

Pravegaa Education

CSIR NET-JRF · GATE · IIT-JAM · JEST · TIFR Physics

GATE 2026 — Physics (PH)

Complete Solutions — Verified against the Official Key

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Organizing Institute: IIT Guwahati | GA: Q1–Q10 | PH: Q11–Q65 | MCQ · MSQ · NAT

Answer Key at a Glance

Matches the official GATE 2026 PH answer key (IIT Guwahati) for all 65 questions

Q	Type	Key	Q	Type	Key	Q	Type	Key
1	MCQ	A	23	MCQ	A	45	MCQ	A
2	MCQ	B	24	MSQ	A; B	46	MCQ	D
3	MCQ	A	25	MSQ	A; D	47	MCQ	A
4	MCQ	B	26	MSQ	A; B	48	MCQ	B
5	MCQ	C	27	MSQ	C; D	49	MCQ	B
6	MCQ	D	28	NAT	0.32–0.34	50	MSQ	B; C
7	MCQ	A	29	NAT	4	51	MSQ	A; C; D
8	MCQ	D	30	NAT	9.0–10.0	52	MSQ	B; C
9	MCQ	C	31	NAT	2.0	53	MSQ	A; C
10	MCQ	C	32	NAT	0.30–0.32	54	MSQ	C; D
11	MCQ	B	33	NAT	21.1–21.3	55	MSQ	B; C; D
12	MCQ	A	34	NAT	4901–4905	56	MSQ	A
13	MCQ	A	35	NAT	7.60–8.00	57	NAT	0.60–0.70
14	MCQ	A	36	MCQ	B	58	NAT	52.0–53.0
15	MCQ	C	37	MCQ	C	59	NAT	10.56
16	MCQ	C	38	MCQ	B	60	NAT	47.5–50.0
17	MCQ	A	39	MCQ	A	61	NAT	0.80–0.90
18	MCQ	B	40	MCQ	A	62	NAT	2.6–3.0
19	MCQ	C	41	MCQ	A	63	NAT	6.5–7.5
20	MCQ	C	42	MCQ	B	64	NAT	2.35–2.45
21	MCQ	A	43	MCQ	A	65	NAT	179.0–180.0
22	MCQ	C	44	MCQ	B			

Marking: MCQ 1-mark –1/3, 2-mark –2/3; MSQ and NAT carry no negative marking. Q1–5, Q11–35: 1 mark; Q6–10, Q36–65: 2 marks.

General Aptitude (GA)

Q1–Q5: 1 mark each

Q6–Q10: 2 marks each

Q1. Words in Context

[MCQ, 1 mark]

Verbal Ability | Vocabulary

“He often _____ the numbers. False claims are not going to help. Honesty _____ trust”, said the manager.

Choose the option with the correct order of words to fill the blanks.

- (A) exaggerates; engenders
- (B) excels; encourages
- (C) aggravates; alleviates
- (D) diminishes; eliminates

Solution

The first sentence criticises the person (“false claims”), so he *exaggerates* (inflates) the numbers. Honesty *engenders* (gives rise to) trust. The other pairs fail: one does not “excel” or “aggravate” numbers, and honesty does not “alleviate” or “eliminate” trust.

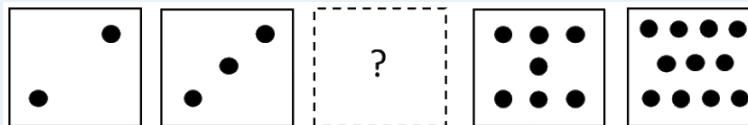
Answer: (A) exaggerates; engenders

Q2. Tile Sequence

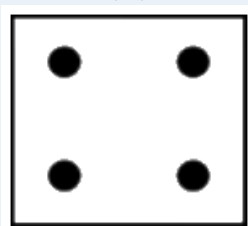
[MCQ, 1 mark]

Analytical Aptitude | Number patterns

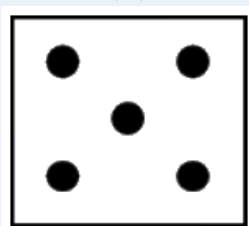
In the sequence of tiles shown below, the missing tile indicated by the question mark should be



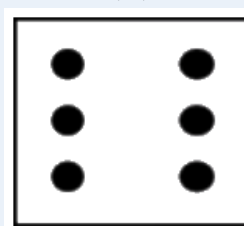
(A)



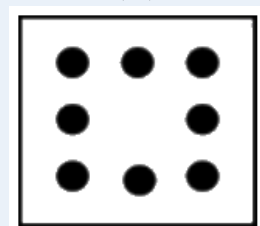
(B)



(C)



(D)



Solution

The tiles contain 2, 3, ?, 7, 11 dots — consecutive prime numbers. The missing tile must have 5 dots; option (B) has 5 dots (option A has 4, C has 6, D has 8).

Answer: (B) 5 dots

Q3. Pigeonhole Reasoning

[MCQ, 1 mark]

Logical Reasoning | *Pigeonhole principle*

A school has 100 students distributed among 1st to 10th standards. Based on this, which one of the following statements is always correct?

- (A) There are at least 10 students who belong to the same standard.
- (B) There is at least one student in each standard.
- (C) There are at most 10 students in 10th standard.
- (D) The total number of students from 1st to 5th standards is at least 50.

Solution

100 students in 10 standards: if every standard had at most 9 students, the total would be at most $90 < 100$. Hence some standard has at least $\lceil 100/10 \rceil = 10$ students. (B), (C), (D) can all fail, e.g. if all 100 students are in the 10th standard.

Answer: (A) at least 10 in one standard

Q4. Counting 3-Digit Numbers

[MCQ, 1 mark]

Quantitative Aptitude | *Permutations*

How many 3-digit numbers can be formed using three distinct single digit prime numbers?

- (A) 64
- (B) 24
- (C) 12
- (D) 4

Solution

Single-digit primes: 2, 3, 5, 7 (four of them, none is 0). Arrangements of 3 distinct ones: ${}^4P_3 = 4 \times 3 \times 2 = 24$.

Answer: (B) 24

Q5. Two-Set Counting

[MCQ, 1 mark]

Quantitative Aptitude | *Venn diagrams*

In a group of students, 10 students like Mathematics, 12 students like English, 4 students like both Mathematics and English, and 6 students like neither Mathematics nor English. The number of students in the group is _____

- (A) 18
- (B) 20
- (C) 24
- (D) 32

Solution

$|M \cup E| = 10 + 12 - 4 = 18$; adding those who like neither: $18 + 6 = 24$.

Answer: (C) 24

Q6. Analogy

[MCQ, 2 marks]

Verbal Ability | Analogies

Charity : P :: Retaliation : Q

Choose the appropriate pair of words P and Q that fit the analogy.

- (A) P = Parsimonious; Q = Vengeful
- (B) P = Altruistic; Q = Amicable
- (C) P = Resentful; Q = Spiteful
- (D) P = Magnanimous; Q = Vindictive

Solution

The relation is “act : the disposition that produces it”. Charity comes from a *magnanimous* (generous) nature; retaliation from a *vindictive* one. In (A) “parsimonious” is the opposite of charitable; in (B) “amicable” does not fit retaliation; in (C) “resentful” does not fit charity.

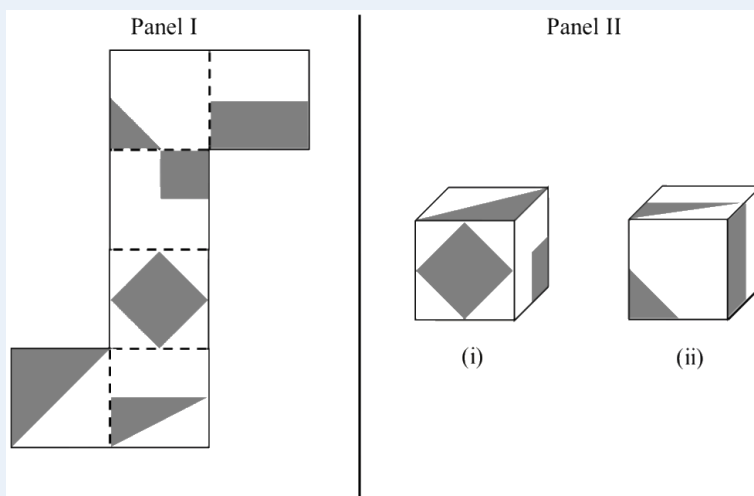
Answer: (D) Magnanimous; Vindictive

Q7. Folding a Cube Net

[MCQ, 2 marks]

Spatial Reasoning | Cube nets

A paper shown in Panel I is folded along the dashed lines (---) to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube. Referring to cubes shown in Panel II, which one of the options is correct?



- (A) Only (i) can correspond to the unfolded cube in Panel I.
- (B) Only (ii) can correspond to the unfolded cube in Panel I.
- (C) Both (i) and (ii) can correspond to the unfolded cube in Panel I.
- (D) Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

Solution

Label the vertical strip of Panel I from top to bottom as S_1 (small corner triangle, lower-left), S_2 (small square), S_3 (diamond), S_4 (thin wedge); the flap to the right of S_1 is R (lower half shaded), and the flap to the left of S_4 is L (half-diagonal triangle). The strip wraps around the cube, so S_1 is opposite S_3 , S_2 is opposite S_4 , and R is opposite L .

Cube (ii): its front face carries a corner triangle at the lower-left — exactly S_1 in its net orientation. Folding keeps the flap R along the right edge of S_1 , with its shaded *lower half* still at the bottom, so the right face of the cube should show a *horizontal* strip along its lower half. Cube (ii) shows a *vertical* strip there, so (ii) is impossible.

Cube (i): front face = S_3 (diamond). The right face shows the lower-half strip of R (consistent, since R is adjacent to every strip face), and the top shows the thin triangular wedge of S_4 , which is folded up along the S_3 – S_4 edge. All visible faces are mutually adjacent in the net, so (i) is realisable.

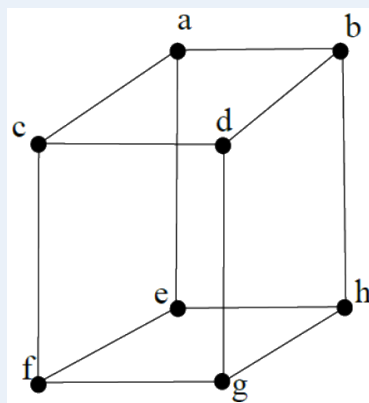
Answer: (A) Only (i)

Q8. Colouring the Corners of a Cube

[MCQ, 2 marks]

Logical Reasoning | Graph colouring

Consider the cube shown below with its 8 corners labelled a, b, c, d, e, f, g, and h. The figure is representative. All corners are to be colored such that any two corners that are connected by an edge must be of different colors. The minimum number of colors required to achieve this is _____



- (A) 8
- (B) 4
- (C) 3
- (D) 2

Solution

Place the cube at $\{0, 1\}^3$. Every edge changes exactly one coordinate, so it joins a vertex with an even coordinate sum to one with an odd sum. Colour “even” corners with one colour and “odd” corners with another: no edge joins two corners of the same colour. One colour is clearly not enough, so the minimum is 2 (the cube graph is bipartite).

Answer: (D) 2

Q9. Positions of Four Hills

[MCQ, 2 marks]

Logical Reasoning | Arrangements

Four hills H1, H2, H3, and H4 are present in an area. The following observations are made about them:

- Neither H2 nor H3 is the easternmost hill.
- Neither H2 nor H3 is the westernmost hill.
- Neither the easternmost hill nor the westernmost hill is the southernmost hill.
- Two hills are located to the west of H2.
- The southernmost hill has at least two hills to its east.

