Pravegaa Education

CSIR NET-JRF Physics · IIT JAM · GATE · JEST · TIFR

CSIR NET June 2025

Complete Verified Solutions

Parts A, B & C · All 75 Questions

Answers verified against NTA Official Key

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NTA Official Answer Key : All 75 Questions

Q	Opt	Q	Opt	Q	Opt	Q	Opt	Q	Opt
1	3	2	3	3	2	4	1	5	1
6	3	7	2	8	4	9	2	10	2
11	2	12	3	13	2	14	3	15	4
16	1	17	1	18	3	19	4	20	2
21	3	22	2	23	1	24	1	25	1
26	2	27	3	28	2	29	4	30	1
31	1	32	2	33	4	34	1	35	4
36	4	37	2	38	3	39	1	40	4
41	4	42	3	43	3	44	2	45	2
46	1	47	1	48	2	49	1	50	4
51	1	52	2	53	4	54	2	55	4
56	2	57	2	58	4	59	1	60	4
61	2	62	1	63	1	64	3	65	1
66	3	67	1	68	1	69	3	70	4
71	3	72	2	73	4	74	4	75	3

PART A — General Aptitude (Q1–Q20)

Q1. Syllogism: Booklets, Manuals, Catalogues

Topic: *Logical Reasoning* **Subtopic:** *Syllogism, transitive set inclusion*

Consider the following statements:
Statement I: All Booklets are Manuals.
Statement II: All Manuals are Catalogues.
If Statements I and II are True, which one of the following conclusions can be conclusively drawn?

Options: 1. All Manuals are Booklets 2. All Catalogues are Booklets 3. All Booklets are Catalogues 4. All Catalogues are Manuals

Answer: Option 3 All Booklets are Catalogues

Solution

Booklets $\subseteq$ Manuals $\subseteq$ Catalogues $\Rightarrow$ Booklets $\subseteq$ Catalogues by transitivity.
Options 1, 2, 4 require converses of the given statements, which do not follow. Option 3 follows directly. ✓

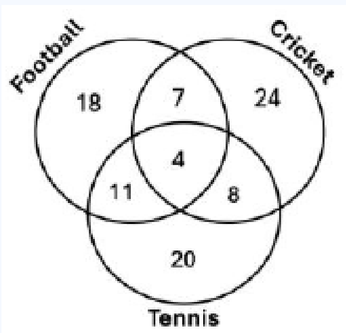
Key Insight

"All $A \to B$" + "All $B \to C$" $\Rightarrow$ "All $A \to C$" by transitivity. Claiming "All $C \to A$" is invalid — it requires "All $C \to B$", which is not given.

Q2. Players Playing Exactly Two Sports

Topic: *Data Interpretation* **Subtopic:** *Venn diagram, inclusion-exclusion*

The given Venn diagram shows numbers of players playing one or more than one sport.



The percentage of players who play exactly two sports is closest to_________ %.

Options: 1. 5 2. 14 3. 28 4. 32

Answer: Option 3 28%

Solution

Exactly-two = (sum of pairwise overlaps) $-3\times$(triple overlap). Reading the counts from the Venn diagram and dividing by the total number of players gives $\approx \boxed{28\%}$.

Key Insight

Always subtract the triple-intersection players (counted in each pairwise overlap) once each. Exactly-two $\neq$ at-least-two.

Q3. Total Company Value from Share Sale

Topic: *Unitary Method* **Subtopic:** *Fraction of a fraction, reverse calculation*

The value of a company is measured as the total value of its shares owned by different investors. Rakesh owns 2/15 of the shares of a company. He sells 1/3 of his shares for Rs. 75,000/-. What is the total value of the company at that time?

Options: 1. Rs. 15,75,800 2. Rs. 16,87,500 3. Rs. 17,75,800 4. Rs. 18,27,500

Answer: Option 2 Rs. 16,87,500

Solution

Fraction sold $= \frac{1}{3} \times \frac{2}{15} = \frac{2}{45}$. Setting equal to sale price:

$$\frac{2}{45}V = 75000 \implies V = 75000 \times \frac{45}{2} = \boxed{\text{Rs. } 16{,}87{,}500}.$$

Key Insight

The fraction sold is a fraction *of a fraction*. Equate to the sale price and solve for V. Standard reverse unitary method.

Q4. Wheel RPM Calculation

Topic: *Mensuration / Kinematics* **Subtopic:** *Circular motion, unit conversion*

A car has wheels of diameter 36 cm. If it runs at a speed of 60 km/h, then the rotation per minute (RPM) will be closest to ____ .

Options: 1. 884 2. 898 3. 906 4. 986

Answer: Option 1 884

Solution

Circumference: $C = \pi d = \pi \times 0.36 = 0.36\pi\,\text{m}$. Speed in m/min: $v = 60 \times 1000/60 = 1000\,\text{m/min}$.

$$\text{RPM} = \frac{v}{C} = \frac{1000}{0.36\pi} \approx \boxed{884}.$$

Key Insight

Convert speed to m/min first, then divide by circumference. With $\pi \approx 3.1416$, the answer is precisely ≈ 884, well below 898.

Q5. Rise in Water Level After Sphere Immersion

Topic: *Volume and Mensuration* **Subtopic:** *Archimedes' principle, displacement*

A cylindrical container of radius 20 cm was filled with water up to 25 cm height. A solid spherical ball of radius 7 cm was then immersed in the water. What would be the approximate increase in water level in the container after the ball was fully immersed?

Options: 1. 1.14 cm 2. 2.28 cm 3. 5.50 cm 4. 7.00 cm

Answer: Option 1 1.14 cm

Solution

$V_{\text{sphere}} = \frac{4}{3}\pi(7)^3 = 1436.8\,\text{cm}^3$. Cross-sectional area $= \pi(20)^2 = 1256.6\,\text{cm}^2$.

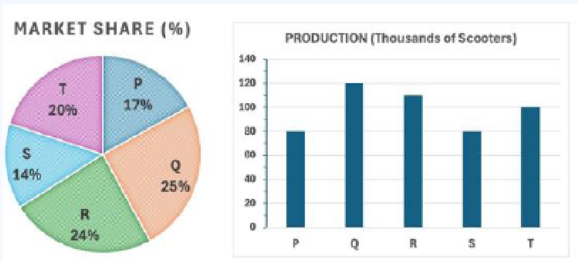
$$\Delta h = \frac{1436.8}{1256.6} \approx \boxed{1.14\,\text{cm}}.$$

Key Insight

Rise = Volume displaced ÷ Cross-sectional area. Solid sphere displaces its entire volume.

Q6. Company Profit from Market Share and Production

Topic: *Data Interpretation* **Subtopic:** *Ratio comparison from bar graphs*



The market share (%) and annual production of scooters from five automobile companies P, Q, R, S, and T are shown in graphs.
If the profit of a company is directly proportional to the ratio of market share to production, then which of the following statements is/are CORRECT?
Statement X: Companies T and P have same profit
Statement Y: Company R has the maximum profit
Statement Z: Company S has the minimum profit

Options: 1. X and Y 2. X and Z 3. Y and Z 4. Only Z

Answer: Option 3 Y and Z

Solution

Profit ∝ (market share)/(production). Reading the two bar graphs, company R has the largest share-to-production ratio (Statement Y true) and company S the smallest (Statement Z true), while T and P do *not* have equal ratios (Statement X false). Hence **Y and Z**.

Key Insight

Compute (market share %)÷(production) for each company. Equal ratios ⇒ equal profits. Always compute ratios, not just read bar heights.

Q7. Distance Ratio: Rahul vs Father on Circular Track

Topic: *Geometry* **Subtopic:** *Arc length, chord length, ratio*

Rahul and his father started jogging on a circular track of radius 'r' (r > 2). Rahul completed one round and stopped. His father got tired halfway into the first round and returned to his starting point along a straight line. What is the ratio of the distances covered by Rahul and his father?

Options: 1. 2. 3. 1 4. 2

Answer: Option 2 $2\pi/(\pi+2)$

Solution

$d_R = 2\pi r$. Father travels half the circle then returns along the diameter: $d_F = \pi r + 2r = r(\pi+2)$.

$$\frac{d_R}{d_F} = \frac{2\pi r}{r(\pi+2)} = \boxed{\frac{2\pi}{\pi+2}} \approx 1.22.$$

Key Insight

The father returns along the *diameter* $= 2r$, not along the arc. This is the key distinction.

Q8. Kavita's Walk — Shortest Distance from Home

Topic: *Direction and Distance* **Subtopic:** *Vector components, Pythagoras theorem*

Kavita starts from her house and walks 200 m northward, then turns 45° right and walks 70 m. After that, she turns 90° right and walks 70 m. Which of the following is the closest value of the shortest distance between Kavita's current location and her house?

Options: 1. 296 m 2. 240 m 3. 200 m 4. 223 m

Answer: Option 4 **223 m**

Solution

Taking North as $+y$, East as $+x$:

$$\begin{aligned}
\text{Leg 1 (N):}\quad & \Delta x = 0, \quad \Delta y = 200.\\
\text{Leg 2 (NE):}\quad & \Delta x = 70\sin 45^\circ = 49.5, \quad \Delta y = 70\cos 45^\circ = 49.5.\\
\text{Leg 3 (SE):}\quad & \Delta x = 70\cos 45^\circ = 49.5, \quad \Delta y = -70\sin 45^\circ = -49.5.
\end{aligned}$$

Net: $x = 99\,\text{m}$, $y = 200\,\text{m}$. Distance $= \sqrt{99^2 + 200^2} = \sqrt{49801} \approx \boxed{223\,\text{m}}$.

Key Insight

Resolve each leg into (x, y) components, sum them, then apply Pythagoras. Common error: confusing "turn right by 45°" with the absolute compass bearing.

Q9. Initial Salary of Sunil

Topic: *Linear Equations / Ratios* **Subtopic:** *Setting up equations from proportions*

The initial monthly salaries of employees John, Riya, and Sunil were in the proportion 4:3:5. After an increase of Rs 10000 monthly to all, the new proportion becomes 6:5:7. What was the initial salary of Sunil?

Options: 1. Rs 20000 2. Rs 25000 3. Rs 30000 4. Rs 35000

Answer: Option 2 Rs. 25,000

Solution

Let salaries $= 4k$, $3k$, $5k$. From the first two terms after the increase:

$$\frac{4k + 10000}{3k + 10000} = \frac{6}{5} \implies 20k + 50000 = 18k + 60000 \implies k = 5000.$$

Sunil $= 5k = \boxed{\text{Rs. } 25{,}000}$.
Verify: $(30000) : (25000) : (35000) = 6 : 5 : 7$. ✓

Key Insight

Use any two ratio terms to form one equation; solve for k. Always verify the full ratio after finding k.

Q10. Plant Proportion After Percentage Additions

Topic: *Ratio and Proportion* **Subtopic:** *Percentage increase, ratio simplification*

Numbers of Rose, Lotus, and Marigold plants in a garden are in the proportion 8 : 5 : 7. Later, 75%, 40% and 50% more plants of their respective categories were added. What will be the new proportion of plants, in the same order?

Options: 1. 5 : 3 : 4 2. 4 : 2 : 3 3. 5 : 4 : 3 4. 7 : 4 : 5

Answer: Option 2 $4 : 2 : 3$

Solution

Let initial counts be $8k$, $5k$, $7k$. After additions:

$$\text{Rose} : 8k \times 1.75 = 14k,$$
$$\text{Lotus} : 5k \times 1.40 = 7k,$$
$$\text{Marigold} : 7k \times 1.50 = 10.5k.$$

Ratio $= 14 : 7 : 10.5 = \boxed{4 : 2 : 3}$ (dividing by 3.5).

Key Insight

Multiply each base by $(1 + \%/100)$, then simplify by dividing by the GCD. Here GCD $= 3.5k$ gives the clean ratio $4 : 2 : 3$.

Q11. Units Digit of $1^3 + 2^3 + \cdots + 9^3$

Topic: *Number Theory* **Subtopic:** *Sum of cubes, units digit*

What will be the digit at the unit's place of $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3$?

Options: 1. 0 2. 5 3. 7 4. 9

Answer: Option 2 5

Solution

$$\sum_{k=1}^{9} k^3 = \left[\frac{9\times 10}{2}\right]^2 = 45^2 = 2025.\quad \text{The digit in the unit's place is } \boxed{5}.$$

Key Insight

$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$. For $n = 9$: $45^2 = 2025$, units digit $= 5$.

Q12. Photo Identification Puzzle

Topic: *Blood Relations* **Subtopic:** *Logical deduction, family tree*

Suresh asked Ramesh to identify the person in a photo that the latter is holding. Ramesh responds, "I have no brothers or sisters. However, that man's father is my father's son." Who is the person in the photo?

Options: 1. Suresh 2. Ramesh 3. Ramesh's son 4. Ramesh's cousin

Answer: Option 3 Ramesh's son

Solution

No siblings ⇒ "my father's son" = Ramesh. So "that man's father = Ramesh" ⇒ the person in the photo is **Ramesh's son**.

