

Pravegaa Education

CSIR NET-JRF · GATE · IIT-JAM · JEST · TIFR Physics

JEST 2025 — Physics

Complete Solutions — Verified against the Official Key

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Part A: 25 MCQ (1 mark) | Part B: 15 MCQ (3 marks) | Part C: 10 Numerical (3 marks)

Answer Key at a Glance

Part A						Part B	
Q1	(b)	Q10	(d)	Q19	(c)	Q1	(a)
Q2	(c)	Q11	(a)	Q20	(a)	Q2	(d)
Q3	(d)	Q12	(d)	Q21	(a)	Q3	(c)
Q4	(a)	Q13	(c)	Q22	(c)	Q4	(c)
Q5	(a)	Q14	(d)	Q23	(b)	Q5	(d)
Q6	(a)	Q15	(a)	Q24	(b)	Q6	(c)
Q7	(a)	Q16	(a)	Q25	(c)	Q7	(c)
Q8	(b)	Q17	(a)			Q8	(a)
Q9	(a)	Q18	(d)			Q9	(a)

Part B (contd.)		Part C	
Q10	(b)	Q1	0
Q11	(a)	Q2	3444
Q12	(b)	Q3	1
Q13	(b)	Q4	6
Q14	(d)	Q5	272
Q15	(c)	Q6	2
		Q7	8
		Q8	2
		Q9	233
		Q10	327

Part A

1-Mark MCQ (Q1–Q25)

Q1. Fraunhofer Diffraction by an Elliptical Aperture

Optics | Fraunhofer diffraction, reciprocal scaling

The Fraunhofer diffraction pattern formed by an elliptical aperture will be

- (a) elliptical with the semi-major axis parallel to that of the aperture.
- (b) elliptical with the semi-major axis perpendicular to that of the aperture.
- (c) circular.
- (d) hyperbolic.

Solution

The far-field amplitude is the 2-D Fourier transform of the aperture function. An ellipse with semi-axes a (along x) and b (along y) is a circle stretched by a and b ; by the Fourier scaling theorem the transform is stretched by $1/a$ and $1/b$. Angular spread along x is $\sim \lambda/a$, along y is $\sim \lambda/b$. If $a > b$, the pattern is *narrower* along x and *wider* along y : an ellipse whose major axis is perpendicular to the aperture's major axis.

Answer: (b) elliptical, major axis $\perp$ aperture's

Insight

Diffraction always “inverts” dimensions: the narrower the aperture in a direction, the broader the spread in that direction.

Q2. Origin Dependence of the Dipole Moment

Electrostatics | Multipole expansion

The electric dipole moment of a charge distribution is independent of the choice of the origin of co-ordinates only if

- (a) the charge distribution is discrete.
- (b) there is no magnetic field present.
- (c) the total charge adds up to zero.
- (d) the charge distribution is not time-dependent.

Solution

Shift the origin by $\vec{a}$: $\vec{r}' = \vec{r} - \vec{a}$.

$$\vec{p}' = \int \rho (\vec{r} - \vec{a}) d\tau = \vec{p} - Q \vec{a}.$$

$\vec{p}' = \vec{p}$ for every $\vec{a}$ iff the total charge $Q = 0$.

Answer: (c) $Q_{\text{tot}} = 0$

Q3. Stone Dropped from a Floating Boat

Fluid Mechanics | Archimedes' principle

A boat is floating in a pond with still water. There is a heavy stone on the boat. If the stone is dropped gently into the water, what happens to the water level in the pond after the stone sinks completely?

- (a) The level goes up or down depending on the size of the stone.
- (b) The level remains the same.
- (c) The level goes up.
- (d) The level goes down.

Solution

Stone (mass m , density $\rho_s > \rho_w$) on the boat: it displaces its *weight* of water, volume $V_1 = m/\rho_w$.

Stone at the bottom: it displaces only its own *volume*, $V_2 = m/\rho_s$.

Since $\rho_s > \rho_w$, $V_2 < V_1$: less water is displaced, so the level falls.

