

Pravegaa Education

CSIR NET-JRF · GATE · IIT-JAM · JEST · TIFR Physics

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Physical Sciences — Complete Solutions

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Exam: 17 July 2026 (3:00–6:00 PM) | Part A: Q1–20 | Part B: Q21–45 | Part C: Q46–75

Answer Key at a Glance

Verified against the NTA provisional answer key (question order as in the official master paper)

Q	Key	Q	Key	Q	Key
1	1	26	4	51	3
2	2	27	3	52	1
3	1	28	1, 4	53	3
4	3	29	3	54	4
5	2	30	4	55	2
6	3	31	3	56	2
7	1	32	3	57	3
8	3	33	1	58	4
9	4	34	2	59	3
10	3	35	2	60	3
11	4	36	2	61	2
12	3	37	1	62	2
13	3	38	1	63	3
14	2	39	3	64	3
15	1	40	1	65	3
16	4	41	3	66	1
17	3	42	1	67	1
18	3	43	1	68	Dropped
19	2	44	1	69	2
20	3	45	1	70	3
21	3	46	1	71	2
22	4	47	2	72	3
23	3	48	2	73	1
24	1	49	1	74	4
25	2	50	1	75	4

Q1–20: Part A (General Aptitude); Q21–45: Part B; Q46–75: Part C. **Q28**: options 1 and 4 are both correct (both accepted by NTA). **Q68**: dropped by NTA — none of the options is correct (derivation included).

Part A

General Aptitude (Q1–Q20)

Q1. Completing a Figure Pattern

[Part A]

General Aptitude | Non-verbal reasoning

Select a figure from amongst the four alternatives which when placed in the blank space of fig (X) would complete the pattern?

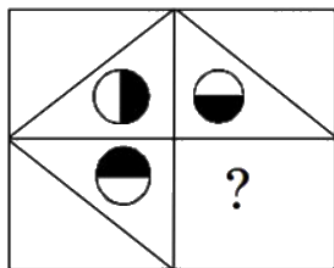


Fig (X)



Solution

- **Lines:** the four diagonal segments join the mid-points of the sides, forming a diamond. The missing quadrant (bottom-right) needs the segment from the right mid-point to the bottom mid-point, a “/” diagonal. This rules out option 3.
- **Circle position:** in each quadrant the circle sits in the corner nearest the centre, so in the bottom-right quadrant it is at the top-left.
- **Shading:** going clockwise TL → TR → BR → BL, the black half turns right → bottom → _ → top, rotating 90° clockwise each step. The missing circle is black on the *left*.

Option 1 (“/” diagonal, circle at top-left, left half black) fits. Option 4 has the right half black, and option 2 the bottom half black.

Answer: (1)

Q2. Letter Series

[Part A]

General Aptitude | *Series*

XUR, TQN, PMJ, __?__, HEB.

Which of the following options will complete the above series by replacing the question mark?

- (1) KHE
- (2) LIF
- (3) LJH
- (4) MJG

Solution

Each letter position moves back by 4 in the alphabet: $X \rightarrow T \rightarrow P \rightarrow \mathbf{L} \rightarrow H$, $U \rightarrow Q \rightarrow M \rightarrow \mathbf{I} \rightarrow E$, $R \rightarrow N \rightarrow J \rightarrow \mathbf{F} \rightarrow B$.

Answer: (2) LIF**Q3. Number Code**

[Part A]

General Aptitude | *Coding-decoding*

In a certain code if '1000' is written as '1728' and '125' is written as '343'. And '343' is written as '729', in the same code '512' will be written as:

- (1) 1000
- (2) 1331
- (3) 990
- (4) 1010

Solution

$1000 = 10^3 \rightarrow 12^3 = 1728$, $125 = 5^3 \rightarrow 7^3 = 343$, $343 = 7^3 \rightarrow 9^3 = 729$: the rule is $n^3 \rightarrow (n+2)^3$.
So $512 = 8^3 \rightarrow 10^3 = 1000$.

Answer: (1) 1000

Q4. Direction Sense

[Part A]

General Aptitude | Directions

There are four towns P, Q, R and T. Q is to the South-West of P, R is to the East of Q and South-East of P and T is to the North of R in line with Q, P. In which direction of P is T located?

- (1) South-East
- (2) North
- (3) North-East
- (4) East

Solution

Place $P = (0, 0)$. Q is South-West: $Q = (-1, -1)$. R is East of Q and South-East of P: $R = (1, -1)$. The line through Q and P is $y = x$. T lies due North of R ($x = 1$) on this line, so $T = (1, 1)$, which is North-East of P.

Answer: (3) North-East

Q5. Coded Family Relations

[Part A]

General Aptitude | Blood relations

Read the following information to answer the question

$P + Q$ means P is the brother of Q;

$P \times Q$ mean P is the father of Q;

$P - Q$ means P is the sister of Q.

Which of the following relation shows that I is the niece of K?

- (1) $K + Y + Z - I$
- (2) $K + Y \times I - Z$
- (3) $Z - I \times Y + K$
- (4) $K \times Y + I - Z$

Solution

For I to be K's niece, I must be female and a child of K's sibling.

Option 2, $K + Y \times I - Z$: K is the brother of Y; Y is the father of I; I is the sister of Z (so I is female). Hence I is the daughter of K's brother, i.e. K's niece. ✓

The others fail. In 1, I's gender is unknown and I is K's sibling. In 3, I is the father of Y. In 4, I is K's grandchild.

Answer: (2) $K + Y \times I - Z$

Q6. Finding the Counterfeit Coin

[Part A]

General Aptitude | *Logical puzzles*

Suppose there are '9' identical gold-coins out of which 8 coins have same weight. One of the coin is counterfeit and weighs slightly less than other coins. You have a classical balance scale for weighing.

What is the minimum number of weighings required to definitively isolate and identify the counterfeit coin?

- (1) 3
- (2) 4
- (3) 2
- (4) 8

Solution

Each weighing has three outcomes (left light, right light, balance), so k weighings can distinguish 3^k cases. With 9 coins we need $3^k \geq 9 \Rightarrow k = 2$.

- Weighing 1: groups of 3 vs 3. The lighter pan holds the fake; if they balance, it is in the third group.
- Weighing 2: from that group of 3, weigh coin vs coin in the same way.

Answer: (3) 2

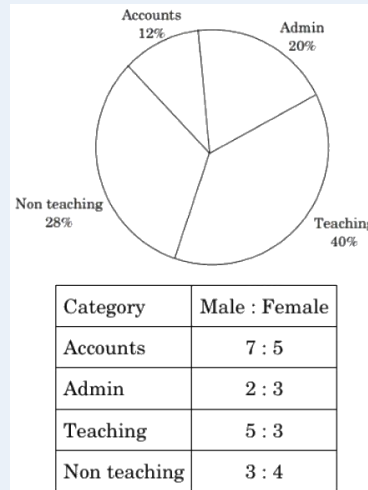
Q7. Pie Chart and Gender Ratios

[Part A]

General Aptitude | Data interpretation

The pie chart shows the percentage of employees working in different departments of a University. The table shows the ratio of males to females in each of these departments.

If the total number of employees = 2000, what is the ratio between female employees in Admin and non teaching departments?



- (1) 3 : 4
- (2) 5 : 3
- (3) 2 : 3
- (4) 4 : 3

Solution

Admin: 20% of 2000 = 400, with M:F = 2 : 3, so there are $\frac{3}{5} \times 400 = 240$ females. Non teaching: 28% of 2000 = 560, with M:F = 3 : 4, so there are $\frac{4}{7} \times 560 = 320$ females. Ratio = 240 : 320 = 3 : 4.

Answer: (1) 3 : 4

Q8. Bar Graph: Growth in Sales

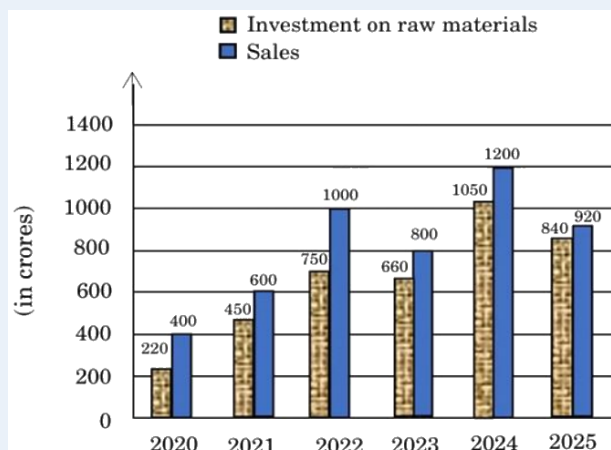
[Part A]

General Aptitude | Data interpretation

The graph represents investment on raw material and sales of an organization from 2020 to 2025 (calendar year)

Study the bar graph and answer the question.

In which year was there a maximum percentage increase in the sales of products as compared the previous year?



- (1) 2021
- (2) 2023
- (3) 2022
- (4) 2024

Solution

Sales (crores): 400, 600, 1000, 800, 1200, 920 for 2020–2025.

Year	2021	2022	2023	2024	2025
Change	+50%	+66.7%	–20%	+50%	–23.3%

The largest percentage increase is in 2022.

Answer: (3) 2022

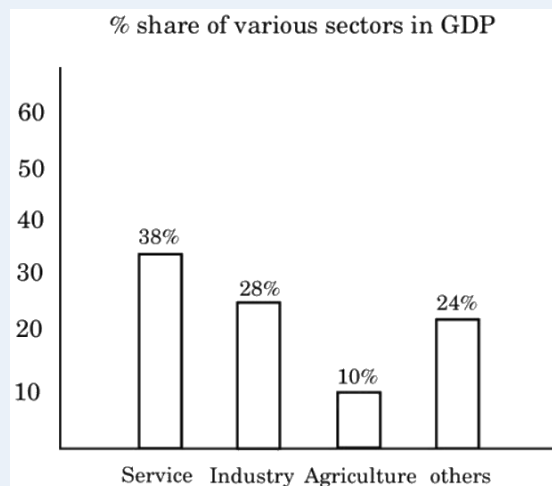
Q9. Sector-wise Share of GDP

[Part A]

General Aptitude | Data interpretation

The following chart shows the sector wise percentage break up of Rs. 650 crore, which is the GDP of a small country with a population of 1 billion people. 80% of the population constitutes its working population engaged in various sector and contributing to GDP.

Which of the following has lowest contribution to GDP?



- (1) Service
- (2) Industry
- (3) Agriculture
- (4) Non working population

Solution

The chart splits the entire GDP (100%) among the working sectors: Service 38%, Industry 28%, Agriculture 10%, others 24%. The remaining 20% of the population is non-working and, by the statement of the question, does not contribute to GDP at all. Its contribution is zero, lower even than agriculture's 10% (= Rs. 65 crore).

Answer: (4) Non working population

Q10. Separating Mixtures

[Part A]

General Aptitude | Everyday science

A process is used to separate substances in chemical mixtures. What is this process known as?

- (1) Distillation
- (2) Filtration
- (3) Chromatography
- (4) Crystallisation

Solution

Chromatography is the general technique for separating the components of a chemical mixture, using their different affinities for a stationary and a mobile phase (e.g. paper, column and gas chromatography). Distillation, filtration and crystallisation are special-purpose methods, based on boiling point, particle size and solubility respectively.