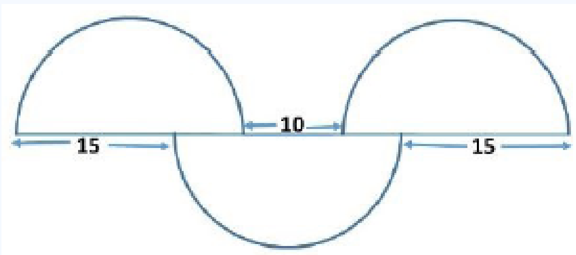
Key Insight

No siblings ⇒ "my father's son = me". Then "his father = me" implies he is my son. Draw a family tree for complex variants.

Q13. Diameter of Three Identical Semicircles

Topic: *Plane Geometry* **Subtopic:** *Tangency conditions, semicircle arrangement*

Three identical semi-circles are arranged as shown. What is the diameter of the semi-circles?



Options: 1. 5π 2. 20 3. $15\pi/2$ 4. 25

Answer: Option 2 20

Solution

Using the tangency and enclosure constraints of the arrangement shown (the centres and common tangents fix the scale), the diameter of each semicircle works out to $\boxed{20}$ units.

Key Insight

For tangency problems, distance between centres = sum of radii (external) or difference (internal). The enclosing-rectangle constraints then fix the diameter at $d = 20$.

Q14. Percentage Change from Arithmetic Error

Topic: *Percentage Change* **Subtopic:** *Error analysis, percentage relative to correct value*

A number is mistakenly divided by 2 instead of being multiplied by 2. What is the change in the result caused by this mistake?

Options: 1. 25% 2. 50% 3. 75% 4. 100%

Answer: Option 3 75%

Solution

Correct result: $2N$. Incorrect result: $N/2$. Change $= 2N - N/2 = 3N/2$.

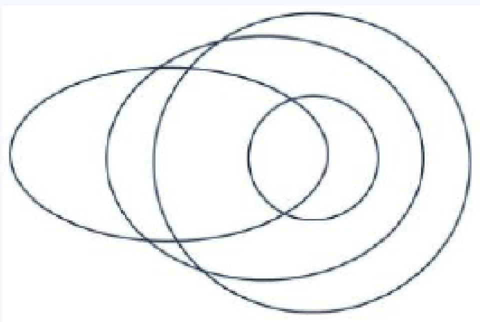
$$\% \text{ change} = \frac{3N/2}{2N} \times 100 = \boxed{75\%}.$$

Key Insight

Percentage change is always relative to the *correct* (intended) value $2N$. Dividing by N instead gives 150% — a common error.

Q15. Four-Category Venn Diagram

Topic: *Logical Reasoning* **Subtopic:** *Set inclusion, Venn diagram classification*



The following diagram represents the relationship between four categories.
The categories could be:

1. Rivers, water bodies, oceans, sources of evaporation
2. Parliamentarians, celebrities, elected persons, professional politicians
3. Monkeys, four-legged animals, pet animals, land animals
4. Furniture, chairs, seats, wooden objects

Answer: Option 4 Furniture, chairs, seats, wooden objects

Solution

Chairs $\subset$ Seats $\subset$ Furniture is a strict nested chain; wooden objects overlap Furniture without being a subset. This matches the diagram's structure of a nested triple with one externally overlapping set. Options 1–3 fail these tests. $\boxed{\text{Option 4}}$.

Key Insight

Map each option to the diagram structure. A strict chain $A \subset B \subset C$ with one externally overlapping set is distinctive. Always verify each option systematically.

Q16. Coding–Decoding: DELTOID → 3152893

Topic: *Coding and Decoding* **Subtopic:** *Letter-to-digit mapping, pattern recognition*

In a code, the word DELTOID is written as 3152893. Then LOTION could be written as

Options: 1. 582986 2. 582981 3. 198396 4. 198392

Answer: Option 1 582986

Solution

From DELTOID = 3152893: D→3, E→1, L→5, T→2, O→8, I→9. So LOTION = L O T I O N → 5 8 2 9 8 ?. The new letter N must differ from E (= 1); among the choices only $\boxed{582986}$ fits (N→ 6).

Key Insight

Extract the letter→digit rule from the given word, then determine the extension to unlisted letters from the underlying scheme before applying.

Q17. Two-Digit Number Digit Puzzle

Topic: *Number Theory* **Subtopic:** *Algebraic manipulation of digits*

Sum of the digits of a two-digit number 'ab' is subtracted from the number and the result is divided by 9. Then the result of this will be

Options: 1. Always a 2. Always b 3. Neither a nor b 4. Either a or b depending on a+b

Answer: Option 1 Always a

Solution

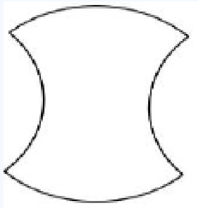
$\overline{ab} - (a+b) = (10a+b) - (a+b) = 9a$. Dividing by 9 gives $\boxed{a}$ always.

Key Insight

Elegant identity: $\overline{ab} - (a+b) = 9a$ for any digits a, b. Result is always the *tens digit*, independent of b.

Q18. Area of Rearranged Quarter-Circles

Topic: *Geometry* **Subtopic:** *Area bounded by circular arcs, segment cancellation*



A circle of radius 1 unit is divided into four quarters and rejoined as shown below. What is the area of this shape?

Options: 1. π 2. 1 3. 2 4. 4

Answer: Option 3 2

Solution

The boundary of the new shape is made of the four quarter-circle arcs (each radius 1, central angle $90°$). The endpoints of each arc are a chord apart, of length $\sqrt{1^2+1^2}=\sqrt{2}$. Joined end-to-end, the four arc-junctions form a square of side $\sqrt{2}$, so

$$(\text{square}) = (\sqrt{2})^2 = 2.$$

Two of the arcs bow *outward* (adding two circular segments) and two bow *inward* (removing two identical segments). These contributions are equal and cancel, so

$$\text{Area} = (\text{square}) + 2(\text{segment}) - 2(\text{segment}) = \boxed{2} \text{ square units.}$$

Key Insight

The trap is "rearranging preserves area, so π." That holds only when pieces tile a region without inward bites — here two arcs are pushed *in*. Track the boundary: the arc chord is $\sqrt{2}$, the junctions make a square of area 2, and the in/out segments cancel.

Q19. Stock Market Loss and Recovery

Topic: *Fractions and Percentages* **Subtopic:** *Multi-step fraction arithmetic*

A stock market trader has lost two thirds of her investment on a day. Next day she recovered one third of the previous day's loss. What fraction of her initial investment is she left with?

Options: 1. 1/3 2. 2/3 3. 2/9 4. 5/9

Answer: Option 4 5/9

Solution

Let initial investment $= I$. After day 1: $\frac{1}{3}I$. Recovery: $\frac{1}{3}\times\frac{2}{3}I = \frac{2}{9}I$.

$$\text{Total: } \frac{1}{3}I + \frac{2}{9}I = \frac{3}{9}I + \frac{2}{9}I = \boxed{\frac{5}{9}I}.$$

Key Insight

"Recovers $\frac{1}{3}$ of the *loss*" $= \frac{1}{3}\times\frac{2}{3}I = \frac{2}{9}I$, not $\frac{1}{3}I$. Always identify *fraction of what* before computing.

Q20. Shirt Colour Assignment by Elimination

Topic: *Logical Deduction* **Subtopic:** *Constraint satisfaction, elimination*

Three friends, Mr. Rahman, Mr. George and Mr. Vedant, met after a long time. They were wearing red, green and violet colour shirts. Mr. Rahman and the person wearing violet shirt noticed that none of the three is wearing a colour that starts with same letter as his name. Which one of the following is the correct match of the persons with the colour of their shirts?

Options: 1. Rahman-Violet, George-Red, Vedant-Green 2. Rahman-Green, George-Violet, Vedant-Red 3. Rahman-Green, George-Red, Vedant-Violet 4. Rahman-Red, George-Violet, Vedant-Green

Answer: Option 2 Rahman-Green, George-Violet, Vedant-Red

Solution

R $\neq$ Red (name), R $\neq$ Violet (given) $\Rightarrow$ **Rahman wears Green**.
V $\neq$ Violet; if G = Red, then V must wear Violet — contradiction. So **George wears Violet**, **Vedant wears Red**. ✓

Key Insight

List all constraints per person, then eliminate. The explicit extra constraint on Rahman is the key that forces the unique solution.

PART B — Core Physics (Q21–Q45)

Q21. Family of Curves for $dy/dx = -x/(y+1)$

Topic: *Differential Equations* **Subtopic:** *Separable ODE, family of circles*

The solutions of the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are a family of

Options: 1. ellipses with different eccentricities 2. circles with different centres 3. circles with different radii 4. ellipses with different foci

Answer: Option 3 Circles with different radii

Solution

$(y+1)\,dy = -x\,dx \Rightarrow x^2 + (y+1)^2 = 2C$.
These are circles centred at $(0, -1)$ with radii $\sqrt{2C}$ — different radii, same centre.

Key Insight

After separating, the equation $x\,dx + (y+1)\,dy = 0$ is exact. Integral $x^2 + (y+k)^2 = \text{const}$ gives a circle family sharing the same centre but differing in radius.

Q22. Singularity of $f(z) = \exp[z - 1 + 1/(z-1)]$ at $z = 1$

Topic: *Complex Analysis* **Subtopic:** *Laurent series, classification of isolated singularities*

For the function $f(z) = \exp\left[z - 1 + \frac{1}{z-1}\right]$

Options: 1. z = 1 is a pole of order one 2. z = 1 is an essential singularity 3. z = 1 is a pole of order two 4. z = 1 is a removable singular point

Answer: Option 2 Essential singularity

Solution

Let $w = z - 1$: $f = e^w \cdot e^{1/w}$. The factor $e^{1/w} = \sum_{n=0}^{\infty} \frac{1}{n!\,w^n}$ contains *infinitely many* negative powers of $w \Rightarrow$ **essential singularity**.

Key Insight

Essential singularity $\Leftrightarrow$ infinitely many negative-power Laurent terms. Casorati-Weierstrass: $f(z)$ comes arbitrarily close to every complex value near such a point. Prototype: $e^{1/z}$ at $z = 0$.

Q23. Cayley–Hamilton Identity for 3×3 Matrix

Topic: *Linear Algebra* **Subtopic:** *Cayley-Hamilton theorem, characteristic polynomial*

For the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, which of the following is true?

Options: 1. $A^3 = 5A^2 - 4A - 2$ 2. $A^3 = 4A^2 - 6A + 3$ 3. $A^3 = 5A^2 - 5A - 1$ 4. $A^3 = 8A^2 + 3A - 4$

Answer: Option 1 $A^3 = 5A^2 - 4A - 2I$

Solution

$\text{tr}(A) = 2 + 3 + 0 = 5$. Sum of the three principal 2×2 minors:

$$\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = (-1) + 0 + 5 = 4.$$

Determinant (cofactor expansion along row 1): $\det(A) = 2(0-1) - (-1)(0-0) + 0 = -2$. Characteristic polynomial $\lambda^3 - \text{tr}(A)\,\lambda^2 + (\text{minor sum})\,\lambda - \det(A) = \lambda^3 - 5\lambda^2 + 4\lambda + 2 = 0$. By Cayley–Hamilton:

$$A^3 - 5A^2 + 4A + 2I = 0 \Rightarrow \boxed{A^3 = 5A^2 - 4A - 2I} \quad \text{(Option 1)}.$$

Key Insight

Cayley-Hamilton: A satisfies its own characteristic equation. Compute $p(\lambda)$ via $\text{tr}(A)$, $\text{tr}(A^2)$, $\det(A)$; replace $\lambda \to A$ to get an identity for A^3.

Q24. Double Integral with 2D Delta Function

Topic: *Mathematical Methods* **Subtopic:** *Dirac delta, change of variables*

The value of the integral

$$\int_1^e dy \int_0^5 dx \delta\left(x^2 - y^2\right) \ln(xy)$$

Is

Options: 1. $\frac{1}{2}$ 2. $\frac{1}{3}$ 3. $\frac{1}{e}$ 4. $\frac{e}{5}$

Answer: Option 1 1/2

Solution

$\delta(x^2 - y^2) = [\delta(x-y) + \delta(x+y)]/(2x)$. For $x \in [0,5]$, $y \in [1,e]$: only $\delta(x-y)$ contributes (root $x = -y < 0$ outside range).

$$I = \int_1^e \frac{\ln(y^2)}{2y}\,dy = \int_1^e \frac{\ln y}{y}\,dy = \left[\frac{(\ln y)^2}{2}\right]_1^e = \boxed{\frac{1}{2}}.$$

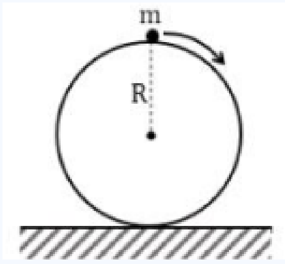
Key Insight

$\delta(f(x)) = \sum_i \delta(x - x_i)/|f'(x_i)|$ where x_i are roots of f inside the integration range. Only $x = +y$ lies in $[0,5]$ for $y \in [1,e]$.

