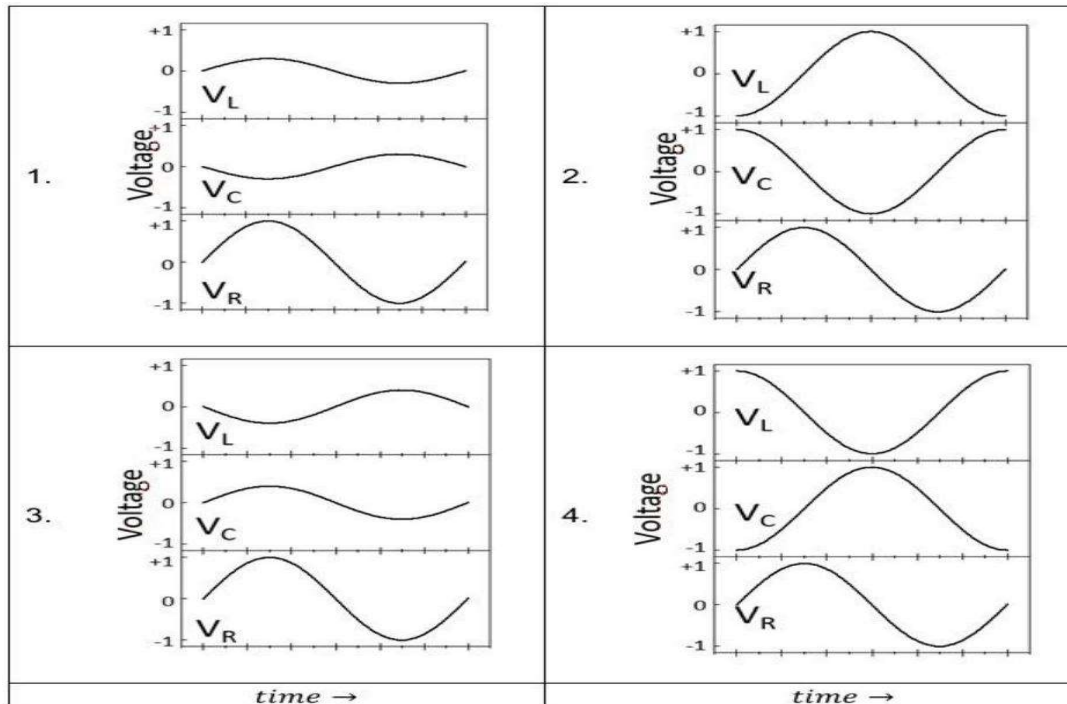
1. $\frac{7}{2}\hbar\omega$ 2. $\frac{3}{2}\hbar\omega$ 3. $4\hbar\omega$ 4. $\frac{5}{2}\hbar\omega$

Q42. A solid sphere of radius R has uniform charge density ρ . A spherical volume of radius $\frac{R}{4}$ is scooped out from the sphere as shown. The electric field at the point marked X is ($\hat{r}$ denotes the unit vector along the radially outward direction)



1. $\frac{2\rho R}{9\epsilon_0} \hat{r}$ 2. $\frac{\rho R}{6\epsilon_0} \hat{r}$ 3. $\frac{\rho R}{3\epsilon_0} \hat{r}$ 4. $\frac{\rho R}{9\epsilon_0} \hat{r}$

- Q43. In an ideal series LCR circuit, which one of the following best represents the steady-state voltage waveforms V_L, V_C, V_R (only one cycle is shown) across L, C and R as a function of time at resonance frequency

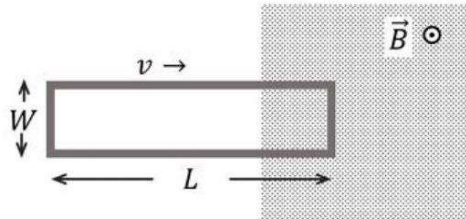


- Q44. Let $p(x)$ be the probability density function for a positive real variable x , and $g(\alpha) = \int_0^\infty p(x)e^{-\alpha x}dx$. If $g'(\alpha)$ and $g''(\alpha)$ are respectively first and second derivatives of $g(\alpha)$ with respect to α , which of the following gives the variance of x ?
1. $g''(0) - [g'(0)]^2$
 2. $g''(0) + [g'(0)]^2$
 3. $[g''(0) - g'(0)]^2$
 4. $\frac{g''(0)}{g'(0)g(0)}$
- Q45. The position and velocity vector of a particle changes from $\vec{R}_1$ to $\vec{R}_2$ and $\vec{V}_1$ to $\vec{V}_2$ as time changes from t_1 to t_2 . If $\vec{r}(t), \vec{a}(t)$ are instantaneous position and acceleration vectors of the particle then the integral $I = \int_{t_1}^{t_2} dt(\vec{r}(t) \times \vec{a}(t))$ is
1. $\vec{R}_2 \times \vec{V}_1 - \vec{R}_1 \times \vec{V}_2$
 2. $\vec{R}_2 \times \vec{V}_2 - \vec{R}_1 \times \vec{V}_1$
 3. $\vec{R}_1 \times \vec{V}_1 - \vec{R}_2 \times \vec{V}_2$
 4. $\vec{R}_1 \times \vec{V}_2 - \vec{R}_2 \times \vec{V}_1$