Answer: (3) Chromatography

Q11. Decrease in Entropy

[Part A]

General Aptitude | *Everyday science*

In which of the following, entropy will decrease?

- (1) Water is boiled
- (2) Sugar is dissolved in water
- (3) Ice is heated to melt
- (4) Egg is boiled in water

Solution

Boiling (liquid $\rightarrow$ vapour), dissolving sugar (mixing), and melting ice (solid $\rightarrow$ liquid) all increase the disorder of the system, so its entropy rises.

When an egg is boiled, the liquid albumen proteins coagulate into a rigid, cross-linked solid network. The system goes from a fluid to a set state: more ordered, with lower entropy. This is the intended answer.

Answer: (4) Egg is boiled in water

Q12. Interactions between Species

[Part A]

General Aptitude | *Ecology*

During the interaction of population of two different species identify the type where only one species is benefited and is detrimental to the other species.

- (1) Commensalism and Amensalism
- (2) Commensalism and Competition
- (3) Parasitism and Predation
- (4) Parasitism and Amensalism

Solution

Interactions of type $(+, -)$ (one species gains, the other is harmed) are parasitism and predation. Commensalism is $(+, 0)$, amensalism $(-, 0)$, and competition $(-, -)$.

Answer: (3) Parasitism and Predation

Q13. Population Growth

[Part A]

General Aptitude | Percentages

The present population of a city is 6000. If the number of males increases by 14% and the number of females increases by 22%, then the population will become 6900. What is the present population of females in the city?

- (1) 900
- (2) 700
- (3) 750
- (4) 850

Solution

Let the numbers of males and females be M and F , with $M + F = 6000$ and $1.14M + 1.22F = 6900$. Then

$$1.14(6000) + 0.08F = 6900 \Rightarrow 0.08F = 60 \Rightarrow F = 750.$$

Answer: (3) 750

Q14. Ratio and Difference

[Part A]

General Aptitude | Ratios

There are two numbers such that the difference between them is 10 and their ratio is 5:3. Find the product of these two numbers

- (1) 150
- (2) 375
- (3) 275
- (4) 80

Solution

The numbers are $5x$ and $3x$ with $2x = 10 \Rightarrow x = 5$. They are 25 and 15, and their product is 375.

Answer: (2) 375

Q15. Alarm Bells

[Part A]

General Aptitude | LCM

Three alarm bells ring at intervals 30 sec, 40 sec and 45 sec respectively. They started ringing simultaneously at 4:00 am in the morning.

At what time will they all ring simultaneously:

- (1) 4:06 am
- (2) 4:36 am
- (3) 4:40 am
- (4) 5:00 am

Solution

$\text{LCM}(30, 40, 45) = 2^3 \cdot 3^2 \cdot 5 = 360 \text{ s} = 6 \text{ min}$. They next ring together at 4:06 am.

Answer: (1) 4:06 am

Q16. Mode and Median

[Part A]

General Aptitude | *Statistics*

There are four positive integers say A, B, C and D such that $A \leq B \leq C \leq D$. Consider the following statements :

- A. The unique mode of the data is 6.
- B. The median score of the data is 7.

What is the minimum possible value of the largest number D?

- (1) 6
- (2) 7
- (3) 8
- (4) 9

Solution

Median = $\frac{B+C}{2} = 7$. For 6 to be the *unique* mode, 6 must occur at least twice; since $B \leq 7$ and the values are sorted, $A = B = 6$, hence $C = 8$. Then $D \geq C = 8$. If $D = 8$, the value 8 also occurs twice and the mode is no longer unique. So $D_{\min} = 9$ (data: 6, 6, 8, 9).

Answer: (4) 9

Q17. Triangle in a Semicircle

[Part A]

General Aptitude | *Mensuration*

The largest possible triangle is inscribed in a semi-circle of diameter 14 cm. What would be the area inside the semi-circle which is not occupied by the triangle?

- (1) 35 sq. cm
- (2) 30 sq. cm
- (3) 28 sq. cm
- (4) 48 sq. cm

Solution

$r = 7 \text{ cm}$. The largest inscribed triangle has the diameter as base and height r : area = $\frac{1}{2}(14)(7) = 49 \text{ cm}^2$. Semicircle area = $\frac{1}{2}\pi r^2 = \frac{1}{2} \cdot \frac{22}{7} \cdot 49 = 77 \text{ cm}^2$. Unoccupied area = $77 - 49 = 28 \text{ cm}^2$.

Answer: (3) 28 cm^2

Q18. Simple Interest

[Part A]

General Aptitude | Simple interest

A certain sum of money becomes double of itself in 5 years at simple rate of interest. In how many years will it become 16 times of itself at the same rate of simple interest?

- (1) 80
- (2) 70
- (3) 75
- (4) 90

Solution

Doubling means the interest equals 100% of the principal in 5 years, i.e. 20% per year. To become 16 times, the interest must be 1500% of the principal: $1500/20 = 75$ years.

Answer: (3) 75 years

Q19. Sum of Cubes

[Part A]

General Aptitude | Algebra

Consider the relationship statement between two natural numbers :

- A. the difference between them is 6.
- B. the difference between their squares is 60.

Find the sum of the cubes of these two numbers.

- (1) 620
- (2) 520
- (3) 720
- (4) 820

Solution

$a - b = 6$ and $a^2 - b^2 = (a - b)(a + b) = 60 \Rightarrow a + b = 10$. So $a = 8$, $b = 2$, and $a^3 + b^3 = 512 + 8 = 520$.

Answer: (2) 520

Q20. Success Rates in a Two-Stage Examination

[Part A]

General Aptitude | Percentages

Candidate in a competitive examination consisted of 60% men and 40% woman. 70% of men and 75% of women cleared the qualifying test and entered the final test where 80% men and 70% women were successful. Which of the following statement is correct?

- (1) Overall success rate is below 50%
- (2) Success rate is higher for women
- (3) More men cleared the examination than women
- (4) Both 1 and 2

Solution

Take 100 candidates: 60 men, 40 women.

- Men: $60 \times 0.70 = 42$ qualify, $42 \times 0.80 = 33.6$ succeed (success rate 56%).

- Women: $40 \times 0.75 = 30$ qualify, $30 \times 0.70 = 21$ succeed (success rate 52.5%).

Overall success = 54.6% > 50%, so 1 is false. Men have the higher rate, so 2 is false and hence 4 is false. More men (33.6) than women (21) cleared the examination, so 3 is correct.

Answer: (3) More men cleared than women

Part B

Physical Sciences (Q21–Q45)

Q21. Free Expansion of a van der Waals-type Gas

[Part B]

Thermodynamics | *Joule expansion*

Internal energy of one mole of a non-ideal gas is given by $U = \frac{3}{2}RT - \frac{a}{V}$, where V is volume of the gas at temperature T and a is a positive constant. An insulated container of volume V_2 contains this gas confined in a volume V_1 at temperature T_1 . The remaining volume $V_2 - V_1$ of this container, which is in vacuum, is separated by a wall. The wall is suddenly removed which makes the gas volume V_2 and temperature T_2 . The temperature T_2 is:

(R is the molar gas constant)

- (1) $T_1 + \frac{2a}{3R} \left(\frac{1}{V_2} + \frac{1}{V_1} \right)$
- (2) $T_1 - \frac{2a}{3R} \left(\frac{1}{V_2} + \frac{1}{V_1} \right)$
- (3) $T_1 + \frac{2a}{3R} \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$
- (4) $T_1 - \frac{2a}{3R} \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$

Solution

Free expansion into vacuum in an insulated container: $Q = 0$ and $W = 0$, so U is conserved.

$$\frac{3}{2}RT_1 - \frac{a}{V_1} = \frac{3}{2}RT_2 - \frac{a}{V_2} \Rightarrow T_2 = T_1 + \frac{2a}{3R} \left(\frac{1}{V_2} - \frac{1}{V_1} \right).$$

Since $V_2 > V_1$ the bracket is negative, so the gas cools (work is done against the attractive forces).

Answer: (3)

Q22. Bead on a Rotating Parabolic Wire

[Part B]

Classical Mechanics | Constraints and conservation laws

A bead of mass M slides along a smooth frictionless and massless wire which is bent in the shape of a parabola $z = cr^2$ in cylindrical coordinates (r, ϕ, z) . Here c is a positive constant. The wire is made to rotate about z-axis at constant angular velocity. Which of the following statements about this system of the wire and the bead is correct?

- (1) Energy is conserved but angular momentum is not conserved
- (2) Energy is not conserved but angular momentum is conserved
- (3) Both energy and angular momentum are conserved
- (4) Both energy and angular momentum are not conserved

Solution

$L_z = Mr^2\omega$ changes whenever the bead moves along the wire ($\dot{r} \neq 0$) while ω is held fixed. An external torque from the driving agent is needed, so angular momentum is not conserved. That torque does work ($P = \tau\omega$), so the energy $E = \frac{1}{2}M(\dot{r}^2 + r^2\omega^2 + \dot{z}^2) + Mgz$ is not conserved either. What is conserved is the Jacobi integral,

$$h = \frac{1}{2}M(\dot{r}^2 + \dot{z}^2) - \frac{1}{2}Mr^2\omega^2 + Mgz,$$

which is not the energy, because the constraint is time-dependent in inertial coordinates.

Answer: (4) neither conserved

Q23. Force Law for a Logarithmic Spiral

[Part B]

Classical Mechanics | Central forces, Binet equation

A particle moves in an orbit $r = k \exp(\alpha\theta)$ under a central force $F(r)$. Here, k and α are constants and $k > 0$. Then $F(r)$ is proportional to:

- (1) $\frac{1}{r}$
- (2) $\frac{1}{r^2}$
- (3) $\frac{1}{r^3}$
- (4) $\frac{1}{r^4}$

Solution

With $u = 1/r = k^{-1}e^{-\alpha\theta}$, we have $\frac{d^2u}{d\theta^2} = \alpha^2u$. The Binet equation gives

$$F = -\frac{\ell^2}{m}u^2 \left(\frac{d^2u}{d\theta^2} + u \right) = -\frac{\ell^2}{m}(1 + \alpha^2)u^3 \propto \frac{1}{r^3}.$$

Answer: (3) $F \propto r^{-3}$

Q24. Hamiltonian from a Lagrangian

[Part B]

Classical Mechanics | Legendre transformation

The Lagrangian of a system is given by $L = x^2 + x\dot{x} + \frac{\dot{x}^2}{2}$. If p is the generalized momentum, then the corresponding Hamiltonian is:

- (1) $\frac{(p^2 - x^2 - 2xp)}{2}$
- (2) $\frac{(p^2 - x^2 + 2xp)}{2}$
- (3) $\frac{(p^2 + x^2 + 2xp)}{2}$
- (4) $\frac{(p^2 + x^2 - 2xp)}{2}$

Solution

$$p = \frac{\partial L}{\partial \dot{x}} = x + \dot{x} \Rightarrow \dot{x} = p - x.$$

$$H = p\dot{x} - L = p(p - x) - x^2 - x(p - x) - \frac{1}{2}(p - x)^2 = \frac{1}{2}(p^2 - 2xp - x^2).$$

Answer: (1) $\frac{1}{2}(p^2 - x^2 - 2xp)$

Q25. Completing a Basis

[Part B]

Mathematical Physics | Linear algebra

In a 4-dimensional vector space V_4 , three linearly independent vectors are given by $\mathbf{e}_1 = (1, 1, 0, 0)$, $\mathbf{e}_2 = (0, 1, 1, 0)$ and $\mathbf{e}_3 = (0, 0, 1, 1)$. Which of the following vector $\mathbf{e}_4$ will make $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$ a basis of V_4 ?