Answer: (d) level goes down

Q4. RC Charging with the Resistor Shorted

Electronics | RC transients

A capacitor with capacitance C is connected in series with a resistor of resistance R and an ideal DC source with voltage V_S . At one instant during the charging of the capacitor if the resistor is replaced by a wire of zero resistance, which of the following statements is true?

- (a) The capacitor immediately attains the source voltage V_S .
- (b) None of the others is true.
- (c) The voltage across the capacitor will increase slowly.
- (d) The voltage across the capacitor will drop immediately to zero.

Solution

During charging $V_C(t) = V_S (1 - e^{-t/RC})$ with time constant $\tau = RC$. Replacing R by an ideal wire makes $\tau \rightarrow 0$: the (ideal) source can supply an unbounded current, so the remaining charge $C(V_S - V_C)$ is delivered instantaneously. Kirchhoff's loop law with $R = 0$ forces $V_C = V_S$ at once.

Answer: (a) $V_C \rightarrow V_S$ immediately

Insight

In any real circuit, wire resistance and the source's internal resistance keep τ small but non-zero; the "instantaneous" jump is the ideal-circuit limit.

Q5. Constants of Motion in a Central Potential

Classical Mechanics | *Symmetries and conservation laws*

The Lagrangian of a two-dimensional system is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - k(x^2 + y^2)^{-1.5}$$

Which of the following is/are the constant(s) of motion?

- | | |
|---------------------------------|---------------------------|
| (a) Energy and angular momentum | (b) None of the others |
| (c) Energy only | (d) Angular momentum only |

Solution

L has no explicit t dependence $\Rightarrow$ energy is conserved.

In polar coordinates $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - kr^{-3}$; θ is cyclic, so $p_\theta = mr^2\dot{\theta}$ (angular momentum) is conserved.

Answer: (a) Energy and angular momentum

Q6. Successive Spin Measurements

Quantum Mechanics | *Measurement postulate, spin- $\frac{1}{2}$*

Three observers successively measure the spin of a given proton along z -axis, x -axis and again z -axis, respectively. The first observer finds the spin projection to be $+\frac{1}{2}$. Assuming no other factors, what is the probability that the third observer finds the spin projection to be $-\frac{1}{2}$?

- | | |
|------------------------|-------|
| (a) 0.5 | (b) 1 |
| (c) None of the others | (d) 0 |

Solution

After the x measurement the state is $|\pm x\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$, whatever the outcome. Then

$$P(-\frac{1}{2} \text{ along } z) = |\langle\downarrow|\pm x\rangle|^2 = \frac{1}{2}.$$

Answer: (a) 0.5

Q7. Eigenvalue of a Differential Operator

Mathematical Physics | Linear operators

Given the differential operator: $D \equiv \frac{d^2}{dx^2} + P \frac{d}{dx} + Q$, where P and Q are constants, what is the eigenvalue corresponding to the eigenfunction $y = e^x$?

- (a) $(1 + P + Q)$ (b) $(1 + Q)$
(c) $(P + Q - 1)$ (d) $(P + Q)$

Solution

$$De^x = e^x + Pe^x + Qe^x = (1 + P + Q)e^x.$$

Answer: (a) $1 + P + Q$

Q8. Generator of a Product of Unitary Evolutions

Quantum Mechanics | Unitary operators

Consider a quantum system that is evolved sequentially with a finite sequence of Hermitian Hamiltonians $\{H_0, H_1, \dots, H_n\}$. The full evolution operator is written as:

$$\mathcal{O} = U_n U_{n-1} \dots U_1 U_0 = e^{-i\mathcal{H}}, \text{ with } U_j = e^{-iH_j} \text{ and, } j = 0, 1, \dots, n$$

Then $\mathcal{H}$ is

- (a) None of the others. (b) a Hermitian operator.
(c) undefined. (d) a unitary operator.

Solution

Each U_j is unitary, and a product of unitaries is unitary: $\mathcal{O}^\dagger \mathcal{O} = \mathbb{I}$. A unitary operator is diagonalisable with eigenvalues $e^{-i\theta_k}$, $\theta_k \in \mathbb{R}$, so $\mathcal{O} = e^{-i\mathcal{H}}$ with $\mathcal{H} = \sum_k \theta_k |k\rangle \langle k|$, which is Hermitian. (In general $\mathcal{H} \neq \sum_j H_j$ because the H_j need not commute — BCH formula — but it is still Hermitian.)