The southernmost hill is _____.

- (A) H1
(B) H2
(C) H3
(D) H4

Solution

By (i) and (ii), the two extreme (east/west) hills are H1 and H4, so H2 and H3 are in the middle. By (iv), H2 is third from the west; hence the west-to-east order is {H1/H4}, H3, H2, {H4/H1}. By (iii), the southernmost hill is H2 or H3. By (v) it has at least two hills to its east: H3 has H2 and one extreme hill to its east, while H2 has only one. So the southernmost hill is H3.

Answer: (C) H3

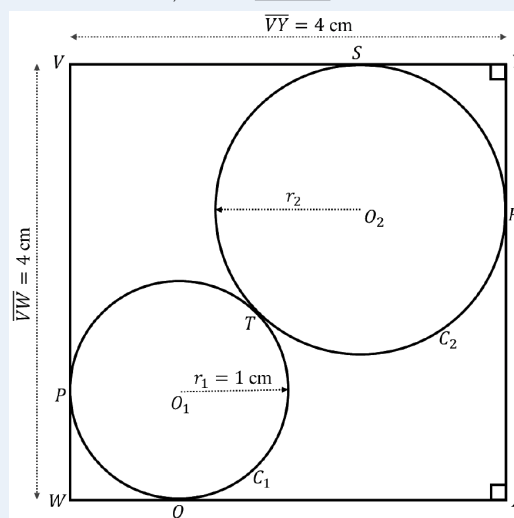
Q10. Two Circles in a Square

[MCQ, 2 marks]

Quantitative Aptitude | Geometry

As shown in the figure, circle C_1 with center O_1 and radius r_1 touches the square $VWXY$ at points P and Q while circle C_2 with center O_2 and radius r_2 touches the square $VWXY$ at points R and S . The two circles touch each other at T .

Given $r_1 = 1$ cm and $\overline{VY} = \overline{VW} = 4$ cm, $r_2 =$ _____ cm.



- (A) $4 - 3\sqrt{2}$ (B) $1 + 2\sqrt{2}$
(C) $7 - 4\sqrt{2}$ (D) $5 + 3\sqrt{2}$

Solution

Take W as the origin, WX along x and WV along y (side 4 cm). C_1 touches WV and WX , so $O_1 = (1, 1)$. C_2 touches VY and YX , so $O_2 = (4 - r_2, 4 - r_2)$. Both centres lie on the diagonal WY , and external tangency gives

$$O_1O_2 = \sqrt{2}(3 - r_2) = 1 + r_2 \Rightarrow r_2 = \frac{3\sqrt{2} - 1}{\sqrt{2} + 1} = (3\sqrt{2} - 1)(\sqrt{2} - 1) = 7 - 4\sqrt{2} \text{ cm.}$$

(Check: $r_2 \approx 1.34$ cm, and $r_1 + r_2 + \sqrt{2}(r_1 + r_2) \approx 5.66 = 4\sqrt{2}$ along the diagonal. ✓)

Answer: (C) $7 - 4\sqrt{2}$

Physics (PH)**Q11–Q35: 1 mark each****Q11. Magnetic Field of a Plane Wave**

[MCQ, 1 mark]

Electrodynamics | Plane EM waves

In free space, an electromagnetic wave is travelling whose wavevector is $\vec{k} = 10(\hat{x} + \sqrt{3}\hat{y}) \text{ m}^{-1}$. The electric field component of this electromagnetic wave is given by $\vec{E}(\vec{r}, t) = \hat{z} 600 \cos(\vec{k} \cdot \vec{r} - \omega t) \text{ V.m}^{-1}$. The speed of light in free space is $c = 3.0 \times 10^8 \text{ m.s}^{-1}$. The corresponding magnetic field $\vec{B}(\vec{r}, t)$ is

- (A) $\vec{B}(\vec{r}, t) = 2 \times 10^{-6}(\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) \text{ V.m}^{-2}.\text{s}$
- (B) $\vec{B}(\vec{r}, t) = 10^{-6}(\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) \text{ V.m}^{-2}.\text{s}$
- (C) $\vec{B}(\vec{r}, t) = 2 \times 10^{-5}(-\sqrt{3}\hat{x} + \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) \text{ V.m}^{-2}.\text{s}$
- (D) $\vec{B}(\vec{r}, t) = 10^{-5}(\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) \text{ V.m}^{-2}.\text{s}$

Solution

For a plane wave in vacuum, $\vec{B} = \frac{1}{c} \hat{k} \times \vec{E}$.

$|\vec{k}| = 10\sqrt{1+3} = 20 \text{ m}^{-1}$, so $\hat{k} = \frac{1}{2}(\hat{x} + \sqrt{3}\hat{y})$ and

$$\hat{k} \times \hat{z} = \frac{1}{2}(\hat{x} \times \hat{z} + \sqrt{3}\hat{y} \times \hat{z}) = \frac{1}{2}(-\hat{y} + \sqrt{3}\hat{x}).$$

Amplitude $B_0 = E_0/c = 600/(3 \times 10^8) = 2 \times 10^{-6} \text{ T}$. Therefore

$$\vec{B} = 2 \times 10^{-6} \cdot \frac{1}{2}(\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) = 10^{-6}(\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t).$$

Check: $\vec{k} \cdot \vec{B} \propto \sqrt{3} - \sqrt{3} = 0$ and $|\vec{B}| = 2 \times 10^{-6} \text{ T}$. ✓

Answer: (B) $10^{-6}(\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t)$

Insight

Option (A) has the right direction but magnitude $4 \times 10^{-6} \text{ T}$ — the trap is forgetting that $(\sqrt{3}\hat{x} - \hat{y})$ is not a unit vector (its length is 2).

Q12. Field across a Charged Sheet

[MCQ, 1 mark]

Electrostatics | Boundary conditions

An infinitely large non-conducting thin sheet in the xy plane ($z = 0$) carries a uniform surface charge density $\sigma = 17.70 \times 10^{-12} \text{ C.m}^{-2}$. The electric field in the region $z < 0$ is $\vec{E}_2 = \hat{x} + 2\hat{y} + 3\hat{z}$. Then, the electric field $\vec{E}_1$ in the region $z > 0$ will be

($\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2.\text{N}^{-1}.\text{m}^{-2}$)

- (A) $\vec{E}_1 = \hat{x} + 2\hat{y} + 5\hat{z}$
- (B) $\vec{E}_1 = \hat{x} + 2\hat{y} + 4\hat{z}$
- (C) $\vec{E}_1 = \hat{x} + 2\hat{y} + 3\hat{z}$
- (D) $\vec{E}_1 = \hat{x} + 4\hat{y} + \hat{z}$

Solution

Tangential components are continuous: $E_{1x} = 1$, $E_{1y} = 2$. The normal component jumps by σ/ϵ_0 :

$$E_{1z} - E_{2z} = \frac{\sigma}{\epsilon_0} = \frac{17.70}{8.85} = 2 \Rightarrow E_{1z} = 3 + 2 = 5.$$

Answer: (A) $\vec{E}_1 = \hat{x} + 2\hat{y} + 5\hat{z}$

Q13. Hermitian Combination

[MCQ, 1 mark]

Quantum Mechanics | Hermitian operators

Consider an operator $\hat{A}$ which is not Hermitian. Find the possible values of c and d such that the operator $(c\hat{A} - d\hat{A}^\dagger)$ is Hermitian.

- (A) $c = i$ and $d = i$
- (B) $c = 1$ and $d = 1$
- (C) $c = -1$ and $d = i$
- (D) $c = i$ and $d = -i$

Solution

$(c\hat{A} - d\hat{A}^\dagger)^\dagger = c^*\hat{A}^\dagger - d^*\hat{A}$. Equating with $c\hat{A} - d\hat{A}^\dagger$ (with $\hat{A}, \hat{A}^\dagger$ independent) requires

$$c = -d^* \quad \text{and} \quad d = -c^*.$$

For $c = d = i$: $-d^* = -(-i) = i = c$ ✓. Indeed $i(\hat{A} - \hat{A}^\dagger)$ is Hermitian. (B) gives the anti-Hermitian $\hat{A} - \hat{A}^\dagger$; (C) and (D) fail the condition.

Answer: (A) $c = i, d = i$

Q14. Curl of a Product

[MCQ, 1 mark]

Mathematical Physics | Vector identities

For a scalar field $\psi(\vec{r})$ and a vector field $\vec{A}(\vec{r})$, $\vec{\nabla} \times (\vec{A}\psi)$ is equivalent to the expression

- (A) $\psi(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla}\psi)$
- (B) $\psi(\vec{\nabla} \times \vec{A}) + \vec{A} \times (\vec{\nabla}\psi)$
- (C) Null vector
- (D) $\vec{A} \times (\vec{\nabla}\psi) - \psi(\vec{\nabla} \times \vec{A})$

Solution

Product rule: $\vec{\nabla} \times (\psi\vec{A}) = \psi(\vec{\nabla} \times \vec{A}) + (\vec{\nabla}\psi) \times \vec{A} = \psi(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla}\psi)$.

Answer: (A) $\psi(\vec{\nabla} \times \vec{A}) - \vec{A} \times \vec{\nabla}\psi$

Q15. Time Reversal of $\vec{E}$ and $\vec{B}$

[MCQ, 1 mark]

Electrodynamics | Discrete symmetries

Which of the following options is correct for transformation of electric field $\vec{E}$ and magnetic field $\vec{B}$ under time reversal, i.e., $t \rightarrow -t$?

- (A) $\vec{E} \rightarrow \vec{E}$ and $\vec{B} \rightarrow \vec{B}$
- (B) $\vec{E} \rightarrow -\vec{E}$ and $\vec{B} \rightarrow \vec{B}$
- (C) $\vec{E} \rightarrow \vec{E}$ and $\vec{B} \rightarrow -\vec{B}$
- (D) $\vec{E} \rightarrow -\vec{E}$ and $\vec{B} \rightarrow -\vec{B}$

Solution

Charges are unchanged under $t \rightarrow -t$, so the Coulomb field $\vec{E}$ is even. Velocities, and hence currents, reverse; so $\vec{B}$ produced by currents is odd. Consistency check with the Lorentz force $m\ddot{\vec{r}} = q(\vec{E} + \vec{v} \times \vec{B})$: $\ddot{\vec{r}}$ is even and $\vec{v}$ is odd, so $\vec{B}$ must be odd.

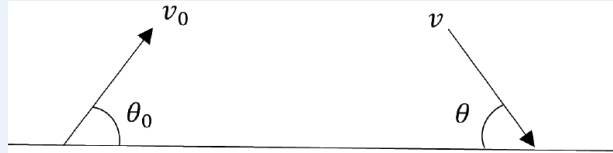
Answer: (C) $\vec{E} \rightarrow \vec{E}, \vec{B} \rightarrow -\vec{B}$

Q16. Projectile with Linear Drag

[MCQ, 1 mark]

Classical Mechanics | Resistive motion

On a horizontal plane, a projectile of mass m is launched from the ground with speed v_0 at an angle θ_0 with the horizontal. In addition to the gravitational force (mg), it also experiences a drag force $\vec{F}_{drag} = -\gamma\vec{v}$, where $\vec{v}$ is its velocity and γ is a constant. It hits the ground at a distance R from the point of launch with its velocity making an angle θ with the horizontal, as shown schematically in the figure. Then which of the following options is correct?