- (1) $(1, 1, 1, 1)$
- (2) $(1, 1, 1, 0)$
- (3) $(1, 0, -1, 0)$
- (4) $(0, 1, 1, 0)$

Solution

A general vector in the span is $a\mathbf{e}_1 + b\mathbf{e}_2 + c\mathbf{e}_3 = (a, a + b, b + c, c)$; its components satisfy $v_1 - v_2 + v_3 - v_4 = 0$.

- $(1, 1, 1, 1)$: $1 - 1 + 1 - 1 = 0$, in the span (it is $\mathbf{e}_1 + \mathbf{e}_3$).
- $(1, 1, 1, 0)$: $1 - 1 + 1 - 0 = 1 \neq 0$, *not* in the span. ✓
- $(1, 0, -1, 0)$: $1 - 0 - 1 - 0 = 0$, in the span ($\mathbf{e}_1 - \mathbf{e}_2$).
- $(0, 1, 1, 0) = \mathbf{e}_2$.

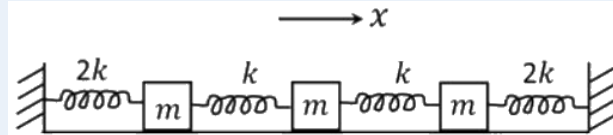
Answer: (2) $(1, 1, 1, 0)$

Q26. Sum of Squared Normal-Mode Frequencies

[Part B]

Classical Mechanics | Coupled oscillations

Three equal masses m are free to move along x -axis on a frictionless surface. The masses are coupled together, and to a rigid wall, by springs as shown in the figure. If $\omega_0 = \sqrt{\frac{k}{m}}$, and the angular frequencies of the three normal modes of this system are ω_1, ω_2 and ω_3 then the value of $\omega_1^2 + \omega_2^2 + \omega_3^2$ is:



- (1) $9\omega_0^2$
- (2) $6\omega_0^2$
- (3) $10\omega_0^2$
- (4) $8\omega_0^2$

Solution

Equations of motion $m\ddot{\mathbf{x}} = -K\mathbf{x}$ with

$$\frac{K}{m} = \omega_0^2 \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

(the diagonal entry is the sum of the springs attached to each mass). The squared normal-mode frequencies are the eigenvalues, so their sum is the trace: $(3 + 2 + 3)\omega_0^2 = 8\omega_0^2$.

(Explicitly: the antisymmetric mode $(1, 0, -1)$ gives $\omega^2 = 3\omega_0^2$, and the symmetric modes $(1, b, 1)$ satisfy $(3 - \lambda)(2 - \lambda) = 2$, so $\omega^2 = \omega_0^2, 4\omega_0^2$. The sum is $1 + 3 + 4 = 8$. ✓)

Answer: (4) $8\omega_0^2$

Q27. Spin-1 Microcanonical Ensemble

[Part B]

Statistical Mechanics | Counting microstates

A micro-canonical ensemble consists of six non-interacting and distinguishable spin 1 particles in a uniform magnetic field. Each particle can thus be in energy states $-E_0, 0$ and E_0 . The number of microstates in the state with total energy zero is:

- (1) 51
- (2) 191
- (3) 141
- (4) 55

Solution

Zero total energy needs equal numbers k of particles in $+E_0$ and in $-E_0$, with $6 - 2k$ in 0:

$$\Omega = \sum_{k=0}^3 \frac{6!}{k! k! (6 - 2k)!} = 1 + 30 + 90 + 20 = 141.$$

Answer: (3) 141**Q28. Point Charge above a Grounded Plane**

[Part B]

Electrostatics | Method of images

Consider the Poisson's equation $\nabla^2 V = -\frac{q}{\epsilon_0} \delta^3(\vec{r} - \vec{a})$ in half-space $z > 0$ with boundary condition $V = 0$ in the $z = 0$ plane. Here, $\vec{a} = a_0 \hat{k}$ with $a_0 > 0$. If $\frac{q}{4\pi\epsilon_0 a_0} = 24$ Volt, then which of the following is correct?

- (1) At $\vec{r} = (0, 0, 2a_0)$, $V = 16$ Volt
- (2) At $\vec{r} = (0, 0, 3a_0)$, $V = 8$ Volt
- (3) At $\vec{r} = (0, 0, 4a_0)$, $V = 4$ Volt
- (4) At $\vec{r} = (0, 0, a_0/2)$, $V = 32$ Volt

Solution

The image charge $-q$ at $-a_0 \hat{k}$ enforces $V = 0$ on $z = 0$. On the z -axis:

$$V(z) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{|z - a_0|} - \frac{1}{z + a_0} \right] = 24 \left[\frac{a_0}{|z - a_0|} - \frac{a_0}{z + a_0} \right] \text{ V.}$$

z	V (V)	Option value	
$2a_0$	$24(1 - \frac{1}{3}) = 16$	16	✓
$3a_0$	$24(\frac{1}{2} - \frac{1}{4}) = 6$	8	×
$4a_0$	$24(\frac{1}{3} - \frac{1}{5}) = 3.2$	4	×
$a_0/2$	$24(2 - \frac{2}{3}) = 32$	32	✓

Both options 1 and 4 are correct; the official key accepts either.

Answer: (1 & 4) $V(2a_0) = 16$ V, $V(a_0/2) = 32$ V**Q29. Pressure Melting of Ice**

[Part B]

Thermodynamics | Clausius–Clapeyron equation

Water, having density 1 gm/cm^3 , freezes into ice at 273 K temperature and 1 atm pressure. The latent heat of melting of ice is 334 J/gm and the density of ice is 0.917 gm/cm^3 . Then for the melting temperature (T_m) which of the following options is correct?

- (1) At 140 atm pressure, $T_m \simeq 274 \text{ K}$.
- (2) At 420 atm pressure, $T_m \simeq 274 \text{ K}$.
- (3) At 140 atm pressure, $T_m \simeq 272 \text{ K}$.
- (4) At 420 atm pressure, $T_m \simeq 272 \text{ K}$.

Solution

Clausius–Clapeyron: $\frac{dT_m}{dP} = \frac{T \Delta v}{L}$ with $\Delta v = v_{\text{water}} - v_{\text{ice}} = 1 - \frac{1}{0.917} = -0.0905 \text{ cm}^3/\text{g} = -9.05 \times 10^{-8} \text{ m}^3/\text{g}$.

$$\frac{dT_m}{dP} = \frac{273 \times (-9.05 \times 10^{-8})}{334} = -7.40 \times 10^{-8} \text{ K/Pa} = -7.5 \times 10^{-3} \text{ K/atm.}$$

Ice contracts on melting, so pressure *lowers* T_m . A drop of 1 K needs $\Delta P \approx 1/(7.5 \times 10^{-3}) \approx 133$ atm. So at ~ 140 atm, $T_m \simeq 272$ K. (At 420 atm the drop would be ≈ 3 K.)

Answer: (3) 140 atm, $T_m \simeq 272$ K

Q30. Time Dependence of Expectation Values

[Part B]

Quantum Mechanics | Heisenberg equation of motion

You are given a time-independent observable O for a system with time independent Hamiltonian H and a general time dependent state $|\phi(t)\rangle$. If the expectation value $\langle O(t) \rangle_\phi = \langle \phi(t) | O | \phi(t) \rangle$, then consider the following statements (A) to (D) and choose the correct option.

- (A) If O commutes with H then $\langle O(t) \rangle_\phi$ is independent of time.
- (B) If $|\phi(t)\rangle$ is an eigen state of H then $\langle O(t) \rangle_\phi$ is independent of time.
- (C) If $|\phi(t)\rangle$ is an eigen state of O then $\langle O(t) \rangle_\phi$ is independent of time.
- (D) If $|\phi(t)\rangle$ is an eigen state of $[O, H]$ then $\langle O(t) \rangle_\phi$ is independent of time.

- (1) Only (A) and (B) are correct
- (2) Only (A) is correct
- (3) Only (B), (C) and (D) are correct
- (4) Only (A), (B) and (C) are correct

Solution

Ehrenfest/Heisenberg: $\frac{d}{dt}\langle O \rangle = \frac{i}{\hbar}\langle [H, O] \rangle$ (since $\partial O / \partial t = 0$).

- (A) $[O, H] = 0 \Rightarrow d\langle O \rangle / dt = 0$. ✓
- (B) A stationary state has $|\phi(t)\rangle = e^{-iEt/\hbar}|E\rangle$. The phase cancels in every expectation value. ✓
- (C) If $|\phi(t)\rangle$ remains an eigenstate of O , then $\langle O \rangle$ equals the eigenvalue. A continuously evolving state cannot jump between discrete eigenvalues, so $\langle O \rangle$ is constant. ✓
- (D) If $[O, H]|\phi\rangle = \lambda|\phi\rangle$, then $\frac{d\langle O \rangle}{dt} = -\frac{i}{\hbar}\lambda$, which is non-zero unless $\lambda = 0$. Not guaranteed. ✗

Answer: (4) (A), (B), (C)

Q31. Survival Probability in a Three-Level System

[Part B]

Quantum Mechanics | Time evolution

A three-state system is described by a certain Hamiltonian having eigen energies E_0 , $2E_0$ and $3E_0$ with the corresponding eigen states $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, respectively. If the state of the system at time $t = 0$ is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, then what is the probability of finding the system in the same state at time $t = \frac{\pi\hbar}{2E_0}$?

- (1) $\frac{1}{3}$
- (2) $\frac{1}{2}$
- (3) $\frac{5}{18}$
- (4) $\frac{2}{9}$

Solution

The normalised eigenvectors are $\frac{1}{\sqrt{3}}(1, 1, 1)$, $\frac{1}{\sqrt{2}}(1, 0, -1)$, $\frac{1}{\sqrt{6}}(1, -2, 1)$. The weights of $(1, 0, 0)$ on them are $|c_n|^2 = \frac{1}{3}, \frac{1}{2}, \frac{1}{6}$.

Survival amplitude $A(t) = \sum_n |c_n|^2 e^{-iE_n t/\hbar}$, with $E_0 t/\hbar = \pi/2$:

$$A = \frac{1}{3}e^{-i\pi/2} + \frac{1}{2}e^{-i\pi} + \frac{1}{6}e^{-i3\pi/2} = -\frac{i}{3} - \frac{1}{2} + \frac{i}{6} = -\frac{1}{2} - \frac{i}{6}.$$

$$P = |A|^2 = \frac{1}{4} + \frac{1}{36} = \frac{10}{36} = \frac{5}{18}.$$

Answer: (3) $5/18$

Q32. Ground Term of Vanadium

[Part B]

Atomic Physics | Hund's rules

Vanadium has electron configuration $[Ar]3d^3 4s^2$. In LS coupling, its ground state configuration is

- (1) ${}^4F_{\frac{9}{2}}$
- (2) ${}^2P_{\frac{1}{2}}$
- (3) ${}^4F_{\frac{3}{2}}$
- (4) ${}^4S_{\frac{3}{2}}$

Solution

$4s^2$ is closed, so only $3d^3$ matters ($l = 2$).