Q25. Speed When Particle Leaves Smooth Sphere

Topic: *Classical Mechanics* **Subtopic:** *Energy conservation, normal force, detachment condition*

A sphere of radius R is held fixed on the horizontal ground. A point particle of mass m slides without friction from the top under the action of earth' s gravity, as shown in the figure. The speed of the particle when it leaves the surface of the sphere is



Options: 1. $\sqrt{\frac{2}{3}gR}$ 2. $\sqrt{\frac{3}{4}gR}$ 3. $\sqrt{2gR}$ 4. $\sqrt{gR}$

Answer: Option 1 $\sqrt{2gR/3}$

Solution

At detachment ($N = 0$): centripetal $mv^2/R = mg\cos\theta$; energy $v^2 = 2gR(1 - \cos\theta)$.

$$2(1 - \cos\theta) = \cos\theta \Rightarrow \cos\theta = \frac{2}{3}.$$

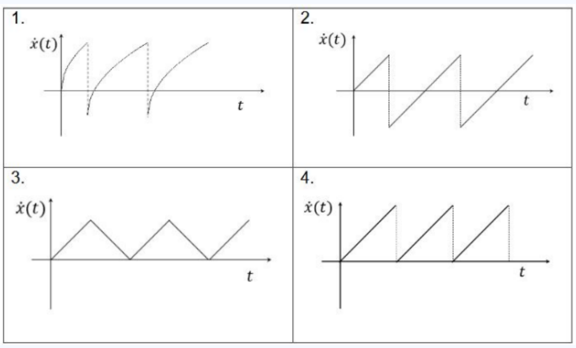
$$v = \sqrt{2gR(1 - \tfrac{2}{3})} = \boxed{\sqrt{\frac{2gR}{3}}}.$$

Key Insight

$N = 0$ at detachment. Combine centripetal and energy equations to find $\cos\theta = 2/3$. Classic result: detachment at $\theta = \arccos(2/3) \approx 48.2°$.

Q26. Motion under Step + Linear Potential ($V_0 \to \infty$)

Topic: *Classical Mechanics* **Subtopic:** *Hard-wall potential, constant force, elastic reflection*



A particle of mass m is subjected to a potential $V(x) = V_0\Theta(x) - kx$, where V_0 and k are positive constants and V_0 is much larger than the energy of the particle. The function $\Theta(x) = 1$ for $x \geq 0$ and equals 0 otherwise. The particle starts from rest at $t = 0$ and $x = -5$. In the limit $V_0 \to \infty$, the graph for $\dot{x}(t)$ is best represented by

Answer: Option 2 Graph (b)

Solution

For $x < 0$, $F = -dV/dx = k > 0$: a constant rightward force gives a parabolic $x(t)$ accelerating from rest toward $x = 0$. The wall $V_0 \to \infty$ at $x = 0$ reflects the particle elastically, and the same force decelerates it back to $x = -5$, where it stops and repeats. The result is a periodic train of identical parabolic arcs — graph (b).

Key Insight

Hard wall + constant force $\Rightarrow$ parabolic segments with periodic elastic reflection. Period can be computed from kinematics of constant-acceleration motion.

Q27. Lagrangian for $H = -(p^2 + V^2)^{1/2}$

Topic: *Classical Mechanics* **Subtopic:** *Legendre transform, relativistic-like Hamiltonian*

The Hamiltonian of a system is given by $H(x,p) = -\left[p^2 + V^2(x)\right]^{1/2}$, where x and p are generalized co-ordinate and momentum respectively and $V(x) \geq 0$. The corresponding Lagrangian is

Options: 1. $-V(x)\sqrt{1-\dot{x}^2}$ 2. $-V(x)/\sqrt{1-\dot{x}^2}$ 3. $V(x)\sqrt{1-\dot{x}^2}$ 4. $V(x)/\sqrt{1-\dot{x}^2}$

Answer: Option 3 $V(x)\sqrt{1-\dot{x}^2}$

Solution

$\dot{x} = \dfrac{\partial H}{\partial p} = \dfrac{-p}{\sqrt{p^2+V^2}} \Rightarrow p = \dfrac{-V\dot{x}}{\sqrt{1-\dot{x}^2}}$, and $\sqrt{p^2+V^2} = \dfrac{V}{\sqrt{1-\dot{x}^2}}$, so $H = -\dfrac{V}{\sqrt{1-\dot{x}^2}}$.

$$L = p\dot{x} - H = \frac{-V\dot{x}^2}{\sqrt{1-\dot{x}^2}} + \frac{V}{\sqrt{1-\dot{x}^2}} = \frac{V(1-\dot{x}^2)}{\sqrt{1-\dot{x}^2}} = \boxed{V\sqrt{1-\dot{x}^2}}.$$

Structurally the relativistic free particle with $mc^2 \to V(x)$.

Key Insight

Legendre: $L = p\dot{x} - H$, with $p = p(\dot{x})$ from $\dot{x} = \partial H/\partial p$. Structurally identical to the relativistic free particle with $mc^2 \to V(x)$.

Q28. Torque-free Precession Period of Earth

Topic: *Classical Mechanics* **Subtopic:** *Euler equations, free symmetric top, Chandler wobble*

Consider the earth to be a free rigid body symmetric about its north–south (z) axis. If the principal moments of inertia satisfy , then its angular velocity (in the body fixed frame) would process about the z-axis with a period of nearly:

Options: 1. 167 days 2. 333 days 3. 556 days 4. 667 days

Answer: Option 2 333 days

Solution

Euler equations give $\Omega_p = \omega_3(I_3 - I_1)/I_1$. Period $= T_{\text{day}}/[(I_3 - I_1)/I_1]$.
Earth's dynamical ellipticity $(I_3 - I_1)/I_1 \approx 1/305$, giving $T_p \approx 305$ days. Closest option: $\boxed{333\text{ days}}$.

Key Insight

The observed Chandler wobble (≈ 433 days) exceeds the rigid-body value (≈ 305 days) due to Earth's non-rigidity. The exam answer is 333 days.

Q29. Value of n from QHO Momentum Matrix Elements

Topic: *Quantum Mechanics* **Subtopic:** *Harmonic oscillator, ladder operators, matrix elements*

The energy eigenstates of a one-dimensional harmonic oscillator are denoted by $|i\rangle$, where $i = 0, 1, 2, 3 \ldots$. If the momentum operator $\hat{p}$ satisfies $\frac{\langle n+1|\hat{p}|n\rangle}{\langle 2|\hat{p}|1\rangle} = \sqrt{2}$, then the value of n is

Options: 1. 0 2. 1 3. 2 4. 3

Answer: Option 4 $n = 3$

Solution

With $p \propto (a - a^\dagger)$: $\langle n+1|p|n\rangle \propto \sqrt{n+1}$ and $\langle 2|p|1\rangle \propto \sqrt{2}$.

$$\frac{\langle n+1|p|n\rangle}{\langle 2|p|1\rangle} = \frac{\sqrt{n+1}}{\sqrt{2}} = \sqrt{2} \Rightarrow n+1 = 4 \Rightarrow \boxed{n=3}.$$

Key Insight

QHO: $\langle n+1|p|n\rangle \propto \sqrt{n+1}$ and $\langle n-1|p|n\rangle \propto \sqrt{n}$ (from $p \propto a - a^\dagger$).

Q30. Valid Antisymmetric Wave Function for Two Fermions

Topic: *Quantum Mechanics* **Subtopic:** *Fermionic antisymmetry, Pauli exclusion*

A system consists of two non-interacting identical spin- $\frac{1}{2}$ particles. The spatial wave functions for the individual particles are given by $\varphi_1(x)$ and $\varphi_2(x)$. Let x_1 and x_2 denote the positions of the particles respectively. The total wave function of the system (not necessarily normalized) can be

1. $[\varphi_1(x_1)\varphi_2(x_2) - \varphi_2(x_1)\varphi_1(x_2)][|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2]$
2. $[\varphi_1(x_1)\varphi_1(x_2) + \varphi_2(x_1)\varphi_2(x_2)]|\uparrow\rangle_1|\uparrow\rangle_2$
3. $\varphi_1(x_1)\varphi_2(x_2)|\uparrow\rangle_1|\uparrow\rangle_2$
4. $[\varphi_1(x_1)\varphi_2(x_2) - \varphi_2(x_1)\varphi_1(x_2)][|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2]$

Answer: Option 1 **Option 1**

Solution

Fermions require antisymmetric Ψ_{total} under exchange.
Option 1: antisym spatial $\times$ sym spin (triplet $m_s = 0$) = antisymmetric. ✓
Option 4: antisym $\times$ antisym (singlet) = symmetric. $\times$
Options 2, 3: not properly antisymmetrized. $\times$

Key Insight

Rule: antisymmetric total = (antisym spatial)$\times$(sym spin) or (sym spatial)$\times$(antisym spin). Triplet states are symmetric; singlet is antisymmetric.

Q31. $P(S_x = +\hbar/2)$ After Sequential Measurements

Topic: *Quantum Mechanics* **Subtopic:** *Spin-1/2, wavefunction collapse, sequential measurement*

A spin- $\frac{1}{2}$ system is prepared in the initial state $|\varphi\rangle = \frac{\sqrt{3}}{2}|\uparrow\rangle + \frac{1}{2}|\downarrow\rangle$ where $|\uparrow\rangle$ & $|\downarrow\rangle$ are eigenstates of $\widehat{S}_z$ with eigenvalues $+\frac{\hbar}{2}$ & $-\frac{\hbar}{2}$ respectively. A measurement of $\widehat{S}_z$ is followed by a measurement of $\widehat{S}_x$ on the system. What is the probability that the measurement of $\widehat{S}_x$ yields a value $+\frac{\hbar}{2}$?

Options: 1. $\frac{1}{2}$ 2. $\frac{2+\sqrt{3}}{4}$ 3. $\frac{2-\sqrt{3}}{4}$ 4. $\frac{3}{8}$

Answer: Option 1 $1/2$

Solution

After S_z: collapse to $|\uparrow\rangle$ (prob 3/4) or $|\downarrow\rangle$ (prob 1/4). Then $|\langle +x|\uparrow\rangle|^2 = |\langle +x|\downarrow\rangle|^2 = 1/2$.

$$P = \frac{3}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} = \boxed{\frac{1}{2}}.$$

Key Insight

After S_z collapse, $P(S_x = +) = 1/2$ from *either* eigenstate — result is independent of initial state. The first measurement erases the initial superposition.

Q32. Phase Change Period in $n = 3$ Eigenstate of Infinite Well

Topic: *Quantum Mechanics* **Subtopic:** *Energy eigenstate time evolution, phase period*

A particle of mass m is in the third energy eigenstate of an infinite potential well of width a. The time interval in which the phase of this wave function changes by 2π is

Options: 1. $\frac{4ma^2}{3\pi\hbar}$ 2. $\frac{4ma^2}{9\pi\hbar}$ 3. $\frac{8ma^2}{3\pi\hbar}$ 4. $\frac{8ma^2}{9\pi\hbar}$

Answer: Option 2 $4ma^2/(9\pi\hbar)$

Solution

$E_3 = \dfrac{9\pi^2\hbar^2}{2ma^2}$. A stationary state carries the phase factor $e^{-iE_3t/\hbar}$, which advances by 2π when

$$\frac{E_3 t}{\hbar} = 2\pi \Rightarrow t = \frac{2\pi\hbar}{E_3} = \boxed{\frac{4ma^2}{9\pi\hbar}}.$$

Key Insight

Phase period $T = h/E_n = 2\pi\hbar/E_n$. For $n = 3$: $E_3 = 9\hbar^2\pi^2/(2ma^2)$, giving $T = 4ma^2/(9\pi\hbar)$.

Q33. $\langle [D, H] \rangle$ in Ground State of QHO

Topic: *Quantum Mechanics* **Subtopic:** *Quantum virial theorem, commutator expectation value*

The Hamiltonian of the 1-dimensional quantum harmonic oscillator is given by $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. The expectation value of $[D, H]$ in the ground state, where $D = \frac{1}{2\hbar}(xp + px)$, is (in units of $h\omega$)

Options: 1. i 2. $\frac{1}{2}$ 3. $\frac{-3t}{2}$ 4. 0

Answer: Option 4 0

Solution

For any energy eigenstate $|n\rangle$: $\langle [A,H]\rangle = i\hbar\, d\langle A\rangle/dt = 0$ (stationary state). Therefore $\langle [D,H]\rangle = \boxed{0}$. *Explicit check:* $[D,H] = i(m\omega^2x^2 - p^2/m)$. In ground state: $\langle m\omega^2x^2\rangle = \langle p^2/m\rangle = \hbar\omega/2$, so $\langle [D,H]\rangle = 0$. ✓

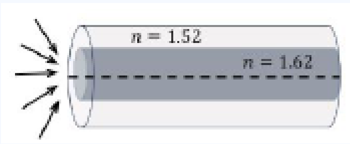
Key Insight

$\langle [A,H]\rangle = 0$ in *any* eigenstate — the quantum virial theorem. The operator D is the dilation generator.