(A) $R = \frac{v_0^2 \sin 2\theta}{g}$ $\theta < \theta_0$

(B) $R < \frac{v_0^2 \sin 2\theta_0}{g}$ $\theta < \theta_0$

(C) $R < \frac{v_0^2 \sin 2\theta_0}{g}$ $\theta > \theta_0$

(D) $R = \frac{v_0^2 \sin 2\theta}{g}$ $\theta > \theta_0$

Solution

Drag removes energy, so the range falls below the vacuum value: $R < v_0^2 \sin 2\theta_0 / g$.

The horizontal velocity only decays, $v_x = v_0 \cos \theta_0 e^{-\gamma t/m}$, while on the descent the vertical motion is still driven by gravity. The trajectory is therefore asymmetric: it comes down more steeply than it went up, so the impact angle exceeds the launch angle, $\theta > \theta_0$. (In the strong-drag limit the projectile falls almost vertically, $\theta \rightarrow 90^\circ$.)

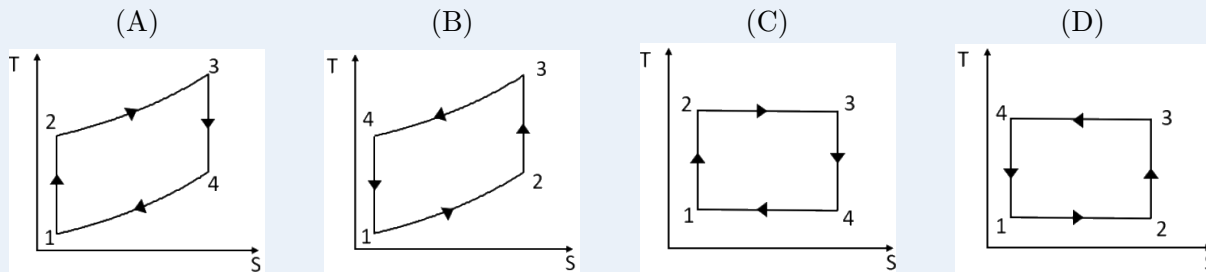
Answer: (C) $R < v_0^2 \sin 2\theta_0 / g$, $\theta > \theta_0$

Q17. Otto Cycle on a T - S Diagram

[MCQ, 1 mark]

Thermodynamics | Heat engines

Consider the Otto cycle for an ideal gas engine consisting of two quasistatic adiabatic and two quasistatic isochoric processes. The correct temperature-entropy (T - S) phase diagram for the cycle is



Solution

- Quasistatic adiabats are isentropes: *vertical* lines on a T - S diagram. Compression $1 \rightarrow 2$ goes up and expansion $3 \rightarrow 4$ goes down.
- On an isochore $dS = nC_V dT/T \Rightarrow T \propto e^{S/nC_V}$: rising *curves* (not horizontal lines). Heat addition $2 \rightarrow 3$ moves right and up; heat rejection $4 \rightarrow 1$ moves left and down.

Rectangles (C), (D) describe a Carnot cycle (isotherms are horizontal). (B) has the right shape but runs anticlockwise (a refrigerator). Only (A) has vertical adiabats, curved isochors and the clockwise (engine) sense $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$.

Answer: (A)

Q18. Two-Dimensional Photon Gas

[MCQ, 1 mark]

Statistical Mechanics | Blackbody radiation

The formula for energy E of a photon gas at temperature T in a two-dimensional box at equilibrium with $g_{2d}(\nu)$ denoting the density of states of photons is given below where symbols ν , h and k_B have their standard meaning. The specific heat (C_V) of this photon gas obeys

$$E = \int_0^\infty d\nu g_{2d}(\nu) \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

(A) $C_V \propto T$

(B) $C_V \propto T^2$

(C) $C_V \propto T^3$

(D) $C_V \propto T^4$

Solution

In 2-D the number of modes up to k is $\propto k^2$, so $g_{2d}(\nu) \propto \nu$. With $x = h\nu/k_B T$:

$$E \propto \int_0^\infty \frac{\nu^2 d\nu}{e^{h\nu/k_B T} - 1} \propto T^3 \int_0^\infty \frac{x^2 dx}{e^x - 1} \Rightarrow C_V = \frac{\partial E}{\partial T} \propto T^2.$$

Answer: (B) $C_V \propto T^2$

Q19. State of Polarisation

[MCQ, 1 mark]

Optics | Polarisation

For the electric field of an electromagnetic wave given below, which of the following statements is correct?

$$\vec{E} = \hat{x} E_0 \cos(\omega t) + \hat{y} 2E_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$

- (A) The electric field is linearly polarised with slope 2.
- (B) The electric field is circularly polarised with radius E_0 .
- (C) The electric field is elliptically polarised with a ratio of major to minor axis being 2.
- (D) The electric field is unpolarised with the two components being phase shifted by $\pi/2$.

Solution

$E_x = E_0 \cos \omega t$, $E_y = -2E_0 \sin \omega t$, so

$$\frac{E_x^2}{E_0^2} + \frac{E_y^2}{(2E_0)^2} = 1.$$

The tip traces an ellipse with semi-axes $2E_0$ (along y) and E_0 (along x): elliptical polarisation with axis ratio 2.

Answer: (C) elliptical, ratio 2

Q20. Chemical Potential of Trapped Bosons at $T = 0$

[MCQ, 1 mark]

Statistical Mechanics | Bose-Einstein condensation

A gas of non-interacting ^4He atoms (of mass m) is in a three-dimensional trap whose energy levels can be approximated by those of a harmonic oscillator potential $V(x, y, z) = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$. The chemical potential of the gas at $T = 0$ K is

- (A) 0
- (B) $\frac{1}{2}\hbar\omega$
- (C) $\frac{3}{2}\hbar\omega$
- (D) $3\hbar\omega$

Solution

^4He atoms are bosons. At $T = 0$ all atoms occupy the single-particle ground state $\epsilon_0 = \frac{3}{2}\hbar\omega$ (zero-point energy of the 3-D oscillator). The Bose occupation $n_0 = 1/(e^{(\epsilon_0 - \mu)/k_B T} - 1)$ can be macroscopic only if $\mu \rightarrow \epsilon_0^-$. Equivalently, adding one more atom at $T = 0$ costs energy ϵ_0 . Hence $\mu = \frac{3}{2}\hbar\omega$.

Answer: (C) $\frac{3}{2}\hbar\omega$

Q21. Tensor Product of Two Qubits

[MCQ, 1 mark]

Quantum Mechanics | Tensor products

Given $|v_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$ and $|v_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$, the tensor product $|v_1\rangle \otimes |v_2\rangle$ is

(A) $\frac{1}{2} \begin{pmatrix} 1 \\ -i \\ i \\ 1 \end{pmatrix}$

(B) $\frac{1}{2} \begin{pmatrix} 1 \\ i \\ -i \\ 1 \end{pmatrix}$

(C) $\frac{1}{2} \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}$

(D) $\frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$

Solution

$$|v_1\rangle \otimes |v_2\rangle = \frac{1}{2} \begin{pmatrix} 1 \cdot (1, -i) \\ i \cdot (1, -i) \end{pmatrix} = \frac{1}{2} (1, -i, i, -i^2)^T = \frac{1}{2} (1, -i, i, 1)^T.$$

(C) and (D) are 2×2 matrices (outer-product type), not a tensor-product vector in $\mathbb{C}^4$.

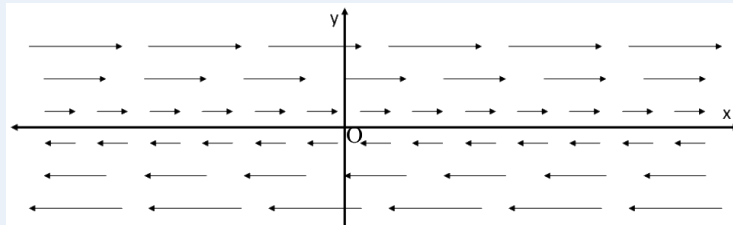
Answer: (A) $\frac{1}{2} (1, -i, i, 1)^T$

Q22. Curl of a Shear Field

[MCQ, 1 mark]

Mathematical Physics | Vector calculus

Sketch of a two-dimensional vector field $\vec{V}$ is shown below. Here, length and arrow head of the arrows denote magnitude and direction of the vector field, respectively. Which of the following statements is correct for $\vec{\nabla} \times \vec{V}$?



- (A) It is zero everywhere in the two-dimensional space.
- (B) Its magnitude is non-zero and its direction is out of the two-dimensional plane.
- (C) Its magnitude is non-zero and its direction is into the two-dimensional plane.
- (D) It points in opposite directions above and below the x-axis.

Solution

The field is a shear flow $\vec{V} = V_x(y) \hat{x}$: arrows point along $+x$ above the x -axis and along $-x$ below it, so V_x increases with y ($\partial V_x / \partial y > 0$). Then

$$\vec{\nabla} \times \vec{V} = \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \hat{z} = -\frac{\partial V_x}{\partial y} \hat{z},$$

which is non-zero and along $-\hat{z}$ (into the plane) on both sides of the axis. A small paddle wheel placed at O would turn clockwise.

Answer: (C) non-zero, into the plane

Q23. An Allowed Process

[MCQ, 1 mark]

Particle Physics | *Conservation laws*

Which one of the following is an allowed process?

(A) $\pi^- + p \rightarrow \pi^0 + n$

(B) $\pi^0 \rightarrow \gamma + \gamma + \gamma$

(C) $p + \bar{p} \rightarrow \Lambda^0 + \Lambda^0$

(D) $p + \bar{p} \rightarrow \gamma$

Solution

- (A) Charge $0 \rightarrow 0$, baryon number $1 \rightarrow 1$, strangeness $0 \rightarrow 0$, isospin conserved: an allowed strong charge-exchange reaction.
- (B) $C(\pi^0) = +1$, but $C(3\gamma) = (-1)^3 = -1$: forbidden by charge conjugation.
- (C) Baryon number $0 \rightarrow 2$ (and strangeness $0 \rightarrow -2$ in a strong process): forbidden.
- (D) A single photon cannot conserve energy and momentum in the CM frame: forbidden.

Answer: (A) $\pi^- + p \rightarrow \pi^0 + n$

Q24. Particles with $Q - S = 0$

[MSQ, 1 mark]

Particle Physics | *Quantum numbers*

Given Q is the electromagnetic charge and S is the strangeness quantum number, identify the particle(s) for which $(Q - S) = 0$ is satisfied.

(A) Σ^{*-}

(B) K^+

(C) Ω^-

(D) Δ^{++}

Solution

Particle (quarks)	Q	S	$Q - S$
$\Sigma^{*-} (dds)$	-1	-1	0 ✓
$K^+ (u\bar{s})$	+1	+1	0 ✓
$\Omega^- (sss)$	-1	-3	2
$\Delta^{++} (uuu)$	+2	0	2

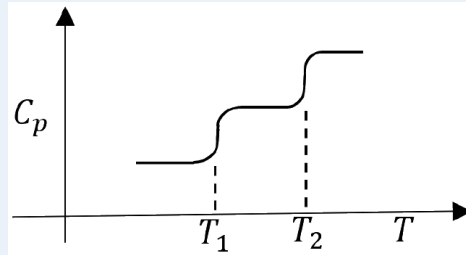
Answer: (A), (B)

Q25. Specific Heat of a Diatomic Gas

[MSQ, 1 mark]

Statistical Mechanics | Equipartition, freezing of modes

Schematic variation of the specific heat C_p of an ideal gas of diatomic molecules with temperature T is shown in the figure below. For rotational energy E_R and vibrational energy E_v of the molecule, which of the following options is/are correct? Here k_B is the Boltzmann constant.