- Hund 1: maximise S . Three parallel spins give $S = \frac{3}{2}$, multiplicity $2S + 1 = 4$.
- Hund 2: maximise L . Fill $m_l = 2, 1, 0$, so $L = 3$ (F).
- Hund 3: the shell is less than half-filled (3 of 10), so $J = |L - S| = \frac{3}{2}$.

Ground term: ${}^4F_{3/2}$.

Answer: $(3) {}^4F_{3/2}$

Q33. Derivative of a Spherical Unit Vector

[Part B]

Mathematical Physics | *Curvilinear coordinates*

In spherical polar coordinates (r, θ, ϕ) , $\frac{\partial \hat{\theta}}{\partial \phi}$ is :

- (1) $\cos \theta \hat{\phi}$
- (2) 0
- (3) $\sin \theta \hat{\phi}$
- (4) $-\cos \theta \hat{r}$

Solution

$\hat{\theta} = (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta)$, so

$$\frac{\partial \hat{\theta}}{\partial \phi} = \cos \theta (-\sin \phi, \cos \phi, 0) = \cos \theta \hat{\phi}.$$

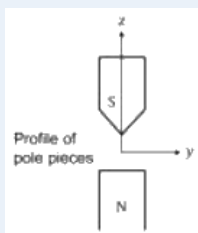
Answer: $(1) \cos \theta \hat{\phi}$

Q34. Stern–Gerlach Force on Copper Atoms

[Part B]

Atomic Physics | *Magnetic moments*

A beam of copper atoms (Cu: $[\text{Ar}]4s^13d^{10}$) is passed through a Stern–Gerlach setup (see figure). If the gradient of the z -component of the magnetic field in the z -direction is 10^3 Tesla/m, then the magnitude of the z -component of the force on an atom is: (μ_B is the Bohr magneton)



- (1) $5 \times 10^2 \mu_B$
- (2) $10^3 \mu_B$
- (3) 0
- (4) $2\pi \times 10^3 \mu_B$

Solution

The closed $3d^{10}$ shell contributes nothing; the single $4s$ electron gives $L = 0$, $S = J = \frac{1}{2}$, $g_J = 2$. Then $\mu_z = -g_J \mu_B m_J = \mp \mu_B$, and

$$|F_z| = |\mu_z| \frac{\partial B_z}{\partial z} = \mu_B \times 10^3 \text{ T/m} = 10^3 \mu_B \text{ (N, with } \mu_B \text{ in J/T)}.$$

The beam splits into two, exactly as for silver in the original experiment.

Answer: $(2) \ 10^3 \mu_B$

Q35. Commutator with an Exponential

[Part B]

Quantum Mechanics | *Operator algebra*

Consider two operators $\hat{R}$ and $\hat{S}$ where $\hat{R}$ commutes with $[\hat{R}, \hat{S}]$. If α is a constant, then $[e^{\alpha\hat{R}}, \hat{S}]$ is:

- (1) $[\hat{R}^\alpha, \hat{S}]$
- (2) $\alpha e^{\alpha\hat{R}} [\hat{R}, \hat{S}]$
- (3) 0
- (4) $\alpha e^{\hat{R}} [e^\alpha \hat{R}, \hat{S}]$

Solution

If $[\hat{R}, [\hat{R}, \hat{S}]] = 0$, induction gives $[\hat{R}^n, \hat{S}] = n\hat{R}^{n-1}[\hat{R}, \hat{S}]$. Summing the exponential series,

$$[e^{\alpha\hat{R}}, \hat{S}] = \sum_n \frac{\alpha^n}{n!} n\hat{R}^{n-1}[\hat{R}, \hat{S}] = \alpha e^{\alpha\hat{R}} [\hat{R}, \hat{S}].$$

(The same rule gives $[\hat{x}, f(\hat{p})] = i\hbar f'(\hat{p})$.)

Answer: $(2) \ \alpha e^{\alpha\hat{R}} [\hat{R}, \hat{S}]$

Q36. Reflection from a Chiral Medium

[Part B]

Electromagnetic Theory | *Optical activity, reflection*

A linearly polarized electromagnetic wave is incident normally on a material of refractive index n_R for right circular polarization and n_L for left circular polarization, where $n_R \neq n_L$. The reflected wave's polarization is :

- (1) Linear
- (2) Elliptical
- (3) Circular
- (4) Random

Solution

Decompose the incident linear wave into equal-amplitude right and left circular components. At normal incidence each circular component reflects with its own Fresnel coefficient:

$$r_R = \frac{1 - n_R}{1 + n_R}, \quad r_L = \frac{1 - n_L}{1 + n_L}, \quad n_R \neq n_L \Rightarrow |r_R| \neq |r_L|.$$

The reflected wave is a superposition of right and left circular waves of *unequal* amplitudes. That is neither linear (equal amplitudes) nor circular (one amplitude zero), but elliptically polarised.

Answer: $(2) \ \text{Elliptical}$

Q37. Counting Two-Particle Microstates

[Part B]

Statistical Mechanics | Quantum statistics

Two particles occupy ten degenerate energy levels. There are three possible scenarios where the two particles are either identical bosons or identical fermions or distinguishable particles, with the associated number of accessible microstates denoted as Ω_B , Ω_F and Ω_C , respectively. Then, which of the following options is correct?

- (1) $\Omega_B = 55$, $\Omega_F = 45$, $\Omega_C = 100$
- (2) $\Omega_B = 65$, $\Omega_F = 55$, $\Omega_C = 100$
- (3) $\Omega_B = 45$, $\Omega_F = 25$, $\Omega_C = 55$
- (4) $\Omega_B = 55$, $\Omega_F = 45$, $\Omega_C = 75$

Solution

With $g = 10$ levels and $N = 2$ particles:

$$\Omega_C = g^2 = 100, \quad \Omega_B = \binom{g+1}{2} = 55, \quad \Omega_F = \binom{g}{2} = 45.$$

(Check: $\Omega_B + \Omega_F = \Omega_C$, as expected for two particles.)

Answer: (1) 55, 45, 100

Q38. Sudden vs. Adiabatic Expansion of a Well

[Part B]

Quantum Mechanics | Sudden and adiabatic approximations

A particle is in the ground state of a one-dimensional infinite well potential with walls at $x = 0$ and at $x = a$. The wall at $x = a$ is moved to $x = 2a$ in two separate, sudden and adiabatic, protocols so that $\langle E \rangle_s$ and $\langle E \rangle_a$ are the respective expectation values of the energy in the final states. Then the ratio $\frac{\langle E \rangle_s}{\langle E \rangle_a}$ is :

- (1) 4:1
- (2) 2:1
- (3) 1:1
- (4) 1:4

Solution

Sudden: the wavefunction is unchanged, $\psi = \sqrt{2/a} \sin(\pi x/a)$ on $[0, a]$ and 0 elsewhere. The new Hamiltonian is still pure kinetic energy there, so $\langle E \rangle_s = \frac{\pi^2 \hbar^2}{2ma^2}$ (no energy is transferred by a sudden change).

Adiabatic: the particle follows the instantaneous ground state, ending in the ground state of the width- $2a$ well: $\langle E \rangle_a = \frac{\pi^2 \hbar^2}{2m(2a)^2} = \frac{\pi^2 \hbar^2}{8ma^2}$.

Ratio = 4 : 1.

Answer: (1) 4 : 1

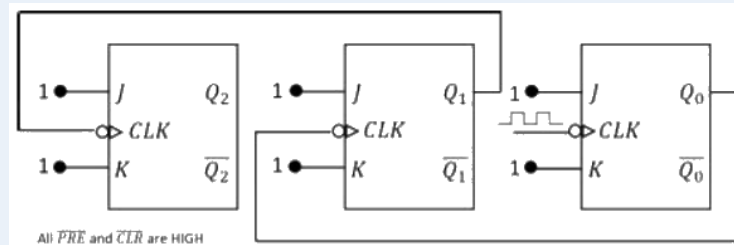
Q39. Ripple Counter of J-K Flip-Flops

[Part B]

Electronics | Sequential logic

Three J-K flip-flops are connected together to make a counter as shown in the figure. The J and K inputs are tied to logical 1 and a clock pulse is applied only to CLK input of the flip flop Q_0 . If the counter is initially in the state $Q_2Q_1Q_0 = 100$, then the state of the counter after 9 clock cycles is:

Figure:



- (1) $Q_2Q_1Q_0 = 001$
- (2) $Q_2Q_1Q_0 = 110$
- (3) $Q_2Q_1Q_0 = 101$
- (4) $Q_2Q_1Q_0 = 111$

Solution

With $J = K = 1$, each flip-flop toggles on the falling edge of its own clock (bubbled CLK). The external clock drives FF₀; the output Q_0 clocks FF₁, and Q_1 clocks FF₂. Each stage toggles when the previous stage goes $1 \rightarrow 0$: this is a 3-bit asynchronous (ripple) *up*-counter, modulo 8. Initial count = $100_2 = 4$. After 9 pulses: $(4 + 9) \bmod 8 = 13 \bmod 8 = 5 = 101_2$.

Answer: (3) $Q_2Q_1Q_0 = 101$

Q40. ADC Threshold Logic

[Part B]

Electronics | Digital logic

An analog-to-digital converter monitors the dc voltage output from a battery. The output of the converter is a three-digit binary number ABC, corresponding to battery voltage in steps of 0.5 V, with A as the most significant bit (MSB). The output of the converter is fed into a logic circuit which produces a HIGH output when the battery voltage exceeds 2 V. The expression for this logic circuit is:

- (1) $AB + AC$
- (2) $AB + BC + AC$
- (3) ABC
- (4) AC

Solution

Voltage = $0.5(4A + 2B + C)$ V. It exceeds 2 V when $4A + 2B + C > 4$, i.e. for codes 101, 110, 111 (5, 6, 7).

$$Y = A\bar{B}C + AB\bar{C} + ABC = A(B + C) = AB + AC.$$

(Code 100 = 2.0 V does not exceed 2 V.)

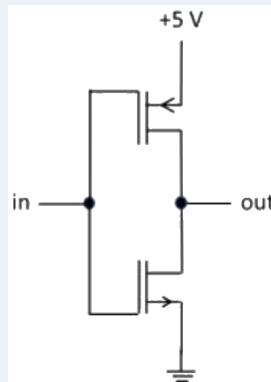
Answer: (1) $AB + AC$

Q41. Identifying a Logic Circuit

[Part B]

Electronics | CMOS logic

The circuit shown below could be used as a:



- (1) TTL NOT Gate
- (2) TTL Buffer
- (3) CMOS NOT Gate
- (4) CMOS Buffer

Solution

The circuit is a complementary MOSFET pair: a p-channel MOSFET from +5 V and an n-channel MOSFET to ground, with gates tied together (input) and drains joined (output). It uses field-effect transistors, not bipolar ones, so it is not TTL.

- in = HIGH: the nMOS is ON and the pMOS OFF, so out = 0 V.
- in = LOW: the pMOS is ON and the nMOS OFF, so out = +5 V.