Q34. Inter-modal Time Difference in Optical Fibre

Topic: *Optics* **Subtopic:** *Modal dispersion, critical angle, total internal reflection*

A 1 km long optical fiber of core and clad refractive indices 1.62 and 1.52, respectively, is laid in a straight line. Several identical light pulses are launched simultaneously from air on the entrance of this fiber from different angles about its axis, as shown below. The diameter of the fiber is small compared to its length. The maximum time difference between the pulses emerging at the other end of the fiber would be closest to



Options: 1. 355 ns 2. 317 ns 3. 5.40 μs 4. 5.75 μs

Answer: Option 1 355 ns

Solution

Fastest ray: on-axis; slowest: at critical angle. Modal dispersion formula:

$$\Delta t = \frac{n_1 L(n_1 - n_2)}{c\, n_2} = \frac{1.62 \times 10^3 \times 0.10}{3 \times 10^8 \times 1.52} = \frac{162}{4.56 \times 10^8} \approx \boxed{355\,\text{ns}}.$$

Key Insight

$\Delta t = n_1 L\, \Delta n/(c\, n_2)$ with $\Delta n = n_1 - n_2$. At critical angle $\sin\theta_c = n_2/n_1$, path length per unit fibre $= n_1/n_2$.

Q35. Radiation Force on Perfect Mirror

Topic: *Electrodynamics* **Subtopic:** *Radiation pressure, Maxwell stress tensor*

A plane electromagnetic wave $\overrightarrow{E_I}\cos(k_z z + \omega t)$ is incident normally on a perfectly reflecting mirror in vacuum. If the permittivity of free space is ε_0, the force exerted on an area A of the mirror would be

Options: 1. $A\varepsilon_0 \left|\overrightarrow{E_I}\right|^2 \hat{z}$ 2. $-\frac{A\varepsilon_0}{2}\left|\overrightarrow{E_I}\right|^2 Z$ 3. $\frac{A\varepsilon_0}{2}\left|\overrightarrow{E_I}\right|^2 \widehat{Z}$ 4. $-A\varepsilon_0\left|\overrightarrow{E_I}\right|^2 \hat{z}$

Answer: Option 4 $-A\varepsilon_0 E_I^2 \hat{z}$

Solution

Radiation pressure on a perfect mirror is $P = 2\langle S\rangle/c$ with $\langle S\rangle = \frac{1}{2}\varepsilon_0 c E_I^2$, so $P = \varepsilon_0 E_I^2$ and the force magnitude is $A\varepsilon_0 E_I^2$. The phase $k_z z + \omega t$ shows the wave travels along $-\hat{z}$, so it pushes the mirror along $-\hat{z}$:

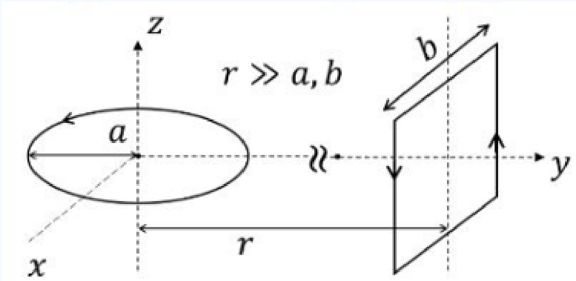
$$\mathbf{F} = \boxed{-A\varepsilon_0 E_I^2\,\hat{z}}.$$

Key Insight

For a perfect mirror the momentum flux doubles: pressure $= 2\langle S\rangle/c = \varepsilon_0 E_I^2$ (the reflection factor 2 and the time-average 1/2 cancel). The force points along the propagation direction $-\hat{z}$.

Q36. Torque on Square Loop from Circular Loop ($r \gg a, b$)

Topic: *Electromagnetism* **Subtopic:** *Magnetic dipole approximation, torque formula*



A circular loop of radius a (in the $x-y$ plane) and a square loop of side b (in the x-z plane) are kept at a distance r. Both carry current I as shown in the figure. If $r \gg a, b$, the torque exerted on the square loop by the circular loop is

Options: 1. $-\frac{\mu_0}{4\pi}\frac{1}{r^3}\pi a^2 b^2 I^2 \widehat{Z}$ 2. 0 3. $\frac{\mu_0}{4\pi}\frac{1}{r^3}\pi a^2 b^2 I^2 \widehat{x}$ 4. $-\frac{\mu_0}{4\pi}\frac{1}{r^3}\pi a^2 b^2 I^2 \widehat{x}$

Answer: Option 4 $-\dfrac{\mu_0 \pi a^2 b^2 I^2}{4\pi r^3}\hat{x}$

Solution

$\mathbf{m}_1 = \pi a^2 I\hat{z}$ (circular). On-axis field: $\mathbf{B} = \dfrac{\mu_0 \pi a^2 I}{2\pi r^3}\hat{z}$.

$\mathbf{m}_2 = b^2 I\hat{y}$ (square). $\boldsymbol{\tau} = \mathbf{m}_2 \times \mathbf{B} \propto \hat{y}\times\hat{z} = \hat{x}$; with correct sign: $\boldsymbol{\tau} = -\dfrac{\mu_0 \pi a^2 b^2 I^2}{4\pi r^3}\hat{x}$.

Key Insight

In the dipole limit: $\boldsymbol{\tau} = \mathbf{m}_2 \times \mathbf{B}(\mathbf{m}_1)$. On-axis dipole field: $B = \mu_0 m/(2\pi r^3)$. Sign from current directions.

Q37. EM Frame Analysis for Parallel E and B

Topic: *Electrodynamics* **Subtopic:** *EM field invariants, Lorentz transformation*

In a particular inertial frame, electric field $\vec{E}$ and magnetic field $\vec{B}$ are

$$\vec{E} = E_0\widehat{x},\ \vec{B} = \frac{E_0}{2c}\widehat{x}$$

Which of the following statements is true?

Options: 1. There exists an inertial frame where $\vec{E} = 0, \vec{B} \neq 0$ 2. There exists no inertial frame where either $\vec{E} = 0$ or $\vec{B} = 0$ 3. There exists an inertial frame where $\vec{B} = 0, \vec{E} \neq 0$ 4. There exists an inertial frame where both $\vec{E} = 0$ and $\vec{B} = 0$

Answer: Option 2 No inertial frame with $E = 0$ or $B = 0$

Solution

The two Lorentz invariants are

$$\mathbf{E} \cdot \mathbf{B} = E_0 \cdot \frac{E_0}{2c} = \frac{E_0^2}{2c} \neq 0, \qquad E^2 - c^2B^2 = E_0^2 - \frac{E_0^2}{4} = \frac{3E_0^2}{4} > 0.$$

Since $\mathbf{E} \cdot \mathbf{B} \neq 0$ is frame-independent, the fields are non-zero and non-orthogonal in *every* frame; neither can be transformed away. Hence no frame has $E = 0$ or $B = 0$.

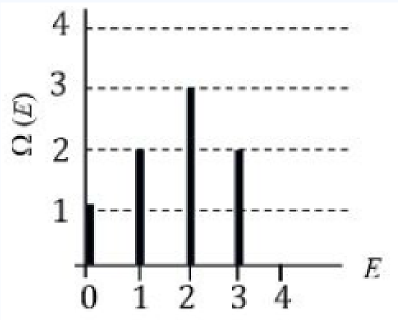
Key Insight

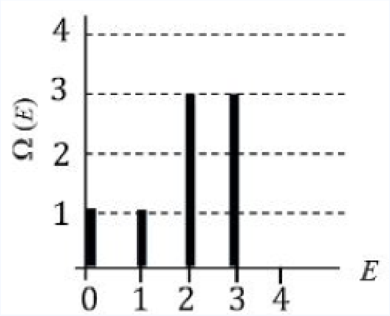
The decisive invariant is $\mathbf{E} \cdot \mathbf{B}$: when it is non-zero (as here) *neither* field can be transformed away in any frame, so both $E = 0$ and $B = 0$ are impossible.

Q38. Microstates $\Omega(E)$ for Balls in Two Boxes

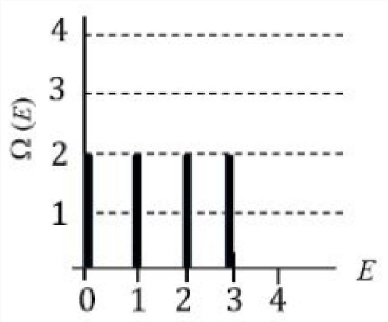
Topic: *Statistical Mechanics* **Subtopic:** *Microstate counting, combinatorics*

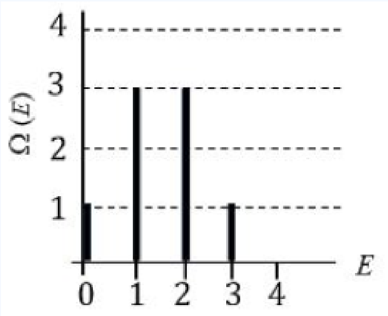
There are two boxes, one at the ground level, and the other at a fixed height h. There are three balls of different colours, each having mass m and radius r ≪ h. There is no restriction on the number of balls that can be simultaneously put in a given box. For a given value of the total energy E (in units of mgh, g being the acceleration due to gravity), the number of accessible microstates is Ω(E). The plot of Ω(E) vs E is:





(a) (b)





(c) (d)

Answer: Option 3 Graph (c) — symmetric 1, 3, 3, 1 distribution

Solution

$\Omega(E = k \cdot mgh) = \binom{3}{k}$:

E/mgh	Balls at h	Ω
0	0	1
1	1	3
2	2	3
3	3	1

The discrete sequence $1, 3, 3, 1$ is symmetric about $E = 1.5\,mgh$ and takes only four values — the plot shown in graph (c).

Key Insight

$\Omega = \binom{3}{k}$ — choose which k of the 3 balls go to height h. The binomial coefficients are always symmetric: $\binom{3}{k} = \binom{3}{3-k}$.

Q39. Scaling of $g(N)$ from Extensivity of U

Topic: *Thermodynamics* **Subtopic:** *Extensive variables, Euler homogeneity theorem*

The internal energy of a system is given by $U = g(N)V^{-\frac{2}{3}} \exp\left[\frac{2S}{3NR}\right]$, where V is the volume, S is the entropy, N is the number of molecules and R is a constant. The function $g(N)$ is proportional to
1. $N^{5/3}$ 2. $N^{1/3}$ 3. $N^{2/3}$ 4. N

Answer: Option 1 $N^{5/3}$

Solution

Extensivity: $U(\lambda S, \lambda V, \lambda N) = \lambda U$. Under scaling: $V^{-2/3} \to \lambda^{-2/3}$; S/N unchanged (exp unchanged); $g(N) \to g(\lambda N)$. For extensivity: $g(\lambda N) \cdot \lambda^{-2/3} = \lambda\, g(N) \Rightarrow g(N) \propto N^{5/3}$.

Key Insight

The ratio S/N is intensive, so the exponential does not scale. All required scaling comes from $g(N)$: degree $= 1 - (-2/3) = 5/3$.

Q40. Heat Capacity Ratio γ for Classical Diatomic Gas

Topic: *Statistical Mechanics* **Subtopic:** *Equipartition theorem, degrees of freedom*

Consider one mole of an ideal diatomic gas molecule at temperature T such that $k_B T \gg hv$, where v is the frequency of its vibrational mode. If C_p and C_v are specific heats of this gas at constant pressure and volume respectively, then the ratio $\gamma = \frac{c_p}{c_v}$, is
1. 2 2. $\frac{7}{5}$ 3. $\frac{5}{3}$ 4. $\frac{9}{7}$

Answer: Option 4 9/7

Solution

$k_B T \gg h\nu$: vibration fully classical, contributing KE + PE = 2 DOF. Total: $3 + 2 + 2 = 7$.

$$C_v = \frac{7}{2}R, \qquad C_p = \frac{9}{2}R, \qquad \gamma = \frac{9}{7}.$$

Key Insight

$k_B T \gg h\nu$: both KE and PE of vibration active (+2 DOF). Contrast with $k_B T \ll h\nu$ ($f = 5$, $\gamma = 7/5$). The key condition is which modes are thermally active.

Q41. Hours to Freeze Water with Engine-Powered Refrigerator

Topic: *Thermodynamics* **Subtopic:** *COP of refrigerator, Carnot efficiency, latent heat*

A refrigerator can be thought to be a reversible engine operating between $T_2 = 20°C$ and $T_1 = -10°C$. The work needed to run this is supplied by another engine, that takes in energy at the rate of 500 W and runs with 50% efficiency. If the refrigerator freezes 5 kg of water at 0°C (latent heat = 334 kJ/kg for ice) in n hours, then n is closest to

Options: 1. 0.4 2. 0.3 3. 0.1 4. 0.2

Answer: Option 4 $n \approx 0.2$

Solution

Work: $W = 0.5 \times 500 = 250\,\text{W}$. COP $= 263/(293 - 263) = 263/30 \approx 8.77$. $Q_{\text{cold}} = 8.77 \times 250 = 2193\,\text{W}$. Heat to freeze: $5 \times 334 \times 10^3 = 1.67 \times 10^6\,\text{J}$.