(A) $E_R \cong k_B T_1$

(B) $E_R \cong k_B T_2$

(C) $E_v \cong k_B T_1$

(D) $E_v \cong k_B T_2$

Solution

A degree of freedom starts contributing to C_p once $k_B T$ becomes comparable to its level spacing. Rotational quanta are much smaller than vibrational quanta ($E_R \ll E_v$). So the first step (at T_1 , $C_p : \frac{5}{2}R \rightarrow \frac{7}{2}R$) is the onset of rotation, $E_R \sim k_B T_1$, and the second step (at T_2 , $\rightarrow \frac{9}{2}R$) is the onset of vibration, $E_v \sim k_B T_2$.

Answer: (A), (D)

Q26. Selection Rules for an x^3 Perturbation

[MSQ, 1 mark]

Quantum Mechanics | Ladder operators

If the perturbation $V = \lambda x^3$ is added to the Hamiltonian of a one-dimensional harmonic oscillator, the matrix element $\langle m|V|0\rangle$ is/are non-zero for which of the following states? Here, the eigenstates of the harmonic oscillator are denoted by $|n\rangle$.

(A) $|m = 3\rangle$

(B) $|m = 1\rangle$

(C) $|m = 2\rangle$

(D) $|m = 5\rangle$

Solution

$x \propto (a + a^\dagger)$, so $x^3|0\rangle$ is a combination of $|1\rangle$ and $|3\rangle$ only:

$$(a + a^\dagger)^3|0\rangle = \sqrt{6}|3\rangle + 3|1\rangle.$$

Parity rules out even m , and three ladder steps cannot reach $m = 5$.

Answer: (A), (B)

Q27. Harmonic Functions

[MSQ, 1 mark]

Mathematical Physics | Laplace equation

For which of the following functions does the Laplacian vanish?

(A) $xe^y - ye^x$

(B) $x \cos(y) - y \cos(x)$

(C) e^{x+iy}

(D) $yx^2 - \frac{y^3}{3} - xy$

Solution

- (A) $\nabla^2 = xe^y - ye^x \neq 0$.
- (B) $\nabla^2 = -x \cos y + y \cos x \neq 0$.
- (C) $e^{x+iy} = e^z$ is analytic, so its real and imaginary parts are harmonic: $\nabla^2 = e^z(1+i^2) = 0$ ✓.
- (D) $\partial_x^2(yx^2) = 2y$, $\partial_y^2(-y^3/3) = -2y$, and xy is harmonic: $\nabla^2 = 0$ ✓.

Answer: (C), (D)

Q28. Hamilton's Action for a Projectile

[NAT, 1 mark]

Classical Mechanics | Action principle

A projectile of mass m is launched from the ground with the initial speed v_0 at an angle 30° from the horizontal. Take the ground to be horizontal. Ignoring the drag, the magnitude of Hamilton's action $\int L dt$ for the particle from the beginning till it hits the ground is $f \times \left(\frac{mv_0^3}{g}\right)$. The value of f (rounded off to two decimal places) is _____

Solution

$v_x = \frac{\sqrt{3}}{2}v_0$, $v_y = \frac{1}{2}v_0 - gt$, $y = \frac{1}{2}v_0 t - \frac{1}{2}gt^2$; time of flight $\tau = v_0/g$. Use units $m = v_0 = g = 1$ (so $\tau = 1$):

$$\int_0^1 \frac{1}{2}(v_x^2 + v_y^2) dt = \frac{1}{2} \cdot \frac{3}{4} + \frac{1}{2} \int_0^1 \left(\frac{1}{2} - t\right)^2 dt = \frac{3}{8} + \frac{1}{24} = \frac{5}{12},$$

$$\int_0^1 y dt = \int_0^1 \left(\frac{t}{2} - \frac{t^2}{2}\right) dt = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}.$$

$$S = \int (T - V) dt = \frac{5}{12} - \frac{1}{12} = \frac{1}{3}, \text{ so } f = 0.33.$$

Answer: $f = \frac{1}{3} \approx 0.33$ (key: 0.32–0.34)

Q29. Most Probable Radius in ψ_{211}

[NAT, 1 mark]

Quantum Mechanics | Hydrogen atom

Consider an electron in the energy eigenstate $\psi_{211}(\vec{r})$ of the hydrogen atom. Given that the radial probability distribution of the electron in such a state takes its maximum value at $r = n_0 a$, where a is the Bohr radius, and n_0 is an integer. The value of n_0 (in integer) is _____

The radial part of the wavefunction $\psi_{211}(\vec{r})$ is given by $R_{21}(r) = \frac{1}{\sqrt{24}a^{5/2}} r e^{-r/2a}$.

Solution

$P(r) = r^2 |R_{21}|^2 \propto r^4 e^{-r/a}$. Setting $\frac{dP}{dr} = \left(4r^3 - \frac{r^4}{a}\right) e^{-r/a} = 0$ gives $r = 4a$ (the Bohr-orbit radius $n^2 a$ for $n = 2$).

Answer: $n_0 = 4$

Q30. Field inside a Uniformly Polarised Sphere

[NAT, 1 mark]

Electrostatics in Matter | Polarisation

A dielectric sphere carries a uniform polarization $P = 26 \mu\text{C}\cdot\text{cm}^{-2}$. The magnitude of the electric field at the center of the sphere is $E \times 10^9 \text{ N}\cdot\text{C}^{-1}$. The value of E (rounded off to one decimal place) is _____
($\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2\cdot\text{N}^{-1}\cdot\text{m}^{-2}$)

Solution

Inside a uniformly polarised sphere the field is uniform: $\vec{E} = -\frac{\vec{P}}{3\epsilon_0}$. $P = 26 \times 10^{-6} \text{ C}/10^{-4} \text{ m}^2 = 0.26 \text{ C}\cdot\text{m}^{-2}$, so

$$E = \frac{0.26}{3 \times 8.85 \times 10^{-12}} = 9.79 \times 10^9 \text{ N}\cdot\text{C}^{-1}.$$

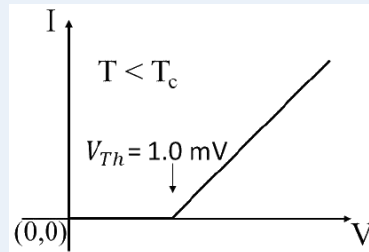
Answer: $E \approx 9.8$ (key: 9.0–10.0)

Q31. Superconducting Gap from Tunnelling

[NAT, 1 mark]

Solid State Physics | Superconductivity, NIS tunnelling

Consider a metal-superconductor junction connected to a dc voltage V . At $T < T_c$, where T_c is the superconductor's transition temperature, the current I versus V behavior of this junction is shown schematically in the figure below. If the superconducting energy gap is D meV. The value of D (rounded off to one decimal place) is _____



Solution

In a normal-metal/insulator/superconductor junction, current starts to flow when eV reaches the edge of the quasiparticle spectrum, which lies Δ above the Fermi level. So $eV_{Th} = \Delta \Rightarrow \Delta = 1.0$ meV.

The superconducting *energy gap* is the full gap between the occupied and empty quasiparticle bands, $E_g = 2\Delta = 2.0$ meV.

Answer: $D = 2.0$ meV

Insight

A threshold of $V_{Th} = \Delta/e$ measures the *half-gap* Δ . In a superconductor–insulator–superconductor junction the threshold would be $2\Delta/e$.

Q32. Effective Mass in a Tight-Binding Band

[NAT, 1 mark]

Solid State Physics | Band structure

For the energy dispersion of an electron in a one-dimensional solid $E(k) = E_0 - 2\gamma \cos(ka)$, the ratio of the effective mass of the electron in the solid to the free electron mass (m_e) at $k = 0$ is R_0 . Taking $\gamma = 0.5$ eV and $a = 0.5$ nm, the value of R_0 (rounded off to two decimal place) is _____

($\hbar = 1.054 \times 10^{-34}$ J.s, $m_e = 9.1 \times 10^{-31}$ kg, electron charge = 1.6×10^{-19} C)

Solution

$$\frac{d^2 E}{dk^2} = 2\gamma a^2 \cos(ka) \Big|_{k=0} = 2\gamma a^2, \text{ so } m^* = \frac{\hbar^2}{2\gamma a^2}.$$

$$2\gamma a^2 = 2(0.5 \times 10^{-19})(0.5 \times 10^{-9})^2 = 2.5 \times 10^{-38} \text{ J.m}^2,$$

$$m^* = \frac{(1.054 \times 10^{-34})^2}{2.5 \times 10^{-38}} = 4.5 \times 10^{-31} \text{ kg}.$$

$$R_0 = m^*/m_e = 0.5, \text{ i.e. } 0.5 \text{ to two decimal places.}$$

Answer: $R_0 \approx 0.5$ (key: 0.50–0.52)

Q33. Entropy from Specific Heat

[NAT, 1 mark]

Thermodynamics | Third law, low- T specific heat

The specific heat $C_p(T)$ of one mole of a material as a function of temperature T is given as $C_p(T) = AT + BT^3$, where $A = 0.695 \text{ mJ.mol}^{-1}.\text{K}^{-2}$ and $B = 0.045 \text{ mJ.mol}^{-1}.\text{K}^{-4}$. When T is changed from 1 K to 10 K at constant pressure, then the change in entropy ΔS in $\text{mJ.mol}^{-1}.\text{K}^{-1}$ (rounded off to one decimal place) is _____

Solution

$$\Delta S = \int_1^{10} \frac{C_p}{T} dT = A(10 - 1) + \frac{B}{3}(10^3 - 1^3) = 0.695(9) + 0.015(999) = 6.255 + 14.985 = 21.24.$$

Answer: $\Delta S \approx 21.2 \text{ mJ.mol}^{-1}.\text{K}^{-1}$ (key: 21.1–21.3)

Q34. Anti-Stokes Raman Line

[NAT, 1 mark]

Atomic & Molecular Physics | Raman spectroscopy

Raman spectrum of a molecule was recorded using a source of wavelength $5000 \text{ \AA}$. The first Stokes line is observed at $5100 \text{ \AA}$. The first anti-Stokes line will appear at a wavelength L (in $\text{\AA}$). The value of L (rounded off to nearest integer) is _____

Solution

Stokes and anti-Stokes lines are symmetric in *wavenumber*: $\bar{\nu}_{AS} = 2\bar{\nu}_0 - \bar{\nu}_S$.

$$\frac{1}{L} = \frac{2}{5000} - \frac{1}{5100} = 2.0392 \times 10^{-4} \text{ \AA}^{-1} \Rightarrow L = 4903.8 \text{ \AA} \approx 4904 \text{ \AA}.$$

(Symmetric *wavelength* spacing would give $4900 \text{ \AA}$, which is outside the key range.)

Answer: $L \approx 4904 \text{ \AA}$ (key: 4901–4905)

Q35. Length Contraction

[NAT, 1 mark]

Special Relativity | Lorentz contraction

A rocket of length 18.0 m is moving at speed $0.9c$ (where c is the speed of light) parallel to its own length, relative to the earth. The length of the rocket measured in meters by an observer on earth (rounded off to two decimal places) is _____

Solution

$$L = L_0 \sqrt{1 - v^2/c^2} = 18.0 \sqrt{1 - 0.81} = 18.0 \times 0.4359 = 7.85 \text{ m}.$$

Answer: $L \approx 7.85 \text{ m}$ (key: 7.60–8.00)

Physics (PH)

Q36–Q65: 2 marks each

Q36. Singular Points of a Rational Function

[MCQ, 2 marks]

Mathematical Physics | Complex analysis

The function $f(z)$ of complex variable z given below,

$$f(z) = \frac{z^2 - 5z + 4}{z^3 + 4z - z^2 - 4}$$

has singular points at $z =$

- (A) 1 and $(2 - i)$ (B) $2i$ and $-2i$
(C) 1 and $(2 + i)$ (D) $(2 + i)$ only

Solution

Factorise: numerator $z^2 - 5z + 4 = (z - 1)(z - 4)$; denominator $z^3 - z^2 + 4z - 4 = z^2(z - 1) + 4(z - 1) = (z - 1)(z^2 + 4)$. Hence

$$f(z) = \frac{z - 4}{z^2 + 4} \quad (z \neq 1).$$

The point $z = 1$ is removable, and the genuine singularities (simple poles) are at $z = \pm 2i$.