PART C

- Q46. Electromagnetic waves of frequency ω are incident on an electron gas, whose relaxation time is τ . Let σ_{low} and σ_{high} represent the respective electrical conductivities of the gas in low frequency ($\omega\tau \ll 1$) and high frequency ($\omega\tau \gg 1$) limits. The ratio ($\sigma_{\text{low}}/\sigma_{\text{high}}$) is
1. inversely proportional to ω^2
 2. directly proportional to ω^2
 3. independent of ω
 4. directly proportional to ω .
- Q47. A hydrogen atom is in a weak external magnetic field $\vec{B}$. Consider an electron of this atom with ($l = 1, s = \frac{1}{2}$) and total $j = \frac{3}{2}$. There are multiple energy levels for this electron due to the magnetic field. The energy spacing between any two adjacent levels (in units of $\mu_B B$) is
1. $\frac{1}{2}$
 2. $\frac{1}{3}$
 3. $\frac{3}{4}$
 4. $\frac{4}{3}$
- Q48. An optical cavity of a laser, formed by two plane mirrors, is filled up with an active medium. The medium emits radiation at wavelengths 450 nm, 600 nm, and 750 nm. If the medium is continuously pumped, at which cavity length among the following, will all three wavelengths be amplified?
1. $750\mu\text{ m}$
 2. $1500\mu\text{ m}$
 3. $600\mu\text{ m}$
 4. $450\mu\text{ m}$
- Q49. A binary alloy consists of N_A number of A-type and N_B number of B-type atoms. The atoms sit on the sites of a simple cubic lattice and the nearest neighbours interact with each other. Assume an attractive interaction energy $-J$ ($J > 0$) between two like neighbours (AA or BB pair) and a repulsive interaction energy $+J$ between two unlike neighbours (AB pair). If N is the total number of sites, then the average energy of the system at a very high temperature ($k_B T \gg J$) is
1. $-3J \frac{(N_A - N_B)^2}{N}$
 2. $3JN$
 3. $3J \frac{(N_A + N_B)^2}{N}$
 4. $-3J(N_A - N_B)$
- Q50. A spherical gaseous ball of radius 15 m was formed with a temperature $T = 3 \times 10^5$ K. The gas expands adiabatically and its temperature drops to 5×10^3 K. Given $\gamma = \frac{5}{3}$ for this gas, the radius of the ball becomes approximately
1. 212 m
 2. 86 m
 3. 137 m
 4. 116 m

- Q51. A long rectangular metallic loop of width W and length $L (>> W)$ starts entering a region, where there is a uniform magnetic field B perpendicular to the plane of the loop. The resistance of the loop is R and its mass is M . If v_0 is the velocity of the loop just before entering the region, then neglecting the self-inductance effect, the velocity at a later time t is



1. $v(t) = \frac{v_0}{1 + \frac{B^2 W^2}{MR} t}$
 2. $v(t) = \frac{v_0}{1 + \left(\frac{B^2 W^2}{MR} t\right)^2}$
 3. $v(t) = v_0 e^{-\frac{B^2 W^2}{MR} t}$
 4. $v(t) = \frac{v_0}{1 + \ln\left(1 + \frac{B^2 W^2}{MR} t\right)}$
- Q52. The excitations of a three-dimensional solid are bosonic in nature and their energy dispersion is given by $\epsilon_k \propto k^2$, in the long wavelength limit. If the chemical potential of the system is zero, the temperature dependence of specific heat of the system at low temperature is proportional to
1. T^3
 2. $T^{\frac{3}{2}}$
 3. $T^{\frac{5}{2}}$
 4. $T^{\frac{1}{2}}$
- Q53. A one-dimensional quantum harmonic oscillator with frequency ω is in its ground state. Its normalised wave function is given by