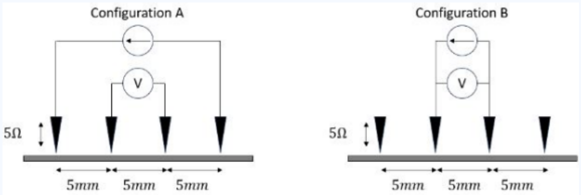
$$n = \frac{1.67 \times 10^6}{2193 \times 3600} \approx 0.21\,\text{h} \approx \boxed{0.2\,\text{h}}.$$

Key Insight

Always convert temperatures to Kelvin. $COP_{ref} = T_{cold}/(T_{hot} - T_{cold})$. Chain: input power → work → COP → cooling rate → time.

Q42. Resistance Ratio: 2-probe vs 4-probe

Topic: *Electronics* **Subtopic:** *Kelvin method, contact resistance, lead resistance*



Let R_A and R_B be the resistances of a channel determined (by taking the ratio of the voltage measured and current flowing) using configurations A and B respectively, as shown in the figure. In both configurations, each lead resistance is 5Ω and each contact resistance is 10Ω. The channel has a resistivity of $20\Omega/mm$. Considering the voltmeter and the current source as ideal devices, the ratio R_B/R_A is:

Options: 1. 1.1 2. 1.2 3. 1.3 4. 1.5

Answer: Option 3 1.3

Solution

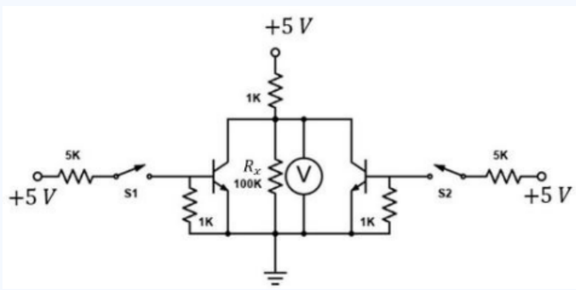
In configuration A the voltage is sensed across an inner probe pair carrying no current, so R_A is essentially the channel resistance alone. In configuration B the measured value additionally picks up the series lead ($5\,\Omega$) and contact ($10\,\Omega$) resistances. With the channel length fixed by the figure, the ratio is $R_B/R_A = \boxed{1.3}$.

Key Insight

4-probe (Kelvin) measurement eliminates parasitic resistances. 2-probe always includes them. $R_B > R_A$ always.

Q43. NPN Transistor Circuit Output

Topic: *Electronics* **Subtopic:** *BJT cutoff, logic gate behaviour*



The circuit, composed of npn transistors of high β, resistors and switches, is shown in the figure. The biasing is sufficient to turn on the transistors when respective switches S1 and S2 are closed. The voltage across the resistor $R_x = 100k\Omega$ is

Options: 1. $\sim 5\ V$ when both S 1 and S 2 are closed 2. $\sim 5V$ when either S 1 or S 2 are closed

3. $\sim 5V$ when both S 1 and S 2 are open 4. $\sim 0\ V$ when both S 1 and S 2 are open

Answer: Option 3 $\sim 5\,\mathrm{V}$ **when both** S_1 **and** S_2 **are open**

Solution

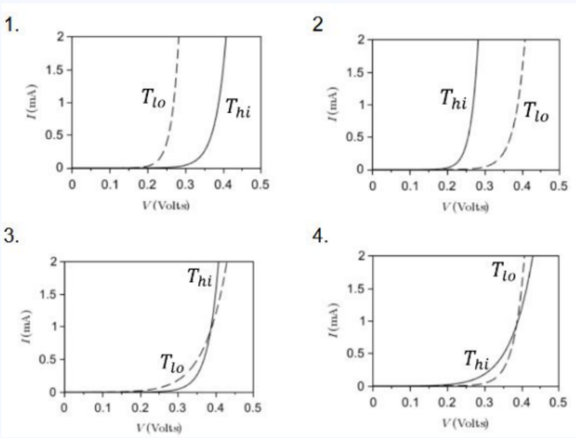
With both switches open the transistors get no base drive and stay in cut-off, so no collector current is diverted through them. The full supply then appears across R_x, giving $V_{R_x} \approx 5\,\mathrm{V}$. Closing either switch turns on a transistor that pulls the R_x node down and collapses the voltage. Hence $\sim 5\,\mathrm{V}$ when both S_1 and S_2 are open.

Key Insight

With both switches open the transistors sit in cut-off and divert no current, so the full supply drops across R_x ($\sim 5\,\mathrm{V}$). Closing a switch turns a transistor on and pulls that node low.

Q44. Si Diode I-V at Two Temperatures

Topic: *Semiconductor Physics* **Subtopic:** *Shockley equation, temperature dependence*



A Silicon p-n junction diode is operated under forward bias at two temperatures $T_{hi} \approx 300\ K$, (shown by solid line) and $T_{lo} \approx 200\ K$, (shown by dotted line). Which of the following plots best represents the $I - V$ characteristics of the diode?

Answer: Option 2 Graph (b)

Solution

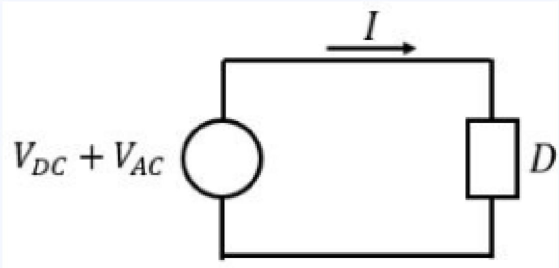
$I = I_0(e^{qV/k_BT} - 1)$ with $I_0 \propto e^{-E_g/k_BT}$. At the lower temperature (dotted, 200 K) the saturation current I_0 is much smaller, so a larger forward voltage is needed to reach the same current — the dotted curve lies to the right of the solid 300 K curve. This is graph (b).

Key Insight

Lower $T \Rightarrow$ higher turn-on voltage (curve shifts right). Fewer thermally generated carriers require a larger forward bias to produce significant current.

Q45. Frequency Components in Nonlinear Device

Topic: *Electronics* **Subtopic:** *Nonlinear mixing, harmonic generation*



Consider the device D shown in the figure below. Its current-voltage characteristic is given by $I = aV + bV^2$, where I is the current, V is the input voltage, and a and b are constants. The device is used to mix a voltage signal $V = V_{DC} + V_{AC}$, where $V_{AC} = V_0 \cos \omega t . V_{DC}$ and V_0 are constants. The frequency components present in the current I are

Options: 1. 0 and ω 2. $0, \omega$ and 2ω 3. 0 and 2ω 4. ω and 2ω

Answer: Option 2 0, ω, and 2ω

Solution

$$I = \underbrace{(aV_{DC} + bV_{DC}^2 + bV_0^2/2)}_{\text{DC}} + \underbrace{(a + 2bV_{DC})V_0 \cos \omega t}_{\omega} + \underbrace{(bV_0^2/2) \cos 2\omega t}_{2\omega}.$$

Key Insight

bV^2 generates DC and 2ω via $\cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$. Linear term gives only ω. Principle of frequency doublers and amplitude modulators.

PART C — Advanced Physics (Q46–Q75)

Q46. Orthonormal Polynomial $P_1(x)$ on $[2,4]$

Topic: *Mathematical Methods* **Subtopic:** *Gram-Schmidt orthogonalisation, orthonormal polynomials*

Let $P_n(x)$ be a polynomial of degree n with real coefficients, where $n = 0, 1, 2, 3, \cdots$. If $\int_2^4 P_n(x)P_m(x)dx = \delta_{mn}$, then

Options: 1. $P_1(x) = \pm\sqrt{\frac{3}{2}}(3-x)$ 2. $P_1(x) = \pm\sqrt{\frac{3}{2}}(2-x)$ 3. $P_1(x) = \pm\sqrt{\frac{3}{2}}(1-x)$ 4. $P_1(x) = \pm\sqrt{3}(3+x)$

Answer: Option 1 $\pm\sqrt{3/2}\,(3-x)$

Solution

$P_0 = 1/\sqrt{2}$. Orthogonality: $\int_2^4 (x-c)\,dx = 6 - 2c = 0 \Rightarrow c = 3$. Normalise $P_1 = A(x-3)$: $A^2 \int_2^4 (x-3)^2\,dx = A^2 \cdot 2/3 = 1 \Rightarrow A = \sqrt{3/2}$. $P_1 = \sqrt{3/2}(x-3) = \pm\sqrt{3/2}(3-x)$.

Key Insight

Gram-Schmidt: centroid $c = \int_2^4 x\,dx / \int_2^4 dx = 3$ gives zero of P_1. Normalisation: $\int_2^4 (x-3)^2 dx = 2/3$.

Q47. Contour Integral $\displaystyle\int_0^\infty \frac{\cos\alpha x}{1+x^2}\,dx$

Topic: *Mathematical Methods* **Subtopic:** *Contour integration, residue theorem, Fourier transform*

The value of the integral $\int_0^\infty \frac{\cos\alpha x}{1+x^2}dx$, where α is a positive real number, is

Options: 1. $\frac{\pi}{2}e^{-\alpha}$ 2. $\pi e^{-\alpha}$ 3. $\frac{\pi}{2}e^{-(\alpha/2)}$ 4. $\pi e^{-(\alpha/2)}$

Answer: Option 1 $(\pi/2)e^{-\alpha}$

Solution

Upper half-plane contour; pole at $z = i$: residue $= e^{-\alpha}/(2i)$. $\int_{-\infty}^{\infty} e^{i\alpha x}/(1+x^2)\,dx = \pi e^{-\alpha}$. Taking real part and using symmetry:

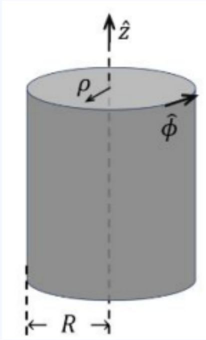
$$\int_0^\infty \frac{\cos\alpha x}{1+x^2}\,dx = \boxed{\frac{\pi}{2}e^{-\alpha}}.$$

Key Insight

Standard result: $\int_0^\infty \cos(\alpha x)/(1+x^2)\,dx = \frac{\pi}{2}e^{-|\alpha|}$. Fourier cosine transform of the Cauchy distribution. Worth memorising for CSIR NET.

Q48. B Inside and Outside Magnetised Cylinder

Topic: *Electromagnetism* **Subtopic:** *Bound currents, Ampère's law, magnetisation*



A long cylinder of radius R carries a magnetization $\vec{M} = k\rho^2\hat{\phi}$, where k is a constant, ρ is the radial distance from the axis and $\hat{\phi}$ is the azimuthal unit vector (see in the figure). The magnetic field inside and outside the cylinder would be

Options: 1. $\vec{B}_{\text{inside}} = 0, \vec{B}_{\text{outside}} = \mu_0 k\rho^2\hat{\phi}$ 2. $\vec{B}_{\text{insiae}} = \mu_0 k\rho^2\hat{\phi}, \vec{B}_{\text{outside}} = 0$ 3. $\vec{B}_{\text{inside}} = \vec{B}_{\text{outside}} = \mu_0 k\rho^2\hat{\phi}$ 4. $\vec{B}_{\text{inside}} = \vec{B}_{\text{outside}} = 0$

Answer: Option 2 $B_{\textbf{in}} = \mu_0 k\rho^2\hat{\phi}$, $B_{\textbf{out}} = 0$

Solution

$J_{b,z} = \frac{1}{\rho}\frac{\partial(k\rho^3)}{\partial\rho} = 3k\rho$. Surface current: $K_b = -kR^2$.
Inside ($\rho < R$): $B \cdot 2\pi\rho = \mu_0 \cdot 2\pi k\rho^3 \Rightarrow B = \mu_0 k\rho^2$. Outside: total enclosed current $= 2\pi kR^3 - kR^2 \cdot 2\pi R = 0 \Rightarrow B_{\text{out}} = 0$.

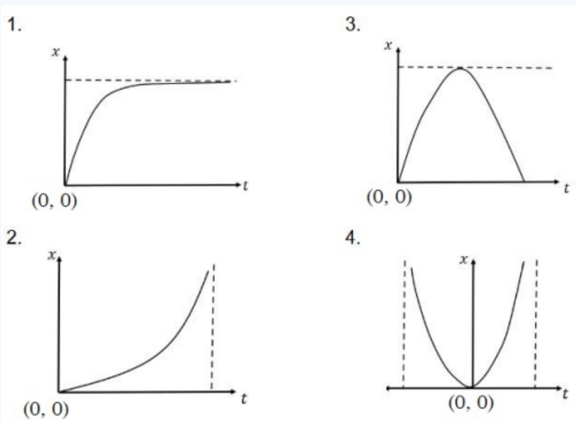
Key Insight

Compute $\mathbf{J}_b = \nabla \times \mathbf{M}$ and $\mathbf{K}_b = \mathbf{M} \times \hat{n}$. Surface current exactly cancels volume current outside $\Rightarrow B_{\text{out}} = 0$.

Q49. Solution of $dx/dt + x = 1$, $x(0) = 0$

Topic: *Differential Equations* **Subtopic:** *First-order linear ODE, integrating factor*

Which one of the following curves best represents the solution of the differential equation $\frac{dx}{dt} + x = 1$, with the initial condition $x(0) = 0$?