Answer: (B) $z = 2i, -2i$

Q37. Trace with a Pauli Commutator

[MCQ, 2 marks]

Mathematical Physics | Pauli matrices

Consider the Pauli matrices $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

The value of $\text{Tr}(\sigma_z [\sigma_x, \sigma_y])$ is

- (A) $2i$ (B) i
(C) $4i$ (D) $\frac{i}{2}$

Solution

$[\sigma_x, \sigma_y] = 2i\sigma_z$, so $\text{Tr}(\sigma_z \cdot 2i\sigma_z) = 2i \text{Tr}(\mathbb{I}_2) = 4i$.

Answer: (C) $4i$

Q38. Which Decay Statement is True?

[MCQ, 2 marks]

Particle Physics | Weak decays, conservation laws

Which one of the following statements is true?

- (A) In the decay $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$, CPT is violated.
- (B) The decay $\Lambda \rightarrow p^+ + \pi^-$ is allowed and strangeness is violated.
- (C) The decay $p^+ \rightarrow e^+ + \gamma$ is allowed.
- (D) The decay $\Omega^- \rightarrow \Xi^0 + K^-$ is allowed.

Solution

- (A) CPT is an exact symmetry of local Lorentz-invariant quantum field theory; muon decay does not violate it.
- (B) $\Lambda(uds) \rightarrow p\pi^-$ is the dominant ($\approx 64\%$) weak decay of the Λ ; $\Delta S = +1$, which the weak interaction permits. ✓
- (C) Violates baryon number (and lepton number): forbidden; the proton is stable.
- (D) $m_{\Xi^0} + m_{K^-} \approx 1315 + 494 = 1809 \text{ MeV} > m_{\Omega^-} = 1672 \text{ MeV}$: energetically forbidden.

Answer: (B)

Q39. Coupled Oscillators by Rotation

[MCQ, 2 marks]

Quantum Mechanics | Normal modes

The Hamiltonian for a quantum particle of mass m is given below, where $\omega < \Omega$. The Schrödinger equation for this system can be solved exactly using the orthogonal transformations: $x = \frac{x_1 - x_2}{\sqrt{2}}$ and $y = \frac{x_1 + x_2}{\sqrt{2}}$.

$$H = -\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] + \frac{1}{2}m\Omega^2(x^2 + y^2) + m\omega^2 xy$$

The ground state energy of this system is

- (A) $\frac{\hbar}{2} \left[\sqrt{\Omega^2 - \omega^2} + \sqrt{\Omega^2 + \omega^2} \right]$
- (B) $\hbar \left[\sqrt{\Omega^2 - \omega^2} + \sqrt{\Omega^2 + \omega^2} \right]$
- (C) $\frac{\hbar}{2} \left[\sqrt{\Omega^2 + \Omega\omega} + \sqrt{\Omega^2 - \Omega\omega} \right]$
- (D) $\hbar \left[\sqrt{\Omega^2 + \Omega\omega} - \sqrt{\Omega^2 - \Omega\omega} \right]$

Solution

The rotation is orthogonal, so the kinetic term keeps its form and $x^2 + y^2 = x_1^2 + x_2^2$. Also

$$xy = \frac{(x_1 - x_2)(x_1 + x_2)}{2} = \frac{x_1^2 - x_2^2}{2}.$$

Therefore

$$V = \frac{1}{2}m(\Omega^2 + \omega^2)x_1^2 + \frac{1}{2}m(\Omega^2 - \omega^2)x_2^2.$$

These are two independent oscillators with frequencies $\omega_{\pm} = \sqrt{\Omega^2 \pm \omega^2}$ (both real since $\omega < \Omega$). The ground-state energy is the sum of the zero-point energies:

$$E_0 = \frac{\hbar}{2} \left[\sqrt{\Omega^2 - \omega^2} + \sqrt{\Omega^2 + \omega^2} \right].$$

Answer: (A)**Q40. Clebsch–Gordan Coefficients**

[MCQ, 2 marks]

Quantum Mechanics | *Addition of angular momenta*

Consider two particles with angular momenta $j_1 = 2\hbar$ and $j_2 = \hbar/2$. If the expression

$$|j = 5/2, m = 3/2\rangle = \begin{cases} c_1 |j_1 = 2, m_1 = 1\rangle |j_2 = 1/2, m_2 = 1/2\rangle + \\ c_2 |j_1 = 2, m_1 = 2\rangle |j_2 = 1/2, m_2 = -1/2\rangle \end{cases}$$

gives an eigenstate of the total angular momentum of the two particles, using standard notation. Which of the following is true?

(Hint: $\hat{J}_\pm |j, m\rangle = \sqrt{j(j+1) - m(m \pm 1)} |j, m \pm 1\rangle$)

(A) $c_1 = \frac{2}{\sqrt{5}}, \quad c_2 = \frac{1}{\sqrt{5}}$

(B) $c_1 = \frac{1}{\sqrt{5}}, \quad c_2 = \frac{2}{\sqrt{5}}$

(C) $c_1 = \frac{1}{\sqrt{2}}, \quad c_2 = \frac{1}{\sqrt{2}}$

(D) $c_1 = 0, \quad c_2 = 1$

Solution

Start from the stretched state $|\frac{5}{2}, \frac{5}{2}\rangle = |2, 2\rangle |\frac{1}{2}, \frac{1}{2}\rangle$ and apply $\hat{J}_- = \hat{J}_{1-} + \hat{J}_{2-}$:

• LHS: $\sqrt{\frac{5}{2} \cdot \frac{7}{2} - \frac{5}{2} \cdot \frac{3}{2}} |\frac{5}{2}, \frac{3}{2}\rangle = \sqrt{5} |\frac{5}{2}, \frac{3}{2}\rangle$.

• RHS: $\sqrt{6-2} |2, 1\rangle |\frac{1}{2}, \frac{1}{2}\rangle + \sqrt{\frac{3}{4} + \frac{1}{4}} |2, 2\rangle |\frac{1}{2}, -\frac{1}{2}\rangle = 2 |2, 1\rangle |\frac{1}{2}, \frac{1}{2}\rangle + |2, 2\rangle |\frac{1}{2}, -\frac{1}{2}\rangle$.

Hence $c_1 = \frac{2}{\sqrt{5}}$ and $c_2 = \frac{1}{\sqrt{5}}$ (normalised: $\frac{4}{5} + \frac{1}{5} = 1$).

Answer: (A) $c_1 = 2/\sqrt{5}, \quad c_2 = 1/\sqrt{5}$ **Q41. 3-D Isotropic Oscillator**

[MCQ, 2 marks]

Quantum Mechanics | *Degeneracy*

The energy E and degeneracy d of the second excited state of a three-dimensional, isotropic quantum harmonic oscillator with angular frequency ω are

(A) $E = \frac{7}{2}\hbar\omega, \quad d = 6$

(B) $E = \frac{7}{2}\hbar\omega, \quad d = 3$

(C) $E = \frac{5}{2}\hbar\omega, \quad d = 3$

(D) $E = \frac{5}{2}\hbar\omega, \quad d = 6$

Solution

$E_N = (N + \frac{3}{2})\hbar\omega$ with $N = n_x + n_y + n_z$ and $d_N = \frac{1}{2}(N+1)(N+2)$. For the second excited state $N = 2$: $E = \frac{7}{2}\hbar\omega, d = 6$ (the states 200, 020, 002, 110, 101, 011).

Answer: (A) $E = \frac{7}{2}\hbar\omega, \quad d = 6$

Q42. Entropy of Two Fermions vs. Two Bosons

[MCQ, 2 marks]

Statistical Mechanics | Microcanonical counting

Two identical particles with a fixed total energy $E = 2\hbar\omega$ are in thermal equilibrium in a one-dimensional harmonic oscillator potential $\frac{1}{2}m\omega^2x^2$. Let the entropy of the particles be denoted by S_F if they are fermions with spin $\frac{1}{2}$ ($s_z = \pm\frac{\hbar}{2}$) and S_B if they are bosons with spin 0. Then, which of the following options is correct? (k_B is the Boltzmann constant)

- (A) $S_F = k_B \ln 2$, $S_B = k_B \ln 2$ (B) $S_F = 2k_B \ln 2$, $S_B = 0$
 (C) $S_F = 4k_B \ln 2$, $S_B = 0$ (D) $S_F = 2k_B \ln 2$, $S_B = k_B \ln 2$

Solution

$(n_1 + \frac{1}{2}) + (n_2 + \frac{1}{2}) = 2 \Rightarrow n_1 + n_2 = 1$: one particle in $n = 0$ and one in $n = 1$.

- Spin-0 bosons: the occupation $\{n = 0, n = 1\}$ is a single symmetric state, so $\Omega_B = 1$ and $S_B = 0$.
- Spin- $\frac{1}{2}$ fermions: the particles are in different orbitals, so each spin is free: $2 \times 2 = 4$ antisymmetrised states. $\Omega_F = 4$ and $S_F = k_B \ln 4 = 2k_B \ln 2$.

Answer: (B) $S_F = 2k_B \ln 2$, $S_B = 0$

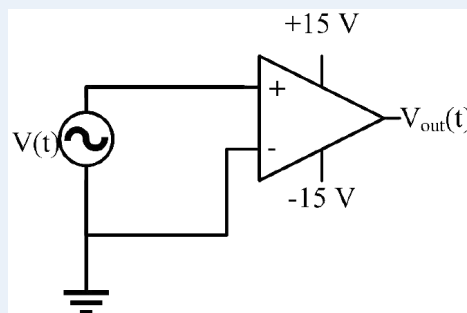
Q43. Op-Amp without Feedback

[MCQ, 2 marks]

Electronics | Comparator

In the circuit shown, $V(t) = 2\sin(2000\pi t)$ Volts, where t is in seconds, which of the following options is correct?

(take the opamp to be ideal)



- (A) $V_{out}(t)$ is square wave with peak-to-peak voltage = 30 V and time period is 1 ms.
 (B) $V_{out}(t)$ is a sine wave with peak-to-peak voltage = 4 V and time period is 1 ms.
 (C) $V_{out}(t)$ is sine wave with peak-to-peak voltage = 30 V and time period of 1 ms.
 (D) $V_{out}(t)$ is square wave with peak-to-peak voltage = 4 V and time period is 1 ms.

Solution

There is no feedback, so the ideal (infinite-gain) op-amp acts as a zero-crossing comparator. $V(t)$ is applied between the inputs, so $V_{out} = +15$ V when $V > 0$ and -15 V when $V < 0$. The output is a square wave of peak-to-peak 30 V with the same period as the input: $T = \frac{2\pi}{2000\pi} = 1$ ms.