The output is the complement of the input: a CMOS inverter. (A CMOS buffer needs two such stages.)

Answer: (3) CMOS NOT gate

Q42. Spin Measurement along $\hat{r}$

[Part B]

Quantum Mechanics | $Spin-\frac{1}{2}$

Consider the spin matrix S_r projected along the radial direction in spherical polar coordinates (r, θ, φ) . The probability that a measurement of S_r on the state $|\beta\rangle = \begin{pmatrix} \sin \frac{\theta}{2} \\ e^{i\varphi} \cos \frac{\theta}{2} \end{pmatrix}$ will yield the value $\frac{\hbar}{2}$ is:

- (1) $\sin^2 \theta$
- (2) $\cos^2 \theta$
- (3) $\frac{1}{2}$
- (4) $\cos^2 \frac{\theta}{2}$

Solution

The $+\frac{\hbar}{2}$ eigenstate of $S_r = \frac{\hbar}{2} \hat{r} \cdot \vec{\sigma}$ is $|+\hat{r}\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\varphi} \sin \frac{\theta}{2} \end{pmatrix}$. Then

$$\langle +\hat{r}|\beta\rangle = \cos \frac{\theta}{2} \sin \frac{\theta}{2} + e^{-i\varphi} e^{i\varphi} \sin \frac{\theta}{2} \cos \frac{\theta}{2} = \sin \theta, \quad P = \sin^2 \theta.$$

Answer: (1) $\sin^2 \theta$

Q43. A Trigonometric Integral by Residues

[Part B]

Mathematical Physics | *Contour integration*

The value of the integral:

$$\oint \frac{d\theta}{5 - 4 \cos \theta}$$

over the circle $|z| = 1$ in the anti-clockwise direction can be calculated by substituting $z = e^{i\theta}$. The value of the integral is:

- (1) $\frac{2\pi}{3}$
- (2) $\frac{\pi}{3}$
- (3) π
- (4) $\frac{4\pi}{3}$

Solution

With $z = e^{i\theta}$, $d\theta = \frac{dz}{iz}$, $\cos \theta = \frac{z + z^{-1}}{2}$:

$$I = \oint \frac{dz}{iz(5 - 2z - 2/z)} = \oint \frac{i dz}{2z^2 - 5z + 2} = \oint \frac{i dz}{(2z - 1)(z - 2)}.$$

Only $z = \frac{1}{2}$ is inside $|z| = 1$. Its residue is $\frac{i}{2(\frac{1}{2} - 2)} = -\frac{i}{3}$, so $I = 2\pi i \left(-\frac{i}{3}\right) = \frac{2\pi}{3}$. (Check:

$$\int_0^{2\pi} \frac{d\theta}{a - b \cos \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}} = \frac{2\pi}{3}.)$$

Answer: (1) $2\pi/3$

Q44. Circulation of the Canonical Momentum

[Part B]

Electrodynamics | Charged particle in a magnetic field

A particle with charge q , which is confined to move in the xy -plane, is subjected to a magnetic field with magnitude B pointing in the $+z$ direction. The particle moves in a circle (C) of radius R . If $\vec{p}$ is the canonical momentum then the following integral

$$\oint_C \vec{p} \cdot d\vec{l}$$

has the magnitude:

($d\vec{l}$ is in the direction of motion)

- (1) $qB\pi R^2$
- (2) $2qB\pi R^2$
- (3) $3qB\pi R^2$
- (4) $4qB\pi R^2$

Solution

$$\vec{p} = m\vec{v} + q\vec{A}.$$

- Kinetic part: $|m\vec{v}| = qBR$ (cyclotron condition), tangent to the path, so $\oint m\vec{v} \cdot d\vec{l} = qBR \cdot 2\pi R = 2\pi qBR^2$.
- Field part: $\oint q\vec{A} \cdot d\vec{l} = q\Phi$. A positive charge in $B\hat{z}$ circulates *clockwise* seen from $+z$, so the flux through the path traversed in the direction of motion is $-B\pi R^2$. This gives $-\pi qBR^2$.

Total: $2\pi qBR^2 - \pi qBR^2 = \pi qBR^2$. (The two terms always have opposite signs, for either sign of q .)

Answer: (1) $qB\pi R^2$

Q45. Energy Expectation in a Box

[Part B]

Quantum Mechanics | Infinite square well

The eigen energies and the normalized eigenfunctions in the n^{th} state of a particle, confined in a one-dimensional infinite potential well, are denoted by E_n and ψ_n , respectively. If $E_1 = 1.2$ eV then the expectation value of energy in the state $\psi = 3\psi_1 + 2\psi_2 + \sqrt{3}\psi_4$ is close to:

- (1) 5.5 eV
- (2) 5.0 eV
- (3) 4.5 eV
- (4) 6.0 eV

Solution

The state is not normalised: $\langle\psi|\psi\rangle = 9 + 4 + 3 = 16$. With $E_n = n^2 E_1$:

$$\langle E \rangle = \frac{9(1) + 4(4) + 3(16)}{16} E_1 = \frac{73}{16} (1.2 \text{ eV}) = 5.48 \text{ eV} \approx 5.5 \text{ eV}.$$

Answer: (1) 5.5 eV

Part C

Physical Sciences (Q46–Q75)

Q46. Standard Error of the Mean

[Part C]

Mathematical Physics | Probability and statistics

In an experiment, a student generates 100 random numbers drawn from a uniform distribution in the interval $(0, 1)$ and calculates their mean. If the experiment is repeated 100 times, then the standard deviation of the means is:

- (1) $\frac{1}{20\sqrt{3}}$
- (2) $\frac{1}{2\sqrt{3}}$
- (3) $\frac{1}{\sqrt{3}}$
- (4) $\frac{1}{200\sqrt{3}}$

Solution

For $U(0, 1)$: $\sigma = \frac{1}{\sqrt{12}} = \frac{1}{2\sqrt{3}}$. The mean of $N = 100$ samples has $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \frac{1}{2\sqrt{3} \cdot 10} = \frac{1}{20\sqrt{3}}$.
(The number of repetitions only sets how well this spread is sampled.)

Answer: (1) $\frac{1}{20\sqrt{3}}$

Q47. Counting Tensor Components

[Part C]

Mathematical Physics | Tensors

In four dimensions, a rank four tensor is given by T_{abcd} . It is symmetric under interchange of the indices a, b and interchange of the indices c, d . Moreover, the tensor is anti-symmetric under the interchange of pair of indices ab and cd . The number of independent components of the tensor is:

- (1) 100
- (2) 45
- (3) 36
- (4) 21

Solution

A symmetric pair (ab) in 4 dimensions takes $\frac{4 \cdot 5}{2} = 10$ values; call it a composite index A . Then T_{AB} is a 10×10 matrix, antisymmetric under $A \leftrightarrow B$: $\frac{10 \cdot 9}{2} = 45$ independent components.

Answer: (2) 45

Q48. A Time-Dependent Constant of Motion

[Part C]

Classical Mechanics | Poisson brackets

For a one-dimensional system the Hamiltonian is given by $H = \frac{p^2}{2} - \frac{1}{2q^2}$. If a constant of motion of the system is $\left(apq + bp^2t + \frac{ct}{q^2}\right)$, where a , b and c are constants and t denotes the time, then which of the following options is correct:

- (1) $a = b = c$
- (2) $a = -b = c$
- (3) $a = -b = -c$
- (4) $a = b = -c$

Solution

Hamilton's equations: $\dot{q} = p$, $\dot{p} = -\partial H/\partial q = -q^{-3}$. For $F = apq + bp^2t + ctq^{-2}$:

$$\begin{aligned}\frac{dF}{dt} &= a(\dot{p}q + p\dot{q}) + b(2p\dot{p}t + p^2) + c(q^{-2} - 2tq^{-3}\dot{q}) \\ &= (a+b)p^2 + (c-a)q^{-2} - 2(b+c)tpq^{-3}.\end{aligned}$$

For this to vanish identically: $b = -a$, $c = a$, $c = -b$, i.e. $a = -b = c$. (This is the dilatation-type invariant of the $1/q^2$ potential, $pq - 2Ht$, up to normalisation.)

Answer: (2) $a = -b = c$

Q49. Capacitor Field Seen by a Moving Observer

[Part C]

Electrodynamics | Transformation of fields

An inertial observer S finds the electric field due to a large rectangular parallel plate capacitor to be directed along the y -axis. For another observer, moving at a relativistic speed with respect to S along the positive x -axis, which of the following statements is correct for the electric field:

- (1) Direction remains same but magnitude increases
- (2) Direction remains same but magnitude decreases
- (3) Direction and magnitude both remain the same
- (4) Direction tilts towards x -axis but the magnitude remains same

Solution

The field E_y is perpendicular to the boost. With $\vec{B} = 0$ in S :

$$E'_y = \gamma(E_y - vB_z) = \gamma E_y, \quad E'_x = E_x = 0.$$

Physically, the plates (which lie parallel to the xz -plane) are Lorentz-contracted along x , so the surface charge density, and hence E , rises by γ . The direction is unchanged (still $\hat{y}$) and the magnitude increases.

Answer: (1) same direction, larger magnitude

Q50. Phase Portrait of a Cubic Potential

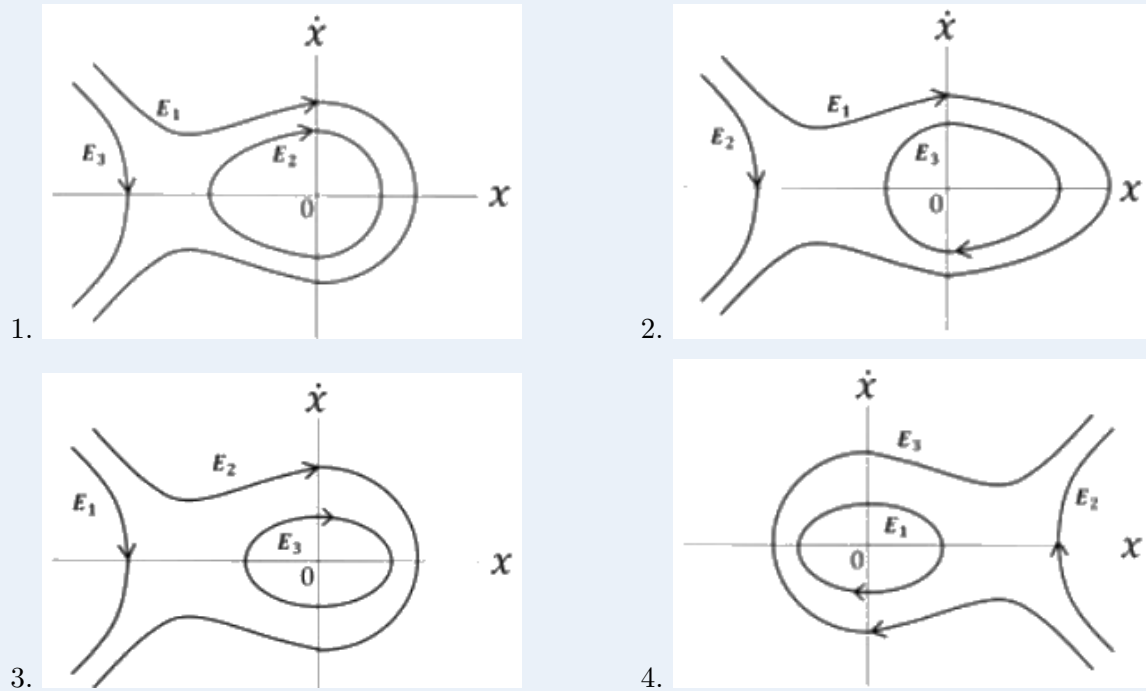
[Part C]

Classical Mechanics | Phase space

A particle is moving in one dimension in the potential

$$V(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3.$$

The schematic phase-space diagrams for the total energies $E_1 > E_2 > E_3$ are best represented by:



Solution

$V'(x) = x(1+x)$. There is a minimum at $x = 0$ ($V = 0$) and a maximum at $x = -1$ ($V = \frac{1}{6}$). $V \rightarrow -\infty$ as $x \rightarrow -\infty$ and $V \rightarrow +\infty$ as $x \rightarrow +\infty$. So the well is on the right and the barrier on the left, which excludes option 4 (the mirror image).