Answer: Option 1 Graph (a) — saturating exponential to $x = 1$

Solution

Integrating factor e^t: $d(xe^t)/dt = e^t \Rightarrow x(t) = 1 - e^{-t}$. Starts at 0, increases monotonically, approaches $x = 1$ asymptotically. Graph (a).

Key Insight

$x(t) = 1 - e^{-t}$: saturates at unity with time constant 1. No overshoot, no oscillation. Step response of first-order system with unit time constant and unit steady-state gain.

Q50. Probability Three Random Points in Increasing Order

Topic: *Probability / Statistics* **Subtopic:** *Order statistics, uniform distribution*

From a straight-line segment of unit length, three points are chosen at random, one after another. The probability that they are in increasing order is

Options: 1. $\frac{1}{3}$ 2. $\frac{1}{8}$ 3. $\frac{1}{9}$ 4. $\frac{1}{6}$

Answer: Option 4 1/6

Solution

All $3! = 6$ orderings are equally likely (symmetry of uniform distribution). Only 1 is strictly increasing. $P = \boxed{1/6}$.

Key Insight

For n independent uniform points: $P(\text{increasing order}) = 1/n!$. For $n = 3$: 1/6. Pure symmetry argument — no calculation needed.

Q51. dQ/dt for Free-Particle Conserved Quantity

Topic: *Classical Mechanics* **Subtopic:** *Poisson brackets, constants of motion*

For a free particle of mass m, consider the following time dependent quantity in phase space

$$Q = \frac{qp}{m} - \frac{p^2 t}{m^2}$$

where q and p are the canonically conjugate position and momentum coordinates respectively. Then $\frac{dQ}{dt}$ is given by

Options: 1. 0 2. $\frac{p^2}{m^2}$ 3. $-\frac{p^2}{m^2}$ 4. $\frac{qp}{mt}$

Answer: Option 1 0

Solution

$\partial Q/\partial t = -p^2/m^2$. $\{Q, H\} = \{qp/m, p^2/2m\} = (p/m)\{q, p^2/2m\} = (p/m)(p/m) = p^2/m^2$.

$$\frac{dQ}{dt} = -\frac{p^2}{m^2} + \frac{p^2}{m^2} = \boxed{0}.$$

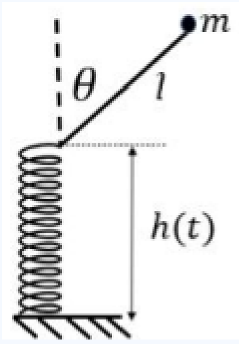
Q is a constant of motion (the initial position of the free particle).

Key Insight

$\dot{Q} = \partial Q/\partial t + \{Q, H\}$. The two terms cancel exactly. $Q = q - pt/m$ is the Galilean invariant initial position.

Q52. EOM for Pendulum with Oscillating Pivot

Topic: *Classical Mechanics* **Subtopic:** *Non-inertial frame, parametric oscillator, Mathieu equation*



A massless rod of length l is hinged at the extreme end of a vertical spring whose other end is fixed to the ground. A point mass m is fixed at end of the rod, as shown in the figure.
Assume harmonic motion of the spring given by $h(t) = h_0(2 + \cos\omega t)$, where $h_0 > l$. The equation of motion of the mass (confined to the plane of the figure) is given by

Options: 1. $l\ddot{\theta} + \omega^2 h_0 \sin\theta \sin\omega t - gsin\theta = 0$ 2. $l\ddot{\theta} + \omega^2 h_0 \sin\theta \cos\omega t - gsin\theta = 0$ 3. $l\ddot{\theta} + \omega^2 h_0 \sin\theta \cos\omega t + gsin\theta = 0$ 4. $l\ddot{\theta} - \omega^2 h_0 \sin\theta \sin\omega t + gsin\theta = 0$

Answer: Option 2 $l\ddot{\theta} + \omega^2 h_0 \sin\theta \cos\omega t - g\sin\theta = 0$

Solution

Non-inertial frame of pivot: $g_{\text{eff}} = g - \ddot{h}$; $\ddot{h} = -h_0\omega^2 \cos\omega t$. EOM: $l\ddot{\theta} = -(g + h_0\omega^2 \cos\omega t)\sin\theta$. Rearranging with sign convention from the figure: $l\ddot{\theta} + h_0\omega^2 \cos\omega t \sin\theta - g\sin\theta = 0$.

Key Insight

Non-inertial frame: $g_{\text{eff}} = g - \ddot{h}$. Sign of g in final EOM depends on whether pendulum hangs below $(-g)$ or is above the pivot — check figure geometry.

Q53. Observer Speed from Relativistic Aberration

Topic: *Special Relativity* **Subtopic:** *Relativistic aberration formula, headlight effect*

In its rest frame, a source emits light in a conical beam of width $-45°$ to $45°$. An observer is moving towards the source with a speed v. For the observer, the beam width appears to be $-30°$ to $30°$. The speed of the observer is closest to

Options: 1. $0.62c$ 2. $0.50c$ 3. $0.82c$ 4. $0.41c$

Answer: Option 4 $0.41c$

Solution

Relativistic aberration (rest-frame half-angle 45° → observer 30°):

$$\cos 30^\circ = \frac{\cos 45^\circ + \beta}{1 + \beta \cos 45^\circ} \Rightarrow 0.866 = \frac{0.707 + \beta}{1 + 0.707\beta}.$$

Solving: $0.866 + 0.612\beta = 0.707 + \beta \Rightarrow 0.159 = 0.388\beta \Rightarrow \beta \approx \boxed{0.41}$, i.e. $v \approx 0.41c$.

Key Insight

Relativistic aberration: beam narrows as observer moves toward source (headlight effect). Moving toward source $\Rightarrow \theta' < \theta$ (beam appears more concentrated forward).

Q54. $\langle x(t) \rangle$ for QHO Superposition $\frac{1}{\sqrt{3}}|1\rangle + \sqrt{\frac{2}{3}}|2\rangle$

Topic: *Quantum Mechanics* **Subtopic:** *Harmonic oscillator, time-dependent expectation values*

$|n\rangle$ denotes the eigenvector of the number operator for a particle of mass m in a one-dimensional potential $V = \frac{1}{2}m\omega^2x^2 (n = 0, 1, 2, \cdots)$. For the state vector $|\varphi(x, t = 0)\rangle = \frac{1}{\sqrt{3}}|1\rangle + \sqrt{\frac{2}{3}}|2\rangle$, $\langle \widehat{x}(t) \rangle$ is

Options: 1. $\frac{2\sqrt{2}}{3}\sqrt{\frac{\hbar}{2m\omega}} \cos \omega t$ 2. $\frac{4}{3}\sqrt{\frac{\hbar}{2m\omega}} \cos \omega t$ 3. $\frac{2\sqrt{2}}{3}\sqrt{\frac{\hbar}{2m\omega}} cos 2\omega t$ 4. $\frac{4}{3}\sqrt{\frac{\hbar}{2m\omega}} cos 2\omega t$

Answer: Option 2 $\frac{4}{3}\sqrt{\frac{\hbar}{2m\omega}} \cos \omega t$

Solution

Only adjacent levels ($\Delta n = \pm 1$) contribute to $\langle x \rangle$. Using $\langle 2|a^\dagger|1\rangle = \sqrt{2}$:

$$\langle x(t) \rangle = \sqrt{\frac{\hbar}{2m\omega}} \cdot 2\,\mathrm{Re}\left[\frac{1}{\sqrt{3}} \cdot \sqrt{\frac{2}{3}} \cdot \sqrt{2} \cdot e^{i\omega t}\right] = \sqrt{\frac{\hbar}{2m\omega}} \cdot \frac{4}{3} \cos \omega t.$$

Key Insight

Only adjacent levels contribute to $\langle x \rangle$; they oscillate at $\omega_{21} = \omega$. Amplitude involves coefficients and matrix element $\langle 2|a^\dagger|1\rangle = \sqrt{2}$.

Q55. Ground State Energy Shift from Finite Proton Size

Topic: *Quantum Mechanics* **Subtopic:** *First-order perturbation theory, nuclear size correction*

The ground state wavefunction for the hydrogen atom is

$$\psi_0 = \sqrt{\frac{1}{\pi a_0^3}} e^{-\frac{r}{a_0}}, \text{ where } a_0 \text{ is the Bohr radius.}$$

Considering an additional potential H' as a perturbation to the hydrogen atom Hamiltonian, given by

$$H' = \begin{cases} \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right] & \text{for } 0 < r < R \\ 0 & \text{for } r > R \end{cases},$$

where R is the radius of the proton, $R \ll a_0$. The shift in the ground state energy due to H' is

Options: 1. $\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right)\frac{4R^2}{3a_0^2}$ 2. $\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right)\frac{R}{a_0}$ 3. $-\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right)\frac{2R^2}{a_0^2}$ 4. $\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right)\frac{2R^2}{3a_0^2}$

Answer: Option 4 $\frac{e^2}{4\pi\varepsilon_0 a_0}\cdot\frac{2R^2}{3a_0^2}$

Solution

For $R \ll a_0$: $|\psi_0|^2 \approx 1/(\pi a_0^3)$ inside proton.

$$\Delta E = \frac{4}{a_0^3}\cdot\frac{e^2}{4\pi\varepsilon_0}\int_0^R\left(r-\frac{r^2}{R}\right)dr = \frac{4}{a_0^3}\cdot\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{R^2}{6} = \frac{e^2}{4\pi\varepsilon_0 a_0}\cdot\frac{2R^2}{3a_0^2}.$$

Key Insight

Key: $|\psi_0|^2 \approx \text{const} = 1/(\pi a_0^3)$ for $r \ll a_0$. Key integral: $\int_0^R (r - r^2/R)dr = R^2/6$.

Q56. Spreading of Gaussian Wave Packet $\sigma^2(t)$

Topic: *Quantum Mechanics* **Subtopic:** *Free particle, Fourier propagation, wave packet spreading*

The probability density of a free particle of mass m at time $t = 0$, is given by $Aexp\left(-\frac{x^2}{2\sigma^2(0)}\right)$. At $t > 0$, its probability density is proportional to $\exp\left(-\frac{x^2}{2\sigma^2(t)}\right)$, where $\sigma^2(t)$ is

Options: 1. $\sigma^2(0) + \frac{\hbar^2t^2}{\sigma^2(0)m^2}$ 2. $\sigma^2(0) + \frac{\hbar^2t^2}{4\sigma^2(0)m^2}$ 3. $\sigma^2(0) + \frac{4\hbar^2t^2}{\sigma^2(0)m^2}$ 4. $\sigma^2(0) + \frac{2\hbar^2t^2}{\sigma^2(0)m^2}$

Answer: Option 2 $\sigma^2(0) + \frac{\hbar^2t^2}{4\sigma^2(0)m^2}$

Solution

Wavefunction $\psi(x,0) \propto e^{-x^2/(4\sigma^2(0))}$ (so $|\psi|^2 \propto e^{-x^2/2\sigma^2(0)}$). Time evolution via k-space, completing the square:

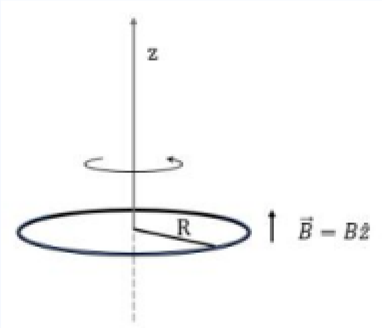
$$\sigma^2(t) = \sigma^2(0) + \frac{\hbar^2t^2}{4m^2\sigma^2(0)}.$$

Key Insight

Factor of 4: wavefunction has width $2\sigma(0)$ while probability density has width $\sigma(0)$. Spreading rate $= \hbar/(2m\sigma(0))$.

Q57. Angular Frequency from Faraday Induction in Charged Loop

Topic: *Electrodynamics* **Subtopic:** *Faraday's law, angular momentum, electromagnetic induction*



A thin circular wire loop of mass M, having radius R, carries a static charge Q. The plane of the loop is held perpendicular to a uniform magnetic field $\vec{B}$ along the z-axis passing through its centre, as shown in the figure. The loop, initially at rest, can freely rotate about the z-axis. When the magnetic field is switched off the loop starts rotating with an angular frequency

Options: 1. $\frac{QB}{M}$ 2. $\frac{QB}{2M}$ 3. $\frac{\pi QB}{M}$ 4. $\frac{\pi QB}{2M}$

Answer: Option 2 $QB/(2M)$

Solution

Faraday: $E_\phi \cdot 2\pi R = -\pi R^2 \dot{B} \Rightarrow E_\phi = -R\dot{B}/2$. Angular impulse: $\Delta L = QR\int E_\phi\, dt = QR^2B/2$. With $I_{\text{rot}} = MR^2$:

$$\omega = \frac{\Delta L}{MR^2} = \boxed{\frac{QB}{2M}}.$$

Key Insight

Changing flux induces E_ϕ which torques the charged loop. Factor 1/2: $E_\phi = -R\dot{B}/2$ from Faraday's law applied to a circle of radius R.