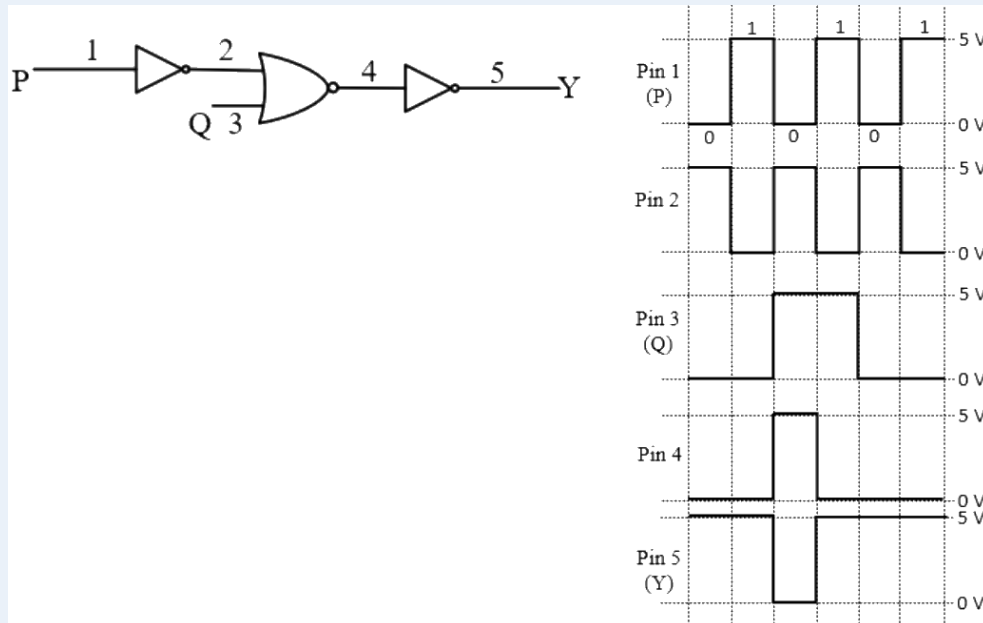
Answer: (A) square, 30 V p-p, 1 ms

Q44. Fault-Finding from Pin Waveforms

[MCQ, 2 marks]

Electronics | Digital logic

Considering the circuit and the associated signals measured at different pins (numbered as 1, 2, 3, 4, 5) shown in the figure, the correct option is



- (A) NOT gate between pins 1 and 2 is faulty.
- (B) NOR gate is faulty.
- (C) NOT gate between pins 4 and 5 is faulty.
- (D) The NOR and output NOT gates, are both faulty.

Solution

Check each gate against its own measured inputs and output:

- **NOT (1→2):** pin 2 is the exact complement of pin 1 in every time slot. Working.
- **NOR (2, 3→4):** pin 4 must be 0 whenever Q (pin 3) is high. But pin 4 is *high* in the slot where pin 2 = 1 and pin 3 = 1. It must also be high when pin 2 = pin 3 = 0, yet it stays low there. Pin 4 actually behaves like pin 2 · pin 3 (AND). **Faulty.**
- **NOT (4→5):** pin 5 is the exact complement of pin 4. Working.

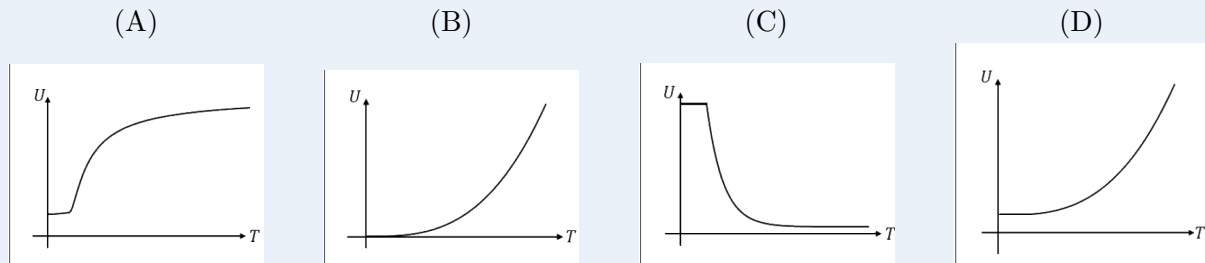
Answer: (B) NOR gate is faulty

Q45. Internal Energy of a Two-Level System

[MCQ, 2 marks]

Statistical Mechanics | Two-level systems

A gas of N classical particles that can occupy energy levels, ϵ_1 and $\epsilon_2 = \epsilon_1 + \Delta$ is in equilibrium with a reservoir at temperature T . From the schematics shown below, choose the correct dependence of the internal energy U on T .



Solution

$$U = N \frac{\epsilon_1 + (\epsilon_1 + \Delta)e^{-\Delta/k_B T}}{1 + e^{-\Delta/k_B T}} = N\epsilon_1 + \frac{N\Delta}{e^{\Delta/k_B T} + 1}.$$

- $T \rightarrow 0$: $U \rightarrow N\epsilon_1$, approached with an exponentially flat start (all derivatives vanish).
 - $T \rightarrow \infty$: $U \rightarrow N\epsilon_1 + \frac{1}{2}N\Delta$, a finite saturation value (the levels become equally populated).
- U rises monotonically from a flat low- T plateau to a saturated high- T plateau. Only curve (A) has this shape; (B) and (D) grow without bound, and (C) decreases.

Answer: (A)

Q46. Adding a Total Time Derivative

[MCQ, 2 marks]

Classical Mechanics | Gauge transformations of L

The Lagrangian

$$L_0 = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega^2 q^2$$

with the generalized coordinate q is transformed to $L = L_0 + \alpha \frac{df(q)}{dt}$. Consider the following statements:

- Expression for the canonical momentum does not change.
- The equation of the motion does not change.

Which of the following options is correct for the above statements?

- Both (i) and (ii) are correct.
- Both (i) and (ii) are not correct.
- (i) is correct and (ii) is not correct.
- (i) is not correct and (ii) is correct.

Solution

$$\alpha \frac{df}{dt} = \alpha f'(q)\dot{q}, \text{ so}$$

$$p = \frac{\partial L}{\partial \dot{q}} = m\dot{q} + \alpha f'(q) \neq m\dot{q} \quad \Rightarrow \quad \text{(i) is false.}$$

A total time derivative does not change the action's variation: the extra Euler–Lagrange terms $\frac{d}{dt}(\alpha f') - \alpha f''\dot{q} = 0$ cancel identically. The equation of motion $m\ddot{q} = -m\omega^2 q$ is unchanged, so (ii) is true.

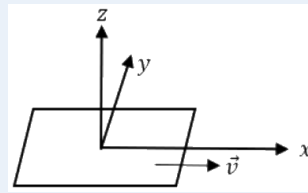
Answer: (D)

Q47. Poynting Vector of a Moving Charged Sheet

[MCQ, 2 marks]

Electrodynamics | Surface currents, energy flow

An infinitely large thin sheet in the xy -plane carries uniform positive charge density and is moving with constant velocity $\vec{v}$ in the $+x$ direction (see figure below). The direction of the corresponding Poynting vector is



- (A) $+x$ for both $z < 0$ and $z > 0$
- (B) $+x$ for $z < 0$ and $-x$ for $z > 0$
- (C) $-x$ for $z < 0$ and $+x$ for $z > 0$
- (D) $-x$ for both $z < 0$ and $z > 0$

Solution

$\vec{E} = \pm \frac{\sigma}{2\epsilon_0} \hat{z}$ (away from the sheet). The moving charge is a surface current $\vec{K} = \sigma v \hat{x}$, which gives $\vec{B} = \frac{\mu_0}{2} \vec{K} \times \hat{n}$, where $\hat{n}$ is the normal pointing into the region:

- $z > 0$: $\vec{E} \parallel +\hat{z}$, $\vec{B} \parallel \hat{x} \times \hat{z} = -\hat{y}$, so $\vec{S} \parallel \hat{z} \times (-\hat{y}) = +\hat{x}$.
- $z < 0$: $\vec{E} \parallel -\hat{z}$, $\vec{B} \parallel \hat{x} \times (-\hat{z}) = +\hat{y}$, so $\vec{S} \parallel (-\hat{z}) \times \hat{y} = +\hat{x}$.

Energy is carried along with the moving sheet on both sides.

Answer: (A) $+x$ for both

Q48. Force on a Dipole near a Point Charge

[MCQ, 2 marks]

Electrostatics | Dipole in a non-uniform field

A positive point charge is fixed at the origin. At some distance from it on the x axis, a point dipole is kept pointing in the $+y$ direction. The force on the dipole is

- (A) 0
- (B) in the $+y$ direction
- (C) in the $-y$ direction
- (D) in the $+x$ direction

Solution

$\vec{F} = (\vec{p} \cdot \vec{\nabla}) \vec{E} = p \partial_y \vec{E}$ at $(d, 0, 0)$, with $\vec{E} = kq \frac{(x, y, z)}{r^3}$:

$$\partial_y E_x = -\frac{3kqxy}{r^5} \Big|_{y=0} = 0, \quad \partial_y E_y = kq \left(\frac{1}{r^3} - \frac{3y^2}{r^5} \right) \Big|_{y=0} = \frac{kq}{d^3}.$$

So $\vec{F} = \frac{kqp}{d^3} \hat{y}$, along $+y$. (Consistent with Newton's third law: the dipole's field at the origin is $-\vec{p}/(4\pi\epsilon_0 d^3)$, pushing the charge along $-y$.)

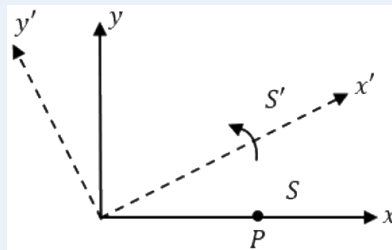
Answer: (B) $+y$

Q49. Fictitious Forces on a Mass at Rest

[MCQ, 2 marks]

Classical Mechanics | Rotating frames

Two frames S (solid lines) and S' (dashed lines) with common origin are shown in the figure below. Frame S is inertial while S' is rotating about the common z -axis. There is a point mass fixed at P on the x -axis of the S frame. The magnitude of the centrifugal force and the Coriolis force experienced by the mass in the S' frame is F_{cen} and F_{cor} , respectively. Which of the following options is correct for these forces?



- (A) $F_{cen} = 0$ and $F_{cor} = 0$
 (B) $F_{cen} \neq 0$ and $F_{cor} \neq 0$ and $F_{cen} = \frac{F_{cor}}{2}$
 (C) $F_{cen} \neq 0$ and $F_{cor} \neq 0$ and $F_{cen} = 2F_{cor}$
 (D) $F_{cen} \neq 0$ and $F_{cor} \neq 0$ and $F_{cen} = F_{cor}$

Solution

In S' the mass moves on a circle with $\vec{v}' = -\vec{\omega} \times \vec{r}$ (speed ωr).

- Centrifugal: $-m\vec{\omega} \times (\vec{\omega} \times \vec{r})$, magnitude $m\omega^2 r$, outward.
- Coriolis: $-2m\vec{\omega} \times \vec{v}' = 2m\vec{\omega} \times (\vec{\omega} \times \vec{r})$, magnitude $2m\omega^2 r$, inward.

The net fictitious force $m\omega^2 r$ (inward) is exactly the centripetal force needed for the circular motion seen in S' . Thus $F_{cen} = F_{cor}/2$.

Answer: (B) $F_{cen} = F_{cor}/2$

Q50. Self-Adjoint Differential Operators

[MSQ, 2 marks]

Mathematical Physics | Sturm–Liouville theory

Which of the following operators is/are self-adjoint?