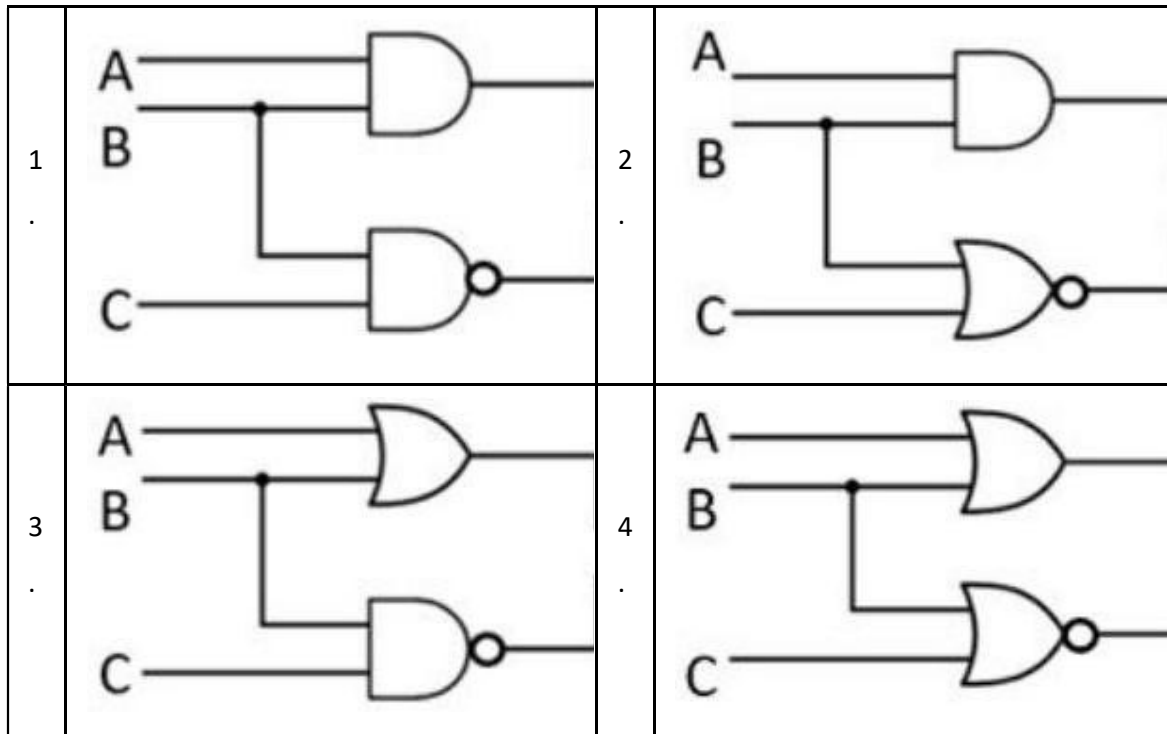
$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left[-\frac{m\omega}{2\hbar} x^2\right].$$

The frequency is suddenly increased to 2ω . The probability of finding the particle in its new ground state is

1. $\frac{2\sqrt{2}}{3}$
2. $\left(\frac{2\sqrt{2}}{3}\right)^{\frac{1}{2}}$
3. $\frac{2}{3}$
4. $\left(\frac{3}{2\sqrt{2}}\right)^{\frac{1}{2}}$

Q54. The digital logic circuit that would give the following truth table

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



Q55. A monochromatic plane wave is incident normally from a dielectric medium A onto another dielectric medium B . The indices of refraction satisfy $n_A < n_B$. One-fourth of the incident energy is reflected back into medium A . Let $\vec{E}$ be the resultant electric field due to the superposition of the incident wave and the reflected wave. Then, the ratio of the two timeaverages $\langle \vec{E}^2 \rangle_{\min} / \langle \vec{E}^2 \rangle_{\max}$ is

1. $\frac{1}{8}$ 2. $\frac{1}{9}$ 3. $\frac{4}{9}$ 4. $\frac{1}{4}$

Q56. The Hamiltonian of a simple pendulum consisting of mass m attached to a massless string of length l is $H = \frac{P_\theta^2}{2ml^2} + mgl(1 - \cos \theta)$. If L denotes the Lagrangian, then $\frac{dL}{dt}$ is

1. $\frac{g}{l} P_\theta \cos \theta$ 2. $\frac{-g}{l} P_\theta \sin \theta$ 3. $\frac{-2g}{l} P_\theta \sin \theta$ 4. $\frac{g}{l} P_\theta \cos (2\theta)$

Q57. Consider a one-dimensional chain of atoms with lattice constant a . The energy of an electron with wave-vector k is $\epsilon(k) = \mu - 2\gamma \cos ka$, where μ and γ are constants. If an electric field $\vec{E}$ is applied along the chain, the time dependent velocity of the electron is proportional to (assume initial wave vector $k = k_0$ at $t = 0$)

1. $\sin^2 \left(k_0 a - \frac{eEa}{\hbar} t \right)$ 2. $\cos \left(k_0 a - \frac{eEa}{\hbar} t \right)$
 2. $\sin \left(k_0 a - \frac{eEa}{\hbar} t \right)$ 4. $\cos^2 \left(k_0 a - \frac{eEa}{\hbar} t \right)$.