- $E > \frac{1}{6}$: unbounded motion. The orbit comes in from $x = -\infty$, turns on the right wall, and goes back: an open curve enclosing the well. This is the highest energy, E_1 .
- $E < \frac{1}{6}$: either a closed orbit (clockwise) around $x = 0$, or an open branch to the left of the barrier. A lower energy gives a smaller closed orbit and a left branch farther from the barrier.

Option 1 has E_1 unbounded, E_2 a large closed orbit near the separatrix, and E_3 (lowest) a left branch far from the barrier. This is consistent, with the correct clockwise flow ($\dot{x} > 0$ on the upper half). Option 3 would need the unbounded E_2 to exceed the bounded E_1 , which contradicts $E_1 > E_2$.

Answer: (1)

Q51. Ground State of $\alpha L^2 + \beta \hbar L_z$

[Part C]

Quantum Mechanics | Angular momentum

The Hamiltonian of a quantum mechanical system is given by $\hat{H} = \alpha \hat{L}^2 + \beta \hbar \hat{L}_z$. Here, α and β are positive constants and $\hat{L}$ represents the orbital angular momentum operator. If l and m are the azimuthal and magnetic quantum numbers then the lowest energy state for this system is:

- (1) $l = 1, m = -1$ for $\alpha = \beta$
- (2) $l = 0, m = 0$ for $\alpha < \frac{\beta}{2}$
- (3) $l = 1, m = -1$ for $\alpha < \frac{\beta}{2}$
- (4) $l = 1, m = +1$ for $\alpha < \frac{\beta}{2}$

Solution

$E_{lm} = \hbar^2 [\alpha l(l+1) + \beta m]$. For fixed l the minimum is at $m = -l$ (since $\beta > 0$), so $E_l^{\min} = \hbar^2 l[\alpha(l+1) - \beta]$.

- $l = 0$: $E = 0$.

- $l = 1, m = -1$: $E = \hbar^2(2\alpha - \beta)$, which is below zero exactly when $\alpha < \beta/2$.

So for $\alpha < \beta/2$ the $l = 1, m = -1$ level lies below $l = 0$ (option 3). Option 4 ($m = +1$) raises the energy, option 2 is not the minimum for $\alpha < \beta/2$, and for $\alpha = \beta$ (option 1) $E_{1,-1} = +\hbar^2\alpha > 0$.

Answer: (3) $l = 1, m = -1$ for $\alpha < \beta/2$

Insight

Strictly, if $\alpha < \beta/4$ the level $l = 2, m = -2$ ($E = \hbar^2(6\alpha - 2\beta)$) is lower still; in general the ground state has $l \approx \beta/2\alpha$. Among the given options, (3) is the only correct statement, valid for $\beta/4 < \alpha < \beta/2$.

Q52. Two-Body Decay of the Neutral Kaon

[Part C]

Special Relativity | Decay kinematics

A kaon (K^0) decays into two neutral pions (π^0). Given that the mass of the K^0 is $495 \text{ MeV}/c^2$ and the mass of the π^0 is $135 \text{ MeV}/c^2$, the final momentum of the π^0 (in MeV/c) in the center of mass frame is approximately:

- (1) 207
- (2) 359
- (3) 157
- (4) 103

Solution

In the kaon rest frame each pion carries $E = \frac{1}{2}M_K c^2 = 247.5 \text{ MeV}$. Then

$$pc = \sqrt{E^2 - (m_\pi c^2)^2} = \sqrt{247.5^2 - 135^2} = \sqrt{43031} \text{ MeV} = 207 \text{ MeV}.$$

Answer: (1) 207 MeV/c

Q53. Harmonic Conjugate

[Part C]

Mathematical Physics | Analytic functions

Consider a complex analytic function $f(x, y) = u(x, y) + i v(x, y)$. If $u(x, y) = x^3 - 3xy^2$, then $v(x, y)$ is:

- (1) $y^3 - 3xy^2$
- (2) $3xy^2 - y^3$
- (3) $3x^2y - y^3$
- (4) $y^3 + 3xy^2$

Solution

$u = \text{Re}(z^3)$, since $z^3 = (x + iy)^3 = x^3 - 3xy^2 + i(3x^2y - y^3)$. Check with Cauchy–Riemann: $v_y = u_x = 3x^2 - 3y^2 \Rightarrow v = 3x^2y - y^3 + g(x)$, and $v_x = 6xy + g' = -u_y = 6xy \Rightarrow g = \text{const.}$

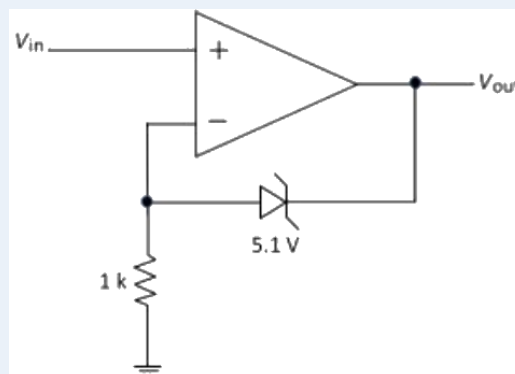
Answer: (3) $3x^2y - y^3$

Q54. Op-Amp with a Zener in the Feedback Path

[Part C]

Electronics | Operational amplifiers

In the circuit shown below, if $V_{\text{in}} = 2.0 \text{ V}$ then V_{out} is



- (1) 2.0 V
- (2) 3.1 V
- (3) 5.1 V
- (4) 7.1 V

Solution

Non-inverting configuration with negative feedback through the Zener. The virtual short gives $V_- = V_{\text{in}} = 2.0 \text{ V}$, so $I = 2.0 \text{ V} / 1 \text{ k}\Omega = 2 \text{ mA}$ flows through the $1 \text{ k}\Omega$ resistor. This current must come from V_{out} through the reverse-biased Zener, which holds its breakdown voltage of 5.1 V :

$$V_{\text{out}} = V_- + V_Z = 2.0 + 5.1 = 7.1 \text{ V.}$$

Answer: (4) 7.1 V

Q55. Particle on a Ring: Degenerate Perturbation

[Part C]

Quantum Mechanics | *Degenerate perturbation theory*

The Hamiltonian of a particle of mass m , moving in a circle of a fixed radius R , is given by $H_0 = \frac{L_z^2}{2mR^2}$, where $L_z = -i\hbar \frac{d}{d\phi}$ and ϕ is the azimuthal angle. This particle is perturbed by $H' = V \cos(2\phi)$ with $V \ll \frac{\hbar^2}{2mR^2}$. Within the first-order degenerate perturbation theory, the magnitude of the energy splitting of the first excited state is:

- (1) $\frac{V}{2}$
- (2) V
- (3) $2V$
- (4) $\frac{V}{4}$

Solution

The first excited level is $m = \pm 1$ (degenerate), with $\psi_{\pm 1} = \frac{e^{\pm i\phi}}{\sqrt{2\pi}}$. Since $\cos 2\phi = \frac{1}{2}(e^{2i\phi} + e^{-2i\phi})$:

$$\langle \pm 1 | H' | \pm 1 \rangle = 0, \quad \langle 1 | H' | -1 \rangle = \frac{V}{2}.$$

The perturbation matrix $\begin{pmatrix} 0 & V/2 \\ V/2 & 0 \end{pmatrix}$ has eigenvalues $\pm V/2$, so the splitting is V .

Answer: (2) V

Q56. Delta-Function Perturbation in a Box

[Part C]

Quantum Mechanics | *Perturbation theory*

Consider a particle of mass m in an infinite potential well with walls at $x = 0$ and $x = 2L$. The particle is perturbed by a weak perturbation $H' = V_0 L \delta(x - 3L/2)$. The first-order correction to its first excited state energy is:

- (1) $2V_0$
- (2) V_0
- (3) $\frac{V_0}{2}$
- (4) 0

Solution

Well of width $2L$: $\psi_n = \sqrt{\frac{1}{L}} \sin \frac{n\pi x}{2L}$. First excited state $n = 2$: $\psi_2 = \sqrt{\frac{1}{L}} \sin \frac{\pi x}{L}$.

$$E_2^{(1)} = V_0 L |\psi_2(3L/2)|^2 = V_0 L \cdot \frac{1}{L} \sin^2 \frac{3\pi}{2} = V_0.$$

Answer: (2) V_0

Q57. Born Cross-Section of a Spherical Well

[Part C]

Quantum Mechanics | Scattering theory

A particle of mass m with momentum $\hbar\vec{k}$ scatters elastically to momentum $\hbar(\vec{k} + \vec{q})$ from a spherical potential

$$V(\vec{r}) = \begin{cases} V_0 & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}.$$

Here, $q^2 = 4k^2 \sin^2 \frac{\theta}{2}$ with θ being the scattering angle. Within the first-order Born approximation and for $qR \ll 1$, the differential scattering cross-section to the leading order in R is proportional to:

- (1) R^2
- (2) R^4
- (3) R^6
- (4) R^8

Solution

$f_B(\theta) = -\frac{2m}{\hbar^2 q} \int_0^\infty r V(r) \sin(qr) dr$. For $qR \ll 1$, $\sin qr \approx qr$:

$$f_B \approx -\frac{2mV_0}{\hbar^2} \int_0^R r^2 dr = -\frac{2mV_0 R^3}{3\hbar^2} \Rightarrow \frac{d\sigma}{d\Omega} = |f_B|^2 \propto R^6.$$

Answer: (3) R^6

Q58. Lowest Degenerate Waveguide Mode

[Part C]

Electrodynamics | Waveguides

Transverse electric modes TE_{mn} propagate in a hollow straight infinite metallic waveguide having rectangular cross section of sides 3 cm and 2 cm. The frequency (in GHz) of the lowest degenerate mode is:

(Given: speed of light = 3×10^8 m/s)

- (1) 5.0
- (2) 7.5
- (3) 9.0
- (4) 15.0

Solution

$f_{mn} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ with $a = 3$ cm, $b = 2$ cm, i.e. $f \propto \sqrt{4m^2 + 9n^2}$ (in units of $c/12$ cm). The values of $4m^2 + 9n^2$ in increasing order are 4(10), 9(01), 13(11), 16(20), 25(21), **36** (30 and 02), ... The first coincidence is TE_{30} and TE_{02} :

$$f = \frac{c}{2} \cdot \frac{3}{3 \text{ cm}} = \frac{3 \times 10^8}{2} \times 100 \text{ Hz} = 15 \text{ GHz}.$$

Answer: (4) 15.0 GHz

Q59. Newton–Raphson Iterations

[Part C]

Mathematical Physics | Numerical methods

Newton-Raphson method is used for finding the roots of the polynomial $(x^2 - 1)(x - 2)$. If the initial trial solutions are chosen as 0 and 0.5, the algorithm converges to p and q , respectively. Then, which of the following is correct?