(A) $x^2 \frac{d^2}{dx^2} + 3x \frac{d}{dx} + x^2$

(B) $(1 - x^2) \frac{d^2}{dx^2} - 2x \frac{d}{dx} + 3x$

(C) $(3x - 4x^3) \frac{d^2}{dx^2} + (3 - 12x^2) \frac{d}{dx} + 12$

(D) $x \frac{d^2}{dx^2} + x^2 \frac{d}{dx} + \frac{5x}{3}$

Solution

$\mathcal{L} = p_0 \frac{d^2}{dx^2} + p_1 \frac{d}{dx} + p_2$ (real coefficients) is formally self-adjoint iff $p'_0 = p_1$, i.e. $\mathcal{L} = \frac{d}{dx} \left(p_0 \frac{d}{dx} \right) + p_2$.

- (A) $p'_0 = 2x \neq 3x$. ×
- (B) $p'_0 = -2x = p_1$ ✓ (Legendre-type operator).
- (C) $p'_0 = 3 - 12x^2 = p_1$ ✓
- (D) $p'_0 = 1 \neq x^2$. ×

Answer: (B), (C)

Q51. Exponential of an Operator

[MSQ, 2 marks]

Quantum Mechanics | Unitary and Hermitian operators

Consider two operators $\hat{A}$ and $\hat{B}$ which are related as $\hat{A} = \exp(i\theta\hat{B})$. If θ is a non-zero real number, which of the following statements is/are true?

- (A) If $\hat{B}$ is Hermitian, then $\hat{A}$ is unitary.
- (B) If $\hat{B}$ is anti-Hermitian, then $\hat{A}$ is unitary.
- (C) If $\hat{B}$ is Hermitian, then $|\text{Det}(\hat{A})| = 1$.
- (D) If $\hat{B}$ is anti-Hermitian, then $\hat{A}$ is Hermitian.

Solution

- (A) $\hat{A}^\dagger = e^{-i\theta\hat{B}^\dagger} = e^{-i\theta\hat{B}} = \hat{A}^{-1}$ ✓
- (C) $\det \hat{A} = e^{i\theta \text{Tr} \hat{B}}$ with $\text{Tr} \hat{B}$ real, so $|\det \hat{A}| = 1$ ✓
- (D) Write $\hat{B} = i\hat{K}$ with $\hat{K}$ Hermitian: $\hat{A} = e^{-\theta\hat{K}}$, and $\hat{A}^\dagger = e^{-\theta\hat{K}} = \hat{A}$ ✓
- (B) The same $\hat{A} = e^{-\theta\hat{K}}$ has real positive eigenvalues $e^{-\theta k} \neq 1$ in general, so it is not unitary. ×

Answer: (A), (C), (D)

Q52. Compatible and Incompatible Observables

[MSQ, 2 marks]

Quantum Mechanics | Commutators, uncertainty

Consider operators $\hat{A}$, $\hat{B}$, and $\hat{C}$ for three observables of a quantum system satisfying $[\hat{A}, \hat{B}] = 0$, $[\hat{B}, \hat{C}] = 0$, and $[\hat{A}, \hat{C}] \neq 0$, with uncertainties ΔA , ΔB , ΔC , respectively. From the options given below, which is/are implied by the commutation relations among $\hat{A}$, $\hat{B}$, and $\hat{C}$?

- (A) $\Delta A \Delta B > 0$ (B) $\Delta A \Delta C > 0$
 (C) $\hat{A}, \hat{B}$ can be simultaneously diagonalized. (D) $\hat{A}, \hat{B}, \hat{C}$ can be simultaneously diagonalized.

Solution

- (C) Commuting Hermitian operators share a complete eigenbasis, so $\hat{A}, \hat{B}$ can be diagonalised together ✓
- (B) $\Delta A \Delta C \geq \frac{1}{2} |\langle [\hat{A}, \hat{C}] \rangle|$. Because $\hat{A}$ and $\hat{C}$ do not commute, they have no complete set of common eigenstates, so A and C cannot both be sharp in general. The commutation relation is what implies this non-zero joint uncertainty ✓ (the intended reading, as in the official key).
- (A) $[\hat{A}, \hat{B}] = 0$ allows $\Delta A = \Delta B = 0$ in a common eigenstate, so $\Delta A \Delta B > 0$ is *not* implied. ×
- (D) Impossible, since $[\hat{A}, \hat{C}] \neq 0$. ×

Answer: (B), (C)

Insight

Strictly, a state with $\langle [\hat{A}, \hat{C}] \rangle = 0$ can make the Robertson bound vanish. The key credits (B) as the statement that non-commuting observables are incompatible (not simultaneously sharp in general).

Q53. Shannon Entropy of Coins and Dice

[MSQ, 2 marks]

Statistical Mechanics | Information entropy

Consider the distribution of outcomes generated by N ($N \gg 1$) independent throws of (i) a coin or (ii) a six-sided dice. A coin (dice) is unbiased if both (all) its sides have equal probability to show up in a throw; it is biased otherwise. For the cases (i) and (ii) above, which of the following statements is/are true?

- (A) The entropy of an unbiased coin is smaller than that of an unbiased dice.
 (B) The entropy of an unbiased coin is greater than that of an unbiased dice.
 (C) The entropy of a biased dice is smaller than that of an unbiased dice.
 (D) The entropy of a biased coin is greater than that of an unbiased coin.

Solution

$S = -N \sum_i p_i \ln p_i$ is maximised by the uniform distribution, with maximum value $N \ln n$ for n outcomes. Unbiased coin: $N \ln 2 < N \ln 6$ (unbiased die), so (A) is true and (B) false. Any bias lowers the entropy below the uniform maximum, so (C) is true and (D) false.

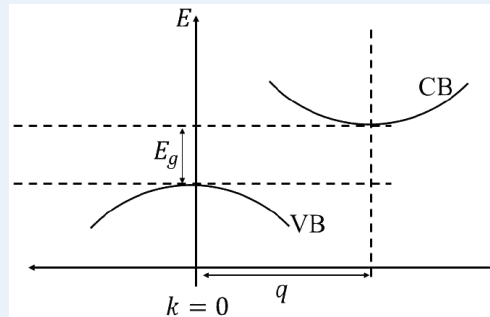
Answer: (A), (C)

Q54. Indirect-Gap Recombination

[MSQ, 2 marks]

Solid State Physics | Semiconductors

The dispersion ($E(k)$) of the conduction band (CB) and valence band (VB) for a semiconductor are shown schematically in the figure. Considering the possibility of an electron making a transition from the bottom of the CB to the top of the VB, which of the following options is/are correct?



- (A) The transition is forbidden.
- (B) A photon can be emitted with an energy exactly equal to E_g .
- (C) A photon can be emitted with an energy less than E_g .
- (D) A phonon can be created with a crystal momentum $\hbar q$.

Solution

The gap is indirect: the CB minimum is at $k = q$ and the VB maximum at $k = 0$. A photon carries negligible momentum, so crystal-momentum conservation requires a phonon of wavevector q to take up $\hbar q$. (D) is true, and the process is allowed (second order), so (A) is false.

Energy conservation for phonon emission: $E_g = \hbar\omega_{\text{photon}} + \hbar\Omega_q$. The emitted photon therefore has energy $E_g - \hbar\Omega_q < E_g$, so (C) is true. A photon of exactly E_g would require a zero-energy phonon of momentum $\hbar q$, which does not exist; (B) is false.

Answer: (C), (D)

Q55. Torque-Free Symmetric Top

[MSQ, 2 marks]

Classical Mechanics | Euler's equations

A symmetric rigid body has moment of inertia I_1, I_2, I_3 about its principal axes 1, 2, and 3, respectively, with $I_1 = I_3 = I_\perp$ and $I_2 \neq I_\perp$. It is rotating in space with no torque on it so that its angular momentum $\vec{L}$ is constant. Let $\omega_1, \omega_2, \omega_3$ be the components of its angular velocity along the principal axes 1, 2, and 3, respectively. Which of the following quantities is/are constant during the motion of this rigid body?

- (A) $\omega_1 + \omega_3$
- (B) $\omega_1^2 + \omega_3^2$
- (C) Angle between axis 2 and $\vec{L}$
- (D) ω_2

Solution

Axis 2 is the symmetry axis. Euler's equations (no torque):

$$I_2 \dot{\omega}_2 = (I_3 - I_1) \omega_3 \omega_1 = 0 \Rightarrow \omega_2 = \text{const (D)};$$

$$\dot{\omega}_1 = \Omega \omega_3, \quad \dot{\omega}_3 = -\Omega \omega_1, \quad \Omega = \frac{(I_2 - I_\perp) \omega_2}{I_\perp}.$$

So (ω_1, ω_3) rotates uniformly: $\omega_1^2 + \omega_3^2 = \text{const}$ (B), while $\omega_1 + \omega_3$ oscillates (A false).
 $L_2 = I_2\omega_2$ and $|\vec{L}|$ are constant, so $\cos \angle(\hat{e}_2, \vec{L}) = L_2/|\vec{L}|$ is constant (C).

Answer: (B), (C), (D)

Q56. Rutherford Distance of Closest Approach

[MSQ, 2 marks]

Classical Mechanics | Central forces, scattering

An α particle moves towards a fixed nucleus carrying charge Ze , with initial speed v_0 and impact parameter b . Starting from a large distance from the nucleus, its distance of closest approach is r_m and its speed there is v_m . Then which of the following options is/are correct?

$$\left(k = \frac{1}{4\pi\epsilon_0} \text{ and } r_0 = k \frac{Ze^2}{mv_0^2} \right)$$

(A) $v_0 b = v_m r_m$

(B) $v_0 b = 2v_m r_0$

(C) For $\frac{b}{r_0} \ll 1$, $r_m = 4r_0 + \frac{b^2}{2r_0}$ ignoring higher order corrections in $\frac{b}{r_0}$

(D) For $\frac{b}{r_0} \ll 1$, $r_m = 4r_0 + \frac{b^2}{8r_0}$ ignoring higher order corrections in $\frac{b}{r_0}$

Solution

Angular momentum (central force): $mv_0 b = mv_m r_m$, and at closest approach the velocity is perpendicular to $\vec{r}$. So (A) is true.

Energy (the α particle has charge $2e$, so $U = \frac{2kZe^2}{r} = \frac{2mv_0^2 r_0}{r}$):

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_m^2 + \frac{2mv_0^2 r_0}{r_m}, \quad v_m = \frac{v_0 b}{r_m} \Rightarrow r_m^2 - 4r_0 r_m - b^2 = 0.$$

$$r_m = 2r_0 + \sqrt{4r_0^2 + b^2} \approx 4r_0 + \frac{b^2}{4r_0} \quad (b \ll r_0).$$

The correction term is $b^2/4r_0$, which matches neither (C) ($b^2/2r_0$) nor (D) ($b^2/8r_0$). (B) would need $r_m = 2r_0$, which is below even the head-on value $4r_0$. Only (A) is correct.