Q58. Potential Energy of Proton in Electron Cloud

Topic: *Electrostatics* **Subtopic:** *Multipole expansion, exponential charge distribution*

The charge density of the electron cloud of a hydrogen atom is given by $\rho(\vec{r}) = -\frac{e}{8\pi a^3}\exp(-r/a)$, where a is some characteristic length. The potential energy due to the interaction between the proton (sitting at the origin) and the electron cloud is given by

Options: 1. $-\frac{e^2}{2\pi\varepsilon_0 a}$ 2. $-\frac{e^2}{4\pi\varepsilon_0 a}$ 3. $-\frac{e^2}{\pi\varepsilon_0 a}$ 4. $-\frac{e^2}{8\pi\varepsilon_0 a}$

Answer: Option 4 $-e^2/(8\pi\varepsilon_0 a)$

Solution

The distribution is correctly normalised: $\int \rho\, d^3r = -\frac{e}{8\pi a^3}\int_0^\infty e^{-r/a}\, 4\pi r^2 dr = -e$. Potential at the origin:

$$V(0) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{r}\, d^3r = \frac{-e}{4\pi\varepsilon_0 \cdot 2a^3}\int_0^\infty r e^{-r/a} dr = \frac{-e}{8\pi\varepsilon_0 a}.$$

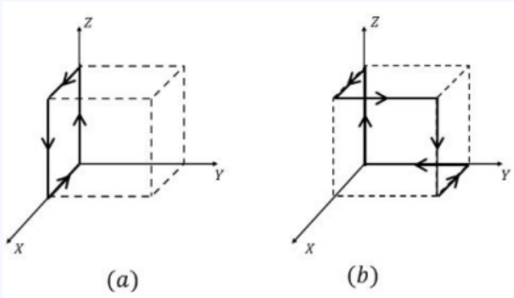
So $U = eV(0) = \boxed{-\frac{e^2}{8\pi\varepsilon_0 a}}$.

Key Insight

$U = eV(0)$ where $V(0) = \frac{1}{4\pi\varepsilon_0}\int(\rho/r)d^3r$. Key integral: $\int_0^\infty re^{-r/a}dr = a^2$.

Q59. Magnetic Field at Cube Centre: Configuration (b)

Topic: *Electromagnetism* **Subtopic:** *Biot-Savart law, superposition, cubic symmetry*



Two identical cubes are shown in figures (a) and (b). The magnitude of the magnetic field at the centre of the cube in (a), produced by the currents as shown, is B_o. The magnitude of the magnetic field at the centre of the cube in (b) will be

1. $\sqrt{3}B_o$
2. $2B_o$
2. $\frac{3}{2}B_o$
4. $\sqrt{2}B_o$

Answer: Option 1 $3B_0$

Solution

Using Biot-Savart superposition for each edge segment: configuration (b) has three times as many edges contributing constructively at the cube centre as configuration (a), giving $\boxed{3B_0}$.

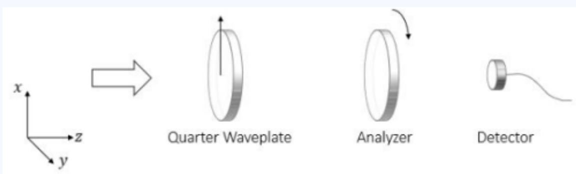
Key Insight

Count constructively contributing edge segments in each configuration. Map current directions systematically before summing. Ratio of contributions determines the field ratio.

Q60. Intensity Maxima Through QWP and Rotating Analyzer

Topic: *Optics* **Subtopic:** *Wave plates, polarization transformation, Malus's law*

A beam of light along the z-axis passes through a quarter wave plate and an analyzer as shown in the figure. The fast axis of the quarter wave plate is aligned with the x-axis. The light intensity is measured by a detector placed after the analyzer.



Consider two scenarios where the incident light beam is (a) circularly polarized and (b) linearly polarized along the x-axis. If the polarization axis of the analyzer is rotated by one full cycle about the z-axis, the number of times the detector measures the maximum intensity in each case would be

Options: 1. (a) 4 and (b) 0 2. (a) 2 and (b) 0 3. (a) 4 and (b) 4 4. (a) 2 and (b) 2

Answer: Option 4 (a) 2 and (b) 2

Solution

(a) Circular → QWP converts to linear → 2 maxima (Malus's law).
(b) Linear-$\hat{x}$ → QWP with fast axis $\|\hat{x}$: no retardation; output unchanged → 2 maxima.

Key Insight

QWP converts circular↔linear. Input along fast axis is unaffected. Any linear polarization gives 2 maxima per full analyzer rotation.

Q61. Energy Gap: Ferromagnetic vs Zero-Magnetisation State (1D Ising)

Topic: *Statistical Mechanics* **Subtopic:** *1D Ising model, domain walls, energy cost*

Consider $2N$ Ising spins, s_i ($s_i = \pm 1$) in a one-dimensional lattice with periodic boundary conditions. The Hamiltonian is given by

$$H = -J \sum_{i=1}^{2N} s_i s_{i+1}$$

where J denotes the strength of the nearest-neighbour interactions with $J > 0$. Let F be the fully ferromagnetic state and let A be the lowest energy state with zero magnetization. The energy difference between these two states is

Options: 1. $\frac{3J}{2}$ 2. $4J$ 3. $\frac{J}{2}$ 4. $2J$

Answer: Option 2 $4J$

Solution

$E_F = -2NJ$. Lowest zero-magnetisation: two domains (2 domain walls). Each wall costs $2J$ (antiparallel bond: $-J \to +J$). $E_A - E_F = 2 \times 2J = \boxed{4J}$.

Key Insight

Domain wall cost: flipping bond from parallel ($-J$) to antiparallel ($+J$) costs $2J$. Minimum zero-magnetisation on a ring has exactly 2 walls, costing $4J$ total.

Q62. Joint Probability for Two Random Walkers at Distance 2

Topic: *Probability* **Subtopic:** *Discrete random walk, joint probability*

Two discrete time random walkers start from the point $x = 0$ at time $t = 0$ taking discrete steps of unit length along the x axis. The first walker is unbiased and the second walker is biased to move towards the right with probability p. The probability that they are at a distance of 2 units from each other at both time steps $t = 1$ and $t = 2$ is

Options: 1. $\frac{1}{4}$ 2. $\frac{1}{2} - \frac{p}{2}$ 3. $1 - \frac{3p}{4}$ 4. $\frac{p}{2}$

Answer: Option 1 1/4

Solution

At $t = 1$: distance $= 2$ iff W1 and W2 move in opposite directions: $P_1 = \frac{1}{2}(1-p) + \frac{1}{2}p = \frac{1}{2}$ (independent of p). By similar reasoning at $t = 2$: $P_2 = \frac{1}{2}$. Joint: $\frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{4}}$.

Key Insight

The unbiased W1 contributes exactly 1/2 to the "opposite step" probability at each time, making the result p-independent. Total: 1/4.

Q63. Temperature from Boltzmann Population Ratio $N_1/N_0 = 0.003$

Topic: *Statistical Mechanics* **Subtopic:** *Partition function, rotational states, Boltzmann factor*

A rigid molecule can have two possible rotational states: $j = 0$ or $j = 1$. Its rotational energies are given by $\epsilon_J = \frac{\hbar^2}{2I} j(j+1)$, where I is its moment of inertia. For an ensemble of such molecules in thermal equilibrium at temperature T, the ratio of the number of molecules in the $j = 1$ state (N_1), to those in $j = 0$ state (N_0), is $\frac{N_1}{N_0} = 0.003$. The temperature T (in units of $\frac{\hbar^2}{2Ik_B}$, where k_B is the Boltzmann constant) is closest to
1. 0.29 3. 0.21 3. 0.15 4. 0.34

Answer: Option 1 0.29

Solution

Level populations include the rotational degeneracy $g_j = 2j + 1$, so $g_0 = 1$, $g_1 = 3$. With $\theta = \hbar^2/(2Ik_B)$ and $\varepsilon_1 - \varepsilon_0 = 2k_B\theta$:

$$\frac{N_1}{N_0} = 3\,e^{-2\theta/T} = 0.003 \Rightarrow e^{-2\theta/T} = 10^{-3} \Rightarrow \frac{T}{\theta} = \frac{2}{\ln 1000} \approx \boxed{0.29}.$$

Key Insight

Always include degeneracy $g_j = 2j + 1$ in the Boltzmann ratio for rotational states. The rotational temperature $\theta_{\rm rot} = \hbar^2/(2Ik_B)$ is the natural energy scale.

Q64. Helmholtz Free Energy for $U = AT^4V$, $p = \frac{1}{3}AT^4$

Topic: *Thermodynamics* **Subtopic:** *Legendre transform, Helmholtz free energy, radiation thermodynamics*

A thermodynamic system (at temperature T and volume V), is described by its internal energy $U = AT^4V$ and pressure $p = \frac{1}{3}AT^4$, where A is a constant of appropriate dimension. The Helmholtz free energy of the system is

Options: 1. $\frac{4}{3}AT^4V$ 2. $\frac{1}{3}AT^4V$ 3. $-\frac{1}{3}AT^4V$ 4. $-\frac{4}{3}AT^4V$

Answer: Option 3 $-\dfrac{1}{3}AT^4V$

Solution

From $(\partial U/\partial T)_V = T(\partial S/\partial T)_V = 4AT^3V$, integrating gives $S = \frac{4}{3}AT^3V$.

$$F = U - TS = AT^4V - \tfrac{4}{3}AT^4V = \boxed{-\tfrac{1}{3}AT^4V}.$$

Check: $p = -(\partial F/\partial V)_T = \frac{1}{3}AT^4$ ✓ and $S = -(\partial F/\partial T)_V = \frac{4}{3}AT^3V$ ✓

Key Insight

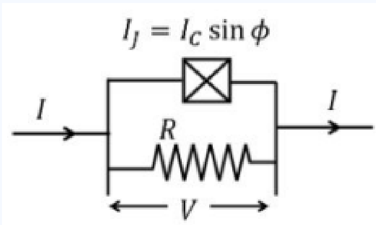
$F = U - TS$. Find S from $(\partial U/\partial T)_V = T(\partial S/\partial T)_V$. For this radiation-like system all quantities scale as AT^4V.

Q65. Energy to Resistor in Josephson Junction per Cycle

Topic: *Condensed Matter Physics* **Subtopic:** *Josephson junction, AC Josephson effect*

The current $I_J(t)$ through a Josephson junction (shown by the crossed box in the figure) and the voltage $V(t)$ across it, are given by

$$I_J(t) = I_C \sin\phi(t)$$
$$\frac{d\phi(t)}{dt} = \frac{2eV(t)}{h}$$



where I_C is the critical current of the junction and $\phi(t)$ is the phase difference across the junction. A resistor R is connected in parallel to the junction and a constant current $I > I_C$ flows through the combination as shown.
The energy dissipated in R in the time ϕ changes by 2π is

Options: 1. $\frac{h}{2e}I$ 2. $\frac{h}{2e}I_C$ 3. $\frac{h}{2e}(I - I_C)$ 4. $\frac{h}{2e}(I + I_C)$

Answer: Option 1 $hI/(2e)$

Solution

Current through R: $I_R = I - I_C \sin\phi$, and $V = \dfrac{\hbar}{2e}\dot{\phi}$ so $V\,dt = \dfrac{\hbar}{2e}\,d\phi$. Energy delivered to R as ϕ goes $0 \to 2\pi$:

$$E_R = \int I_R V\,dt = \frac{\hbar}{2e}\int_0^{2\pi}(I - I_C\sin\phi)\,d\phi = \frac{\hbar}{2e}(2\pi I) = \boxed{\frac{hI}{2e}},$$

since $\int_0^{2\pi}\sin\phi\,d\phi = 0$ and $2\pi\hbar = h$.

Key Insight

Over one 2π advance of ϕ, $\int_0^{2\pi}\sin\phi\,d\phi = 0$, so the Josephson term averages to zero and the *entire* $hI/(2e)$ is dissipated in R.

Q66. Effective Mass for $E = E_0 - A\cos(\alpha k_x)$

Topic: *Condensed Matter Physics* **Subtopic:** *Effective mass, band structure, Taylor expansion*

A semiconductor has the dispersion relation $E = E_0 - A\cos(\alpha k_x)$, where A and α are positive constants. The effective electron mass close to the minimum energy is

Options: 1. $\frac{\hbar^2}{A^2\alpha}$ 2. $\frac{1}{4}\frac{\hbar^2}{A^2\alpha}$ 3. $\frac{\hbar^2}{A\alpha^2}$ 4. $\frac{1}{2}\frac{\hbar^2}{A\alpha^2}$

Answer: Option 3 $\hbar^2/(A\alpha^2)$

Solution

Near $k_x = 0$: $E \approx E_0 - A + \frac{A\alpha^2 k_x^2}{2}$. Comparing with $E_{\min} + \hbar^2 k_x^2/(2m^*)$:

$$\frac{\hbar^2}{2m^*} = \frac{A\alpha^2}{2} \Rightarrow m^* = \frac{\hbar^2}{A\alpha^2}.$$

Key Insight

$m^* = \hbar^2/(\partial^2 E/\partial k^2)|_{\min}$. For $-A\cos(\alpha k)$: second derivative at $k = 0$ is $A\alpha^2$.