- (1) $p = 1, q = -1$
- (2) $p = -1, q = 1$
- (3) $p = 2, q = 1$
- (4) $p = 1, q = 2$

Solution

$f = x^3 - 2x^2 - x + 2$, $f' = 3x^2 - 4x - 1$, and $x_{n+1} = x_n - f/f'$.

- $x_0 = 0$: $f = 2$, $f' = -1$, so $x_1 = 0 - \frac{2}{-1} = 2$, which is exactly a root. So $p = 2$.
- $x_0 = 0.5$: $f = (-0.75)(-1.5) = 1.125$, $f' = 0.75 - 2 - 1 = -2.25$, so $x_1 = 0.5 + 0.5 = 1$, exactly a root. So $q = 1$.

Answer: (3) $p = 2, q = 1$

Q60. Fixed Point of a Nonlinear Oscillator

[Part C]

Classical Mechanics | Nonlinear dynamics

A dynamical system is described by a differential equation $\ddot{x} - \dot{x} + x^2 - 2x = 0$. The nature of the fixed point at the origin is:

- (1) unstable spiral
- (2) stable node
- (3) unstable hyperbola
- (4) stable spiral

Solution

Linearise about $x = 0$ (drop x^2): $\ddot{x} - \dot{x} - 2x = 0$, so $\lambda^2 - \lambda - 2 = 0 \Rightarrow \lambda = 2, -1$. The eigenvalues are real and of opposite sign, so the origin is a saddle point, whose nearby trajectories are hyperbolic: an unstable “hyperbolic” fixed point.

Answer: (3) saddle (unstable hyperbola)

Q61. Mean Free Path in a Metal

[Part C]

Solid State Physics | Drude–Sommerfeld model

Resistivity of a monovalent metal, having atomic density 4×10^{22} atoms/cm³, is found to be $1 \mu\Omega\text{-cm}$. The approximate value of the mean free path of electrons in this metal is:

(Given: $\hbar = 10^{-34}$ J-s and $e = 1.6 \times 10^{-19}$ C)

- (1) 1 nm
- (2) 100 nm
- (3) 10^4 nm
- (4) 10^6 nm

Solution

$n = 4 \times 10^{28} \text{ m}^{-3}$ (one electron per atom), $\rho = 10^{-8} \Omega\text{m}$. From $\rho = \frac{m}{ne^2\tau}$ and $\ell = v_F\tau = \frac{\hbar k_F}{m}\tau$, the mass cancels:

$$\ell = \frac{\hbar k_F}{ne^2\rho}, \quad k_F = (3\pi^2 n)^{1/3} = 1.06 \times 10^{10} \text{ m}^{-1}.$$

$$\ell = \frac{10^{-34} \times 1.06 \times 10^{10}}{4 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 10^{-8}} = 1.0 \times 10^{-7} \text{ m} \approx 100 \text{ nm}.$$

Answer: (2) $\approx 100 \text{ nm}$

Q62. London Equation

[Part C]

Solid State Physics | Superconductivity

In a superconductor, the current density $\vec{j}$ and magnetic field $\vec{B}$ are related by the equation: (Here, n is the number density of electrons, e is electron charge, m_e is electron mass and μ_0 is permeability of vacuum.)

- (1) $\frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{j} - \frac{ne^2}{m_e} \vec{B} \right) = 0$
- (2) $\left(\vec{\nabla} \times \vec{j} + \frac{ne^2}{m_e} \vec{B} \right) = 0$
- (3) $\left(\nabla^2 \vec{B} - \mu_0 \vec{\nabla} \times \vec{j} \right) = 0$
- (4) $\left(\nabla^2 \vec{j} - \mu_0 \vec{\nabla} \times \vec{B} \right) = 0$

Solution

The second London equation is

$$\vec{\nabla} \times \vec{j} = -\frac{ne^2}{m_e} \vec{B} \iff \vec{\nabla} \times \vec{j} + \frac{ne^2}{m_e} \vec{B} = 0.$$

Combined with $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$, it gives $\nabla^2 \vec{B} = \vec{B}/\lambda_L^2$ (Meissner effect). Option 1 has the wrong sign and only the weaker time-derivative (perfect-conductor) form. Options 3 and 4 are not the London relation.

Answer: (2)

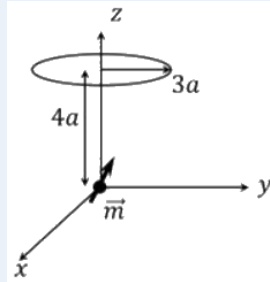
Q63. EMF from a Spinning Magnetic Dipole

[Part C]

Electrodynamics | Faraday's law, dipole flux

A magnetic dipole $\vec{m}$ is kept at the origin with its direction along $+z$ -axis. At time $t = 0$ it starts spinning about x -axis with angular speed ω . A wire loop of radius $3a$ is kept with its plane parallel to xy -plane and centered at $z = 4a$ as shown in the figure. The emf E induced in the loop for $t > 0$ is:

(Note that the magnetic flux through the given loop will be the same as through an appropriate section of a spherical shell of radius $5a$ with its center at the origin.)



- (1) $\left(\frac{9\mu_0 m \omega}{500a}\right) \sin \omega t$
- (2) $\left(\frac{9\mu_0 m \omega}{500a}\right) \cos \omega t$
- (3) $\left(\frac{9\mu_0 m \omega}{250a}\right) \sin \omega t$
- (4) $\left(\frac{9\mu_0 m \omega}{250a}\right) \cos \omega t$

Solution

Rotating about x : $\vec{m}(t) = m(0, \pm \sin \omega t, \cos \omega t)$. By symmetry of the loop about the z -axis, only m_z contributes net flux.

Flux of a z -dipole through the cap of the sphere $r = 5a$ with half-angle θ_0 ($\sin \theta_0 = 3/5$), using

$$B_r = \frac{\mu_0}{4\pi} \frac{2m_z \cos \theta}{r^3}:$$

$$\Phi = \int_0^{\theta_0} B_r 2\pi r^2 \sin \theta d\theta = \frac{\mu_0 m_z}{2r} \sin^2 \theta_0 = \frac{\mu_0 m \cos \omega t}{10a} \cdot \frac{9}{25} = \frac{9\mu_0 m \cos \omega t}{250a}.$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = \frac{9\mu_0 m \omega}{250a} \sin \omega t.$$

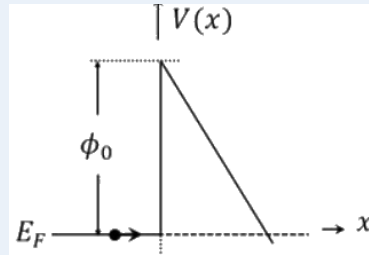
Answer: (3) $\frac{9\mu_0 m \omega}{250a} \sin \omega t$

Q64. Field Emission in the WKB Approximation

[Part C]

Quantum Mechanics | WKB tunnelling

The surface of a metal of work function ϕ_0 is subjected to an electric field $\mathcal{E}$. As a result, the electric potential profile just outside the surface is given by $V(x) = \phi_0 - x\mathcal{E}$ as shown in the figure. For electric field $\mathcal{E}$, the probability $p(\mathcal{E})$ of tunneling of an electron moving at the Fermi energy (E_F) can be found using the WKB approximation. If $\lambda = \sqrt{\frac{2me\phi_0^3}{\hbar^2}}$, then the value of the ratio $\frac{p(\mathcal{E} = 2\lambda)}{p(\mathcal{E} = \lambda)}$ will be:
(Here, e and m are electron's charge and mass, respectively)



- (1) 1.36
- (2) 2.73
- (3) 1.94
- (4) 5.13

Solution

Measured from E_F , the barrier is $e(\phi_0 - x\mathcal{E})$ for $0 < x < \phi_0/\mathcal{E}$ (triangular barrier). WKB gives

$$p = \exp \left[-\frac{2}{\hbar} \int_0^{\phi_0/\mathcal{E}} \sqrt{2me(\phi_0 - x\mathcal{E})} dx \right] = \exp \left[-\frac{4}{3} \frac{\sqrt{2me} \phi_0^{3/2}}{\hbar \mathcal{E}} \right] = \exp \left(-\frac{4\lambda}{3\mathcal{E}} \right).$$

(Fowler–Nordheim form.) Hence

$$\frac{p(2\lambda)}{p(\lambda)} = \frac{e^{-2/3}}{e^{-4/3}} = e^{2/3} = 1.948 \approx 1.94.$$

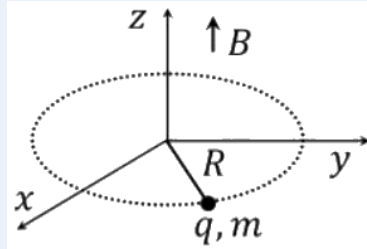
Answer: (3) $e^{2/3} \approx 1.94$

Q65. Force in a Rod under a Growing Magnetic Field

[Part C]

Electrodynamics | Faraday induction, Lorentz force

A particle of mass m and charge q is fixed at one end of a rigid insulating rod of length R . The other end of the rod is pivoted such that it can rotate about the z -axis in the xy -plane as shown in the figure. A time dependent magnetic field $\vec{B} = \beta t \hat{z}$ is turned on at $t = 0$. Here β is a constant. Then the force in the rod at time t will be:



- (1) Compressive and of magnitude $q^2 \beta^2 t^2 R / m$
- (2) Zero
- (3) Compressive and of magnitude $q^2 \beta^2 t^2 R / (4m)$
- (4) Tensile and of magnitude $q^2 \beta^2 t^2 R / (4m)$

Solution

Induced field: $\oint \vec{E} \cdot d\vec{l} = -\dot{\Phi} \Rightarrow E_\phi(2\pi R) = -\beta\pi R^2$, so $E_\phi = -\frac{1}{2}\beta R$.

Speed: $m\dot{v}_\phi = qE_\phi \Rightarrow v_\phi = -\frac{q\beta R}{2m}t$ (magnitude $v = \frac{q\beta R t}{2m}$).

Radial balance: the magnetic force $q\vec{v} \times \vec{B} = qv_\phi B (\hat{\phi} \times \hat{z}) = qv_\phi B \hat{r}$ is *inward*, with magnitude

$$qvB = q \cdot \frac{q\beta R t}{2m} \cdot \beta t = \frac{q^2 \beta^2 t^2 R}{2m},$$

twice the required centripetal force $\frac{mv^2}{R} = \frac{q^2 \beta^2 t^2 R}{4m}$. The rod must push the charge *outward* with the difference, $\frac{q^2 \beta^2 t^2 R}{4m}$. A rod that pushes outward on its end is in compression.