Answer: (A)

Q57. Crossed $\vec{E}$ and $\vec{B}$ Fields

[NAT, 2 marks]

Electrodynamics | Cycloidal motion

Consider a particle of mass $m = 9.0 \times 10^{-5}$ kg and charge $q = 3.0 \times 10^{-4}$ C in a uniform electromagnetic field $\vec{E} = 2\hat{x}$ V.m⁻¹, $\vec{B} = 3\hat{z}$ V.m⁻².s. The particle is released from the coordinates (0, 5 m, 0) at time $t = 0$. Starting from initial speed zero, it comes back to the y -axis for the first time at time t . The value of t in seconds (rounded off to two decimal places) is _____

Solution

Starting from rest in crossed fields, the particle moves on a cycloid:

$$x(t) = \frac{E}{B\omega_c}(1 - \cos \omega_c t), \quad y(t) = 5 - \frac{E}{B\omega_c}(\omega_c t - \sin \omega_c t),$$

drifting along $\vec{E} \times \vec{B} \parallel -\hat{y}$ with cyclotron frequency $\omega_c = \frac{qB}{m} = \frac{(3 \times 10^{-4})(3)}{9 \times 10^{-5}} = 10$ rad/s. It returns to $x = 0$ (the y -axis) when $\omega_c t = 2\pi$:

$$t = \frac{2\pi}{10} = 0.628 \text{ s} \approx 0.63 \text{ s}.$$

Answer: $t \approx 0.63 \text{ s}$ (key: 0.60–0.70)

Q58. Energy from Stimulated Emission

[NAT, 2 marks]

Atomic Physics | Lasers, population inversion

An atom has two energy levels with energy difference 2.2 eV between them. A gas of these atoms is with 8×10^{20} atoms in the upper state and 5×10^{20} atoms in the lower state. Ignoring spontaneous emission, the maximum possible energy released by this gas of atoms by stimulated emission is E Joules. The value of E (rounded off to one decimal place) is _____ ($e = 1.6 \times 10^{-19}$ C)

Solution

Stimulated emission and absorption have equal rates per atom, so net emission continues only while the upper state has more atoms. It stops at equal populations, $N_u = N_l = 6.5 \times 10^{20}$. Net number of photons released $= (8 - 6.5) \times 10^{20} = 1.5 \times 10^{20}$:

$$E = 1.5 \times 10^{20} \times 2.2 \times 1.6 \times 10^{-19} \text{ J} = 52.8 \text{ J}.$$

Answer: $E \approx 52.8 \text{ J}$ (key: 52.0–53.0)

Q59. Singlet–Triplet Splitting

[NAT, 2 marks]

Quantum Mechanics | Spin coupling

The Hamiltonian of two interacting spin-1/2 particles is $H = \frac{A}{\hbar^2} \vec{S}_1 \cdot \vec{S}_2$, where $\vec{S}_1$ and $\vec{S}_2$ are the spin angular momenta of particles 1 and 2, respectively. Here, $A = 10.56$ eV. The energy in eV required to induce an excitation from the ground state to the excited state (rounded off to two decimal places) is _____

Solution

$\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2} (S^2 - S_1^2 - S_2^2)$ gives $+\frac{1}{4}\hbar^2$ for the triplet ($S = 1$) and $-\frac{3}{4}\hbar^2$ for the singlet ($S = 0$). With $A > 0$ the singlet ($-\frac{3}{4}A$) is the ground state and the triplet ($+\frac{1}{4}A$) the excited state. The excitation energy is $\frac{1}{4}A + \frac{3}{4}A = A = 10.56$ eV.

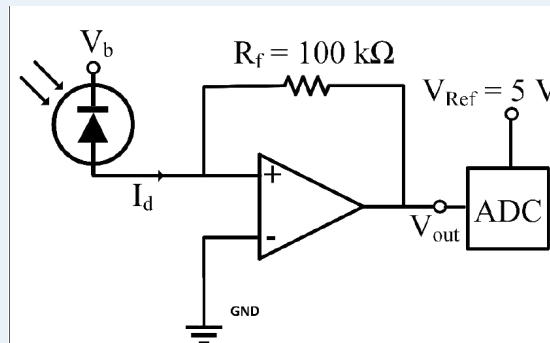
Answer: 10.56 eV

Q60. Smallest Resolvable Photocurrent

[NAT, 2 marks]

Electronics | Transimpedance amplifier, ADC

The output signal (current I_d) of a reversed biased (with a voltage V_b) photodiode, on which light is incident, is fed to an amplifier (see figure). The output voltage is digitized by a 10 bit Analogue to Digital convertor (ADC) which has a reference voltage of 5 V. The smallest current I_d which can be measured by the circuit in nano-Amperes (rounded off to one decimal place) is _____



Solution

The op-amp with feedback resistor R_f is a transimpedance amplifier: $|V_{out}| = I_d R_f$. The ADC resolution is $V_{LSB} = \frac{V_{Ref}}{2^{10}} = \frac{5}{1024} = 4.883$ mV. The smallest measurable current is therefore

$$I_{d,\min} = \frac{V_{LSB}}{R_f} = \frac{4.883 \times 10^{-3}}{10^5} = 48.8 \text{ nA}.$$

(Using $2^{10} - 1$ levels gives 48.9 nA; both lie in the key range.)

Answer: $I_d \approx 48.8$ nA (key: 47.5–50.0)

Q61. Magnetic Field inside a Charging Capacitor

[NAT, 2 marks]

Electrodynamics | Displacement current

A capacitor is made of two circular metal plates of radius 1 m separated by a distance of $d = 1$ mm. The space between them is filled by a dielectric with permittivity $\epsilon_r = 5$. The capacitor is connected to a voltage $V = 10 \sin(2\pi \times 10^6 \times t)$ volts, where t is in seconds. The maximum value of the magnetic field in between the plates at a radial distance $r = 0.5$ m from the centre of the capacitor is $B \times 10^{-6}$ T. The value of B (rounded off to two decimal places) is ____
(Speed of light in vacuum $c = 3 \times 10^8$ m.s⁻¹)

Solution

Ampère–Maxwell on a circle of radius r inside the plates, with $E = V/d$:

$$B(2\pi r) = \mu_0 \epsilon_0 \epsilon_r \pi r^2 \frac{1}{d} \frac{dV}{dt} \Rightarrow B = \frac{\epsilon_r r}{2c^2 d} \frac{dV}{dt}.$$

$$\left(\frac{dV}{dt}\right)_{\max} = 10 \times 2\pi \times 10^6 = 6.283 \times 10^7 \text{ V/s, so}$$

$$B_{\max} = \frac{5 \times 0.5}{2 \times 9 \times 10^{16} \times 10^{-3}} \times 6.283 \times 10^7 = 8.73 \times 10^{-7} \text{ T} = 0.87 \times 10^{-6} \text{ T}.$$

Answer: $B \approx 0.87$ (key: 0.80–0.90)

Q62. Moment of Inertia from a Rotational Spectrum

[NAT, 2 marks]

Molecular Physics | Rigid rotor

Rotational spectrum of a diatomic molecule consists of lines of equal spacing with an interval of 20.0 cm^{-1} . Its moment of inertia is found to be $I_0 \times 10^{-47} \text{ kg.m}^2$, the value of I_0 (rounded off to one decimal place) is ____
($h = 6.6 \times 10^{-34}$ J.s, speed of light in vacuum $c = 3 \times 10^8$ m.s⁻¹)

Solution

Line spacing $= 2\tilde{B} = 20.0 \text{ cm}^{-1}$, so $\tilde{B} = 10.0 \text{ cm}^{-1} = 1000 \text{ m}^{-1}$. With $\tilde{B} = \frac{h}{8\pi^2 c I}$:

$$I = \frac{6.6 \times 10^{-34}}{8\pi^2 (3 \times 10^8)(1000)} = 2.79 \times 10^{-47} \text{ kg.m}^2.$$

Answer: $I_0 \approx 2.8$ (key: 2.6–3.0)

Q63. Fermi Energy of Copper

[NAT, 2 marks]

Solid State Physics | Free-electron model

Copper has an electron number density of $8.3 \times 10^{28} \text{ m}^{-3}$. Its Fermi energy in eV (rounded off to one decimal place) is _____

($\hbar = 1.06 \times 10^{-34} \text{ J.s}$, mass of electron $m_e = 9.10 \times 10^{-31} \text{ kg}$, charge of electron = $1.60 \times 10^{-19} \text{ C}$)

Solution

$$k_F = (3\pi^2 n)^{1/3} = (3\pi^2 \times 8.3 \times 10^{28})^{1/3} = 1.35 \times 10^{10} \text{ m}^{-1}.$$

$$E_F = \frac{\hbar^2 k_F^2}{2m_e} = \frac{(1.06 \times 10^{-34})^2 (1.35 \times 10^{10})^2}{2 \times 9.10 \times 10^{-31}} = 1.12 \times 10^{-18} \text{ J} = 7.0 \text{ eV}.$$

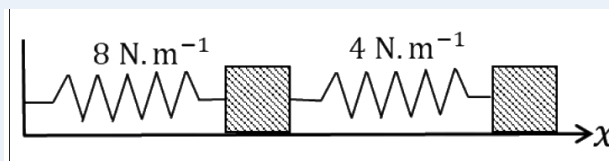
Answer: $E_F \approx 7.0 \text{ eV}$ (key: 6.5–7.5)

Q64. Normal Modes of Two Blocks and Two Springs

[NAT, 2 marks]

Classical Mechanics | Coupled oscillations

Two 1 kg blocks are connected to two massless springs of spring constants 8 N.m^{-1} and 4 N.m^{-1} . The system is kept on a frictionless horizontal floor with one end of a spring attached to a wall (see figure below). They are performing oscillatory motion along the x -axis with the normal mode frequencies ω_H and ω_L ($\omega_H > \omega_L$). The ratio $\frac{\omega_H}{\omega_L}$ (rounded off to two decimal places) is _____



Solution

With $k_1 = 8$ (wall spring), $k_2 = 4$ and $m = 1$:

$$\ddot{x}_1 = -(k_1 + k_2)x_1 + k_2x_2, \quad \ddot{x}_2 = k_2x_1 - k_2x_2 \Rightarrow \begin{vmatrix} 12 - \omega^2 & -4 \\ -4 & 4 - \omega^2 \end{vmatrix} = 0.$$

$\omega^4 - 16\omega^2 + 32 = 0 \Rightarrow \omega^2 = 8 \pm 4\sqrt{2}$. Hence

$$\frac{\omega_H}{\omega_L} = \sqrt{\frac{8 + 4\sqrt{2}}{8 - 4\sqrt{2}}} = \sqrt{\frac{2 + \sqrt{2}}{2 - \sqrt{2}}} = 1 + \sqrt{2} = 2.41.$$

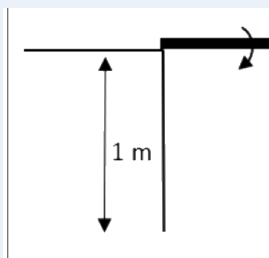
Answer: $\omega_H/\omega_L \approx 2.41$ (key: 2.35–2.45)

Q65. Falling, Rotating Scale

[NAT, 2 marks]

Classical Mechanics | Rigid-body rotation

A 15 cm long scale is held horizontally with one of its ends on the edge of a 1 m high table and the other end resting on one's index finger. As the finger is removed (see figure below), the scale starts rotating about its end on the table. After 0.1 s, during which it has rotated by a negligibly small angle but has gained a rotational speed as it leaves the table and falls vertically towards the ground. When its centre of mass has fallen by 0.5 m, it has rotated by an angle θ . The value of θ in degrees (rounded off to one decimal place) is ____
($g = 9.8 \text{ m.s}^{-2}$)



Solution

Pivoting stage. For a rod pivoted at one end ($I = \frac{1}{3}ML^2$), the torque $Mg\frac{L}{2}$ gives

$$\alpha = \frac{3g}{2L} = \frac{3(9.8)}{2(0.15)} = 98 \text{ rad.s}^{-2}, \quad \omega = \alpha t = 98 \times 0.1 = 9.8 \text{ rad.s}^{-1}.$$

Free fall. After leaving the table there is no torque about the centre of mass, so ω stays constant. The rotation angle during this stage is negligible, so the centre of mass starts falling essentially from rest:

$$t_f = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.5)}{9.8}} = 0.3194 \text{ s}, \quad \theta = \omega t_f = 9.8 \times 0.3194 = 3.130 \text{ rad} = 179.4^\circ.$$

Answer: $\theta \approx 179.4^\circ$ (key: 179.0–180.0)