Q58. The bond dissociation energy of OH molecule is 4.18 eV with rotational constant 18.8 cm^{-1} . For rotational induced dissociation, the minimum value of rotational quantum number is closest to

1. 114 2. 454 3. 45 4. 90

Q59. For a particle in the angular momentum state $|l = 4, m_l = 2\rangle$, the expectation value of the operator $L_x L_y$ is

1. $-\hbar^2$ 2. $\hbar^2$ 3. $-i\hbar^2$ 4. $i\hbar^2$

Q60. Find the curve that extremizes the functional

$$I(y) = \int_0^1 \left[\left(\frac{dy}{dx} \right)^2 + 12xy \right] dx$$

for the given boundary conditions $y(0) = 0$ and $y(1) = 1$

1. $y = x^3$ 2. $y = x^2$ 3. $y = 2x^2 - x$ 4. $y = 3x^3 - 2x^2$

Q61. In a one-dimensional chain of atoms, the phonon energy dispersion is given by $E = A|\sin ka|$. Here, A is a constant, k is a vector in the reciprocal space and a is lattice spacing. The density of states is proportional to

1. $\frac{1}{\sqrt{A^2 - E^2}}$ 2. $\frac{1}{\sqrt{A^2 + E^2}}$ 3. $\frac{1}{\sqrt{A - E}}$ 4. $\frac{1}{\sqrt{A + E}}$

Q62. A sequence of polynomial $Q_n(x)$ [$n = 0, 1, 2, \dots$] satisfies the recursion relation

$$Q_{n+1}(x) - 2xQ_n(x) + 2nQ_{n-1}(x) = 0, \text{ for all } n \geq 0 \text{ [here } Q_{-1}(x) = 0 \text{]}$$

The generating function for the polynomials, $g(x, t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} Q_n(x)$, satisfies

1. $\frac{\partial g}{\partial t} = 2(t+x)g$ 2. $\frac{\partial g}{\partial t} = 2(x-t)g$ 3. $\frac{\partial g}{\partial t} = \frac{2(x-t)}{t}g$ 4. $\frac{\partial g}{\partial t} = 2 + (x+t)g$

Q63. For a spherical nucleus, consider the interior charge distribution to be

$$\rho(r) = \frac{\rho_0}{1 + \exp [(r - R)/a]}$$

where ρ_0 , R and a are constants of appropriate dimensions. In the limit $a \rightarrow 0^+$, the number of protons (charge e) inside a sphere of radius $2R$ is given by

1. $\frac{2\rho_0}{e} \left(\frac{4}{3} \pi R^3 \right)$ 2. $\frac{\rho_0}{e} \left(\frac{4}{3} \pi R^3 \right)$ 3. $\frac{8\rho_0}{e} \left(\frac{4}{3} \pi R^3 \right)$ 4. $\frac{4\rho_0}{e} \left(\frac{4}{3} \pi R^3 \right)$

Q64. Consider a one-dimensional lattice (with lattice spacing a) along X-axis with sites labelled by $x = 0, 1, 2, \dots, L$. A particle carrying a charge $-q$ can occupy any one of these sites. An electric field of strength E is applied in the positive x-direction. The average energy of the particle at a temperature T (in the limit $L \rightarrow \infty$) is $\left(\beta = \frac{1}{k_B T} \right)$

1. $\frac{Eq a}{e^{\beta Eq a} - 1}$ 2. $\frac{Eq a}{1 + e^{\beta Eq a}}$ 3. $\frac{Eq a}{2}$ 4. $-Eq a$

Q65. A cubic sample of edge length L is maintained at a temperature of 4 K. The speed of sound in the material of the sample is 5×10^3 m/s. The minimum value of L required to excite the lowest frequency phonon mode is closest to

1. 10 nm 2. 30 nm 3. 20 nm 4. 5 nm

Q66. Consider the cross-sections

$$\sigma_1 = \sigma(p + n \rightarrow \Delta^+ + n) \text{ and } \sigma_2 = \sigma(p + n \rightarrow \Delta^0 + p)$$

where the (Δ^+, Δ^0) are part of the baryon decuplet. Then

1. one of the $\sigma_{1,2}$ vanishes identically 2. $\sigma_1 \gg \sigma_2$, with both being non-zero
3. $\sigma_1 \ll \sigma_2$, with both being non-zero 4. $\sigma_1 \approx \sigma_2$