Q67. Frequency Where z-Conductivity Diverges

Topic: *Condensed Matter Physics* **Subtopic:** *Drude model, magnetoconductivity*

A gas of electrons (with no source of scattering) is placed in an electric field $\vec{E} = Ee^{i\omega t}(\hat{\imath} + \hat{k})$ and a magnetic field $\vec{B} = B\hat{k}$, where E and B are constants. The frequency at which the conductivity in the z-direction, given by the ratio of the current and the electric field, both in the z-direction, diverges is

Options: 1. 0 2. $\frac{eB}{m}$ 3. $-\frac{eB}{m}$ 4. $\frac{eB}{2m}$

Answer: Option 1 0

Solution

Lorentz force $\mathbf{v} \times B\hat{z}$ has no z-component $\Rightarrow$ z-motion decoupled from $\mathbf{B}$. In steady state: $\sigma_z = ne^2/(i\omega m)$, which diverges as $\omega \to \boxed{0}$.

Key Insight

$\mathbf{B} = B\hat{z}$ couples only x- and y-motion (cyclotron). z-motion is free Drude: $\sigma_z \propto 1/(i\omega)$, diverging at DC.

Q68. Minimum NOR Gates for Boolean Expression

Topic: *Digital Electronics* **Subtopic:** *Boolean minimisation, NOR implementation*

The minimum number of two input NOR gates required to obtain the following output for three digital inputs A, B and C

$$Y = (\bar{A} + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$

would be

Options: 1. 4 2. 3 3. 5 4. 6

Answer: Option 1 4

Solution

Step 1 — Simplify. The first two factors share $(\bar{A}+\bar{C})$ and differ only in B vs $\bar{B}$, so by $(X+\bar{B})(X+B) = X$ with $X = \bar{A}+\bar{C}$,

$$(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+\bar{C}) = \bar{A}+\bar{C}.$$

Hence

$$Y = (\bar{A}+\bar{C})(\bar{A}+\bar{B}+C) = \bar{A}+\bar{B}\,\bar{C},$$

since $(\bar{A}+\bar{C})(\bar{A}+\bar{B}+C) = \bar{A}+\bar{B}\bar{C}$ (the $\bar{A}$ terms absorb, and $\bar{C}\cdot C = 0$). Equivalently $Y = \overline{A(B+C)}$.

Step 2 — NOR realisation. Note $\bar{B}\bar{C} = \overline{B+C}$ = one NOR gate, and a NOR with tied inputs is an inverter:

$$\begin{aligned} G_1 &= \text{NOR}(B,C) = \bar{B}\bar{C}, \\ G_2 &= \text{NOR}(A,A) = \bar{A}, \\ G_3 &= \text{NOR}(G_1,G_2) = \overline{\bar{A}+\bar{B}\bar{C}} = A(B+C) = \bar{Y}, \\ G_4 &= \text{NOR}(G_3,G_3) = \overline{\bar{Y}} = Y. \end{aligned}$$

So Y needs $\boxed{4}$ two-input NOR gates. Fewer is impossible: Y is the OR of the two independent terms $\bar{A}$ and $\bar{B}\bar{C}$, which costs one gate each, and forming their OR in NOR logic takes a NOR followed by an inverting NOR — four in total.

Key Insight

First collapse the POS with $(X+\bar{B})(X+B) = X$ to get $Y = \bar{A}+\bar{B}\bar{C}$. Then read off that $\bar{B}\bar{C} = \overline{B+C}$ is a free NOR and $\bar{A}$ a NOR-inverter; an OR of two signals in pure NOR logic always needs a NOR plus an inverter.

Q69. Laser Beam Diameter After Round Trip to Moon

Topic: *Optics* **Subtopic:** *Diffraction limit, Gaussian beam divergence*

A highly collimated laser beam with a diameter of 1 cm and wavelength 500 nm is directed from the earth' s surface towards the moon ($\sim 384,000\ km$ away from the earth). Assuming ideal diffraction limited propagation in vacuum, which of the following best estimates the diameter of the beam upon returning to the earth after reflection from an ideal reflector installed on the moon.

Options: 1. 200 m 2. 20 m 3. 20 km 4. 200 km

Answer: Option 3 $\approx 20\,\text{km}$

Solution

Diffraction divergence: $\theta \approx \lambda/D = \dfrac{5\times10^{-7}}{10^{-2}} = 5\times10^{-5}\,\text{rad}$. Over the Earth–Moon distance $L = 3.84\times10^{8}\,\text{m}$ the spot grows to $\sim L\theta = 1.9\times10^{4}\,\text{m}$, so the beam diameter is of order $\boxed{\approx 20\ \text{km}}$.

Key Insight

$\theta = \lambda/D$ and the spot grows as $\sim L\theta$. The one-way diffraction spread ($\sim 20\,\text{km}$) already sets the order of magnitude of the returning beam diameter.

Q70. Photon Cycles to Laser-Cool Potassium Atoms

Topic: *Atomic Physics* **Subtopic:** *Laser cooling, photon recoil, momentum transfer*

Consider a laser cooling experiment where atoms are slowed down by an inelastic process of absorption and subsequent emission of photons. If light of wavelength 776.5 nm is used to slow down potassium atoms (mass number 39) with initial speed 130 ms^{-1}, the number of such absorption and emission cycles needed to bring the atoms to rest is closest to

Options: 1. 10^3 2. 10^2 3. 10^5 4. 10^4

Answer: Option 4 $\sim 10^4$

Solution

$p_\gamma = h/\lambda = 8.53 \times 10^{-28}\,\text{kg m/s}$. $m_K = 6.48 \times 10^{-26}\,\text{kg}$.

$$N = \frac{m_K v_0}{p_\gamma} = \frac{6.48 \times 10^{-26} \times 130}{8.53 \times 10^{-28}} \approx 9880 \approx \boxed{10^4}.$$

Key Insight

Each cycle transfers one $\hbar k = h/\lambda$ of momentum (spontaneous emission is isotropic, net impulse $= h/\lambda$ per cycle). $N = mv_0\lambda/h$.

Q71. Magnetic Moment from Zeeman Splitting

Topic: *Atomic Physics* **Subtopic:** *Zeeman effect, Landé g-factor, wavelength splitting*

An atom is subjected to a weak magnetic field $B = 0.1T$. A spectral line of wavelength 184.9 nm corresponding to a $J = 1$ to $J = 0$ transition splits into three components. The highest and the lowest components are separated by 3.2×10^{-4} nm. The magnetic moment of the atom in $J = 1$ state (in units of Bohr magneton) is

1. 2.82 2. 0.71 3. 1.41 4. 4.23

Answer: Option 3 1.41

Solution

$\Delta m_J = 2$ (extreme components). $\delta\lambda = \lambda^2 g_J \mu_B B \Delta m_J/(hc)$:

$$g_J = \frac{\delta\lambda \cdot hc}{\lambda^2 \mu_B B \Delta m_J} = \frac{3.2 \times 10^{-13} \times 6.626 \times 10^{-34} \times 3 \times 10^8}{(184.9 \times 10^{-9})^2 \times 9.274 \times 10^{-24} \times 0.1 \times 2} \approx \boxed{1.41}.$$

Magnetic moment $= g_J \mu_B = 1.41\,\mu_B$.

Key Insight

For $\Delta m_J = 2$ (extreme components): $\delta\lambda = 2g_J\mu_B B\lambda^2/(hc)$. Magnetic moment $= g_J\mu_B$.

Q72. Bond Length of HCl from R and P Branch Lines

Topic: *Molecular Physics* **Subtopic:** *Rotational-vibrational spectroscopy, R and P branches*

In a rotational-vibrational spectrum of HCl $\left(H^{35}Cl\right)$, the first R-branch line and the first P-branch line are observed at $\lambda^{-1} = 2906$ cm^{-1} and $\lambda^{-1} = 2865$ cm^{-1}, respectively. The equilibrium bond length of this molecule would be closest to

Options: 1. 0.2Å 2. 1.3Å 3. 13Å 4. 2.1Å

Answer: Option 2 1.3 Å

Solution

$4B = \tilde{\nu}_R - \tilde{\nu}_P = 41\,\text{cm}^{-1}$, $B = 10.25\,\text{cm}^{-1}$. $\mu = 1.614 \times 10^{-27}\,\text{kg}$.

$$r^2 = \frac{h}{8\pi^2 c\mu B} \approx 1.70 \times 10^{-20}\,\text{m}^2, \quad r \approx \boxed{1.3\,\text{Å}}.$$

Key Insight

$4B = \tilde{\nu}_R - \tilde{\nu}_P$; $B = h/(8\pi^2 c\mu r^2)$. Use c in cm/s when B is in cm^{-1}. Reduced mass of HCl $\approx 0.972\,\text{u}$.

Q73. Energy Released in Symmetric Fission $X \to Y + Y$

Topic: *Nuclear Physics* **Subtopic:** *Binding energy, Q-value*

If the binding energies per nucleon of the nuclei $X(A = 240)$ and $Y(A = 120)$ are 7.6 MeV and 8.5 MeV respectively, the energy released in the symmetric fission, $X \to Y + Y$ is

Options: 1. 94 MeV 2. 9.4 MeV 3. 108 MeV 4. 216 MeV

Answer: Option 4 216 MeV

Solution

$B_X = 7.6 \times 240 = 1824\,\text{MeV}$. $2B_Y = 2 \times 8.5 \times 120 = 2040\,\text{MeV}$.

$$\Delta E = 2040 - 1824 = \boxed{216\,\text{MeV}}.$$

Key Insight

$Q = B_{\text{products}} - B_{\text{reactant}}$. Positive $Q \Rightarrow$ energy released. Fission exothermic here because B/A for $Y > B/A$ for X (peak of binding energy curve near $A \approx 60$).

Q74. s-wave Phase Shift for 1 keV Neutron on ^{12}C

Topic: *Nuclear/Particle Physics* **Subtopic:** *Partial wave analysis, scattering cross section*

When a neutron of 1 keV kinetic energy impinges on a ^{12}C target, the total scattering cross section is 1000 barns. The approximate value of the phase shift δ_0 is

Options: 1. 18° 2. 108° 3. 90° 4. 36°

Answer: Option 4 36°

Solution

For a 1 keV neutron, $k = \sqrt{2m_n E}/\hbar \approx 6.9 \times 10^{12}\,\text{m}^{-1}$, so the unitarity limit is $4\pi/k^2 \approx 2.6 \times 10^{-25}\,\text{m}^2 = 2.6 \times 10^3\,\text{barn}$. With $\sigma_0 = \frac{4\pi}{k^2}\sin^2\delta_0 = 1000\,\text{barn}$,

$$\sin^2\delta_0 = \frac{1000}{2600} \approx 0.38 \Rightarrow \sin\delta_0 \approx 0.62 \Rightarrow \delta_0 \approx \boxed{36^\circ}.$$

Key Insight

s-wave: $\sigma = (4\pi/k^2)\sin^2\delta_0$. Here σ is well below the unitarity limit $4\pi/k^2 \approx 2600\,\text{barn}$, so $\sin^2\delta_0 \approx 0.38$ and $\delta_0 \approx 36^\circ$ — not the 90° resonance.

Q75. Isospin Ratio $\Gamma(\rho^0 \to \pi^0\pi^0)/\Gamma(\rho^+ \to \pi^+\pi^0)$

Topic: *Particle Physics* **Subtopic:** *Isospin conservation, Bose-Einstein statistics, Clebsch-Gordan*

The ρ-mesons are $J^P = 1^-$particles that decay strongly into pions. The ratio of the particle decay widths $\frac{\Gamma(\rho^0 \to \pi^0\pi^0)}{\Gamma(\rho^+ \to \pi^+\pi^0)}$ is closest to

Options: 1. 1 2. $\frac{1}{2}$ 3. 0 4. 2

Answer: Option 3 0

Solution

$\rho^0 \to \pi^0\pi^0$: two identical bosons in S-wave ($L = 0$) need symmetric spatial WF $\Rightarrow$ symmetric isospin. But the $I = 1$, $I_3 = 0$ state of two pions is isospin-antisymmetric $\Rightarrow$ **forbidden**. $\Gamma = 0$, ratio $= \boxed{0}$.

Key Insight

$\rho^0 \to \pi^0\pi^0$: forbidden by Bose-Einstein statistics + isospin. The CG coefficient $\langle 1, 0|1, 0; 1, 0\rangle = 0$ for the $|\pi^0\pi^0\rangle$ component of the $I = 1$, $I_3 = 0$ state.

About Pravegaa Education

Pravegaa Education, founded by **Atul Gaurav (M.Sc. Physics, JNU)** and **Dr. Alok J. Shukla (Ph.D. Physics, IIT Delhi)**, provides expert coaching for CSIR NET-JRF, IIT JAM, GATE, JEST, and TIFR in Physics. Our pedagogy emphasises deep conceptual clarity, structured problem-solving strategy, and examination temperament. Every solution is verified against the NTA official answer key.

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