Answer: (3) compressive, $q^2 \beta^2 t^2 R / (4m)$

Q66. Einstein Coefficients and Radiation Density

[Part C]

Atomic Physics | Einstein A and B coefficients

A container has a gas at temperature T with its N_1 molecules in the ground state and N_2 molecules in the excited state of energy $h\nu$. Both the levels are non-degenerate and A and B are the Einstein coefficients for spontaneous and stimulated emissions, respectively, between the two levels. Then the spectral density $\rho(\nu, T)$ of radiation inside the container is proportional to:

- (1) $\frac{A}{B} \left(\frac{N_2}{N_1 - N_2} \right)$
- (2) $\frac{A}{B} \left(\frac{N_2}{N_1 + N_2} \right)$
- (3) $\frac{N_2}{\left(\frac{A}{B}\right) N_1 - N_2}$
- (4) $\frac{N_2}{\left(\frac{A}{B}\right) N_1 + N_2}$

Solution

Non-degenerate levels give $B_{12} = B_{21} = B$. Detailed balance in equilibrium:

$$N_1 B \rho = N_2 A + N_2 B \rho \Rightarrow \rho = \frac{A}{B} \frac{N_2}{N_1 - N_2}.$$

(With $N_1/N_2 = e^{h\nu/k_B T}$ this is Planck's law.)

Answer: (1) $\frac{A}{B} \frac{N_2}{N_1 - N_2}$

Q67. Biased Random Walk

[Part C]

Mathematical Physics | Binomial distribution

A biased random walker confined to the x -axis moves, at each step, with probability 0.6 to the right and with probability 0.4 to the left. If the walker takes 1000 steps starting from the origin, the probability that the walker is found exactly 200 steps to the right of the origin is close to:

- (1) 0.026
- (2) 0.125
- (3) 0.250
- (4) 0.375

Solution

A net displacement of +200 with $R + L = 1000$ requires $R = 600$, $L = 400$. This is exactly the mean $Np = 600$, so the binomial probability is the peak value:

$$P(R = 600) \simeq \frac{1}{\sqrt{2\pi Npq}} = \frac{1}{\sqrt{2\pi(1000)(0.6)(0.4)}} = \frac{1}{\sqrt{1508}} = 0.0257 \approx 0.026.$$

Answer: (1) 0.026

Q68. Two Coupled Ising Spins

[Part C — Dropped]

Statistical Mechanics | Ising model

There is a ferromagnetic interaction J between two Ising spins, $S_1, S_2 = \pm 1$, and they are under the influence of an external magnetic field h . The Hamiltonian of this system is,

$$H = -JS_1S_2 - hS_1 - hS_2.$$

The spins are in equilibrium at temperature T and $\beta = 1/k_B T$ with k_B as Boltzmann constant. The average value of either spin, for small h , is:

- (1) $\frac{4h \exp(\beta J)}{\cosh(\beta J)}$
- (2) $\frac{h \exp(2\beta J)}{\cos(\beta J)}$
- (3) $\frac{8h \exp(4\beta J)}{\sinh(\beta J)}$
- (4) $\frac{2h \exp(2\beta J)}{\sin(\beta J)}$

Solution

States: $(++)$: $E = -J - 2h$; $(--)$: $E = -J + 2h$; $(+-)$, $(-+)$: $E = +J$.

$$Z = 2e^{\beta J} \cosh(2\beta h) + 2e^{-\beta J}, \quad \langle S_1 \rangle = \frac{e^{\beta J} \sinh(2\beta h)}{e^{\beta J} \cosh(2\beta h) + e^{-\beta J}}.$$

For small h : $\langle S_1 \rangle \approx \frac{2\beta h e^{\beta J}}{2 \cosh \beta J} = \frac{\beta h e^{\beta J}}{\cosh(\beta J)}$.

None of the options matches: each lacks the factor β , and has the wrong numerical prefactor or the wrong function. Option 1 is dimensionally inconsistent (it has the dimensions of energy, whereas $\langle S \rangle$ is dimensionless) and too large by $4/\beta$.

Answer: Dropped by NTA — correct result $\langle S \rangle \approx \beta h e^{\beta J} / \cosh(\beta J)$

Insight

Full marks are awarded to all candidates for a dropped question. The derivation is included for completeness; option 1 has the right structure $e^{\beta J} / \cosh(\beta J)$ but not the right prefactor.

Q69. Why Modulate?

[Part C]

Experimental Techniques | Noise, lock-in detection

Modulation techniques are used to shift a signal to higher frequencies to

- (1) remove the effects of white noise.
- (2) reduce the effects of $1/f$ -noise and drift.
- (3) reduce shot noise and Johnson noise.
- (4) smoothen the signal and amplify it.

Solution

Flicker ($1/f$) noise and slow drifts dominate near dc. Modulating the signal (chopping, lock-in detection) moves it to a frequency where the noise floor is the flat white-noise level, and narrow-band detection then rejects the low-frequency drift. White noise (Johnson, shot) is frequency-independent, so shifting frequency does not reduce it.

Answer: (2) reduce $1/f$ noise and drift

Q70. Magnetic Moment of ^{17}O

[Part C]

Nuclear Physics | Shell model, Schmidt values

The table shows the g -factors of proton (p) and neutron (n) for orbital (g_l) and spin (g_s) angular momentum.

	g_l	g_s
p	1	5.58
n	0	-3.82

The magnetic moment of ^{17}O nucleus, according to the single particle shell model, in units of nuclear magneton (μ_N) is:

- (1) 0.55
- (2) 4.79
- (3) -1.91
- (4) -3.82

Solution

^{17}O = doubly-magic ^{16}O + one neutron in $1d_{5/2}$ ($l = 2$, $j = l + \frac{1}{2} = \frac{5}{2}$). Schmidt value for $j = l + \frac{1}{2}$:

$$\mu = [g_l l + \frac{1}{2}g_s] \mu_N = [0 + \frac{1}{2}(-3.82)] \mu_N = -1.91 \mu_N.$$

(Experiment: $-1.89 \mu_N$.)

Answer: (3) $-1.91 \mu_N$

Q71. Coulomb Energy of Mirror Nuclei

[Part C]

Nuclear Physics | Liquid-drop model

Assume a spherically uniform charge distribution in a nucleus having mass number A and radius $R = R_0 A^{1/3}$ with $R_0 \approx 1.5$ fm. If the difference in electrostatic energy between two mirror nuclei, ${}_Z^AX$ and ${}_{Z-1}^AY$, is 3.53 MeV, then the value of atomic number Z is closest to:

(Given: $\frac{e^2}{4\pi\epsilon_0} \approx 1.44$ MeV.fm and $1 \text{ fm} = 10^{-15} \text{ m}$)

- (1) 6
- (2) 8
- (3) 10
- (4) 12

Solution

$$E_C = \frac{3}{5} \frac{Z^2 e^2}{4\pi\epsilon_0 R}, \text{ so}$$

$$\Delta E_C = \frac{3}{5} \cdot \frac{1.44}{1.5 A^{1/3}} [Z^2 - (Z-1)^2] = 0.576 \frac{2Z-1}{A^{1/3}} \text{ MeV.}$$

For mirror nuclei $A = 2Z - 1$, so $\Delta E_C = 0.576 A^{2/3}$ MeV. Then

$$A^{2/3} = \frac{3.53}{0.576} = 6.13 \Rightarrow A \approx 15, \quad Z = \frac{A+1}{2} = 8 \quad ({}^{15}\text{O}/{}^{15}\text{N}).$$

(Check: $0.576 \times 15^{2/3} = 3.50$ MeV. ✓)

Answer: (2) $Z = 8$

Q72. Decays of the ρ^0

[Part C]

Particle Physics | Angular momentum and parity

Consider the following decay processes of ρ^0 :

(A) $\rho^0 \rightarrow \pi^+ + \pi^-$

(B) $\rho^0 \rightarrow \pi^0 + \pi^0$

For the conservation of angular momentum and parity, which of the following options is correct?

- (1) Both processes (A) and (B) are allowed
- (2) Both processes (A) and (B) are forbidden
- (3) Only process (A) is allowed
- (4) Only process (B) is allowed

Solution

ρ^0 : $J^P = 1^-$; pions are spin-0 with $P = -1$. Angular momentum requires orbital $l = 1$, which gives parity $(-1)(-1)(-1)^1 = -1$. ✓

- (A) $\pi^+\pi^-$ in $l = 1$: allowed. This is the dominant ($\sim 100\%$) decay.
- (B) $\pi^0\pi^0$ are identical bosons, so their spatial wavefunction must be symmetric (even l). $l = 1$ is antisymmetric: forbidden.

Answer: (3) Only (A)

Q73. Simpson vs. Trapezoidal Rule

[Part C]

Mathematical Physics | Numerical integration

The integral

$$\int_0^4 x^4 dx$$

when computed numerically using Simpson's $1/3^{\text{rd}}$ method and trapezoidal method, each with unit step size, yields values I_s and I_t , respectively. Then which of the following options is correct?

- (1) $I_s = 205.3, \quad I_t = 226.0$
- (2) $I_s = 205.3, \quad I_t = 205.3$
- (3) $I_s = 204.8, \quad I_t = 226.0$
- (4) $I_s = 226.0, \quad I_t = 226.0$

Solution

$f(0..4) = 0, 1, 16, 81, 256$ with $h = 1$.

$$I_s = \frac{1}{3} [0 + 4(1) + 2(16) + 4(81) + 256] = \frac{616}{3} = 205.33, \quad I_t = \frac{1}{2} (0 + 256) + (1 + 16 + 81) = 226.0.$$

(Exact value: $4^5/5 = 204.8$.)

Answer: (1) $I_s = 205.3, I_t = 226.0$

Q74. X-Ray Peaks of KCl

[Part C]

Solid State Physics | Structure factor

Potassium chloride (KCl) crystallizes in NaCl structure. The atomic form factors of K^+ and Cl^- are the same. Then the first four X-ray diffraction peaks for KCl are:

- (1) (100), (110), (111), (200)
- (2) (110), (200), (222), (310)
- (3) (111), (200), (220), (311)
- (4) (200), (220), (222), (400)

Solution

NaCl structure: fcc lattice with basis at 0 and $\frac{a}{2}(1, 1, 1)$. The structure factor for unmixed (hkl) is

$$S = 4 \left[f_K + f_{Cl} e^{i\pi(h+k+l)} \right].$$

All odd (hkl) give $4(f_K - f_{Cl}) = 0$ (since $f_K = f_{Cl}$); all even give $8f$. K^+ and Cl^- are isoelectronic (18 electrons), so the crystal diffracts like a simple cubic lattice of side $a/2$. The first four peaks are (200), (220), (222), (400).

Answer: (4)

Q75. Reduced Chi-Square

[Part C]

Experimental Techniques | *Data analysis*

Given is a set of 5 data points of the form (x_i, y_i, σ_i) with σ_i as the estimated uncertainty in y_i . This data is fitted to the form $y = ax + b$ and the minimized χ^2 (chi-square) is found to be 5.1. The reduced χ^2 for this fitting is:

- (1) 1.0
- (2) 1.2
- (3) 1.5
- (4) 1.7

Solution

Degrees of freedom $\nu = N - (\text{fit parameters}) = 5 - 2 = 3$, so $\chi_\nu^2 = 5.1/3 = 1.7$.

Answer: (4) 1.7