Q67. In a heap of 20 biased coins, 17 have a 60% probability of showing heads while the other three special coins have a 90% probability of doing so. A coin is selected at random and tossed. If the result is a head, the probability that it was one of the three special coins is best approximated by

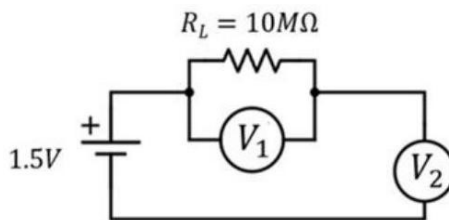
1. 0.18 2. 0.14 3. 0.21 4. 0.26

Q68. The Lagrangian $L = L(x, y, \dot{x}, \dot{y})$ is invariant under the transformation $x \rightarrow x + \epsilon y$ and $y \rightarrow y + \epsilon x$, for any infinitesimal real parameter ϵ .

If P_x, P_y denote canonically conjugate momenta corresponding to x, y respectively, then the corresponding conserved quantity is

1. $yP_x - xP_y$ 2. $yP_x + xP_y$ 3. $xP_x + yP_y$ 4. $xP_x - yP_y$

Q69. In the circuit shown below, the input impedance of voltmeters V_1 and V_2 are $10\text{M}\Omega$. If $R_L = 10\text{M}\Omega$ and $V_{\text{in}} = 1.5\text{ V}$.



The measured voltages by V_1 and V_2 are closest to

1. 0.5 V and 1.0 V, respectively 2. 0 V and 1.5 V, respectively
3. 1.5 V and 0 V, respectively 4. 1.0 V and 0.5 V, respectively

Q70. Suppose that the volume and the surface terms are the most dominant ones in the semi-empirical formula for the binding energy of a nucleus. Let C_s and C_v be the coefficients of the surface and volume terms. Which of the following is a criterion for stability of the nucleus?

1. $A > \left(\frac{C_s}{C_v}\right)^3$ 2. $A < \left(\frac{C_s}{C_v}\right)^3$ 3. $A > \left(\frac{2C_s}{3C_v}\right)^3$ 4. $A < \left(\frac{2C_s}{3C_v}\right)^3$

Q71. The Lagrangian of a two-particle system is given by

$$L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_1\dot{q}_2) - \frac{1}{2}m\omega^2\left(q_1^2 + q_2^2 + \frac{1}{2}q_1q_2\right).$$

The normal mode frequencies (in units of ω) are

1. $\sqrt{\frac{5}{3}}, \frac{1}{2}$ 2. $\sqrt{\frac{5}{6}}, \sqrt{\frac{3}{2}}$ 3. $\sqrt{\frac{6}{5}}, \sqrt{2}$ 4. $\sqrt{\frac{5}{6}}, \sqrt{2}$

Q72. A thermistor measures an object's temperature T , by measuring its resistance R according to $R = AT^{-n}$, where A and n are positive constants. The observed resistances for different values of temperature (including environmental and instrumental sources of error) are

$T(K)$	$R(\Omega)$
250	140
300	110
350	90

The estimated value of the exponent n , from the above data, is closest to

1. 2.0 2. 0.8 3. 1.3 4. 2.7

Q73. In a high energy scattering experiment involving two identical particles, each of rest mass m_0 , one particle is initially at rest, while the other one is incident upon it with energy E and momentum p . The total energy of the two-particle system in the centre-of-mass frame, in the limit $E \gg m_0 c^2$, is approximately given by

1. E 2. $2E$ 3. $\sqrt{\frac{Em_0 c^2}{2}}$ 4. $\sqrt{2Em_0 c^2}$

Q74. Consider an emission line of wavelength $\lambda = 550 \text{ nm}$ of Argon ($A = 40, Z = 18$) at a temperature 400 K . The full Doppler width of the emission line will be closest to

1. 10^{-2} nm 2. 10^{-1} nm 3. 10^{-3} nm 4. 10^{-5} nm

Q75. Consider the one-dimensional motion of a particle of positive charge q confined to an infinite potential well

$$V(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq \pi \\ \infty & \text{otherwise} \end{cases}$$

which is subjected to a perturbing electric field $\vec{E} = E_0 \hat{x}$. The shift in the ground state energy, to the first order in q , is

1. $\frac{q\pi}{2}$ 2. $-\frac{q\pi}{2}$ 3. $q\pi E_0$ 4. $-q\pi E_0$