Answer: (b) $\mathcal{H}$ is Hermitian

Q9. Photon–Matter Interactions

Modern Physics | Photoelectric, Compton, pair production, bremsstrahlung

Match the following statements.

A1	Photoelectric effect	B1	Involves loosely bound or free electrons
A2	Compton scattering	B2	Inverse photoelectric effect
A3	Pair production	B3	Needs a minimum of 1.02 MeV of energy for the incident radiation
A4	Bremsstrahlung	B4	Involves bound electrons and depends on the specifics of the material

(a) A1-B4, A2-B1, A3-B3, A4-B2

(b) A1-B3, A2-B4, A3-B2, A4-B1

(c) A1-B4, A2-B3, A3-B2, A4-B1

(d) A1-B3, A2-B2, A3-B1, A4-B4

Solution

- Photoelectric: absorption by a *bound* electron; work function is material-specific $\Rightarrow$ B4.
- Compton: scattering off effectively free/loosely bound electrons $\Rightarrow$ B1.
- Pair production: threshold $2m_e c^2 = 1.02$ MeV $\Rightarrow$ B3.
- Bremsstrahlung (X-ray production): electron KE $\rightarrow$ photon, the inverse of photo-emission $\Rightarrow$ B2.

Answer: (a) A1-B4, A2-B1, A3-B3, A4-B2

Q10. Relativistic Force and Acceleration

Special Relativity | Relativistic dynamics

For a relativistic point particle the momentum is $\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - v^2/c^2}}$, where $\vec{v}$ is its velocity as measured by an inertial observer. Then the acceleration is in the same direction as the applied force

- only when force is neither parallel nor perpendicular to the velocity.
- always.
- never.
- only when force is parallel or perpendicular to the velocity.

Solution

$$\vec{F} = \frac{d}{dt}(\gamma m_0 \vec{v}) = \gamma m_0 \vec{a} + \gamma^3 m_0 \frac{(\vec{v} \cdot \vec{a})}{c^2} \vec{v}. \text{ Decomposing,}$$

$$\vec{F}_{\parallel} = \gamma^3 m_0 \vec{a}_{\parallel}, \quad \vec{F}_{\perp} = \gamma m_0 \vec{a}_{\perp}.$$

The “longitudinal” and “transverse” masses differ, so $\vec{a} \parallel \vec{F}$ only if one component vanishes, i.e. $\vec{F} \parallel \vec{v}$ or $\vec{F} \perp \vec{v}$.

Answer: (d) only when $\vec{F} \parallel \vec{v}$ or $\vec{F} \perp \vec{v}$

Q11. Plane EM Wave in an Isotropic Medium

Electrodynamics | *Plane waves*

For a plane electromagnetic wave propagating with wave vector $\vec{k}$ in a homogeneous and isotropic medium, which of the following holds?

- (a) $\vec{E} \cdot \vec{B} = 0$ (b) $\vec{E} \times \vec{B} = \vec{0}$
 (c) $\vec{k} \cdot (\vec{E} \times \vec{B}) = 0$ (d) None of the others.

Solution

From Faraday's law, $\vec{B} = \frac{1}{\omega} \vec{k} \times \vec{E}$, so $\vec{E}, \vec{B}, \vec{k}$ are mutually perpendicular: $\vec{E} \cdot \vec{B} = 0$. Also $\vec{E} \times \vec{B} \parallel \vec{k}$ (Poynting vector), so (b) and (c) are false.

Answer: (a) $\vec{E} \cdot \vec{B} = 0$

Q12. Displaced Charged Ball in Equilibrium

Electrostatics | *Coulomb force, stability*

Consider two identical charged balls, each of mass m and charge q . One of them is initially held fixed on a frictionless insulating horizontal surface and the other is carefully placed above the first one at a height h from the surface, such that the gravitational force on it is balanced by the Coulomb repulsion. The upper ball is now shifted horizontally by a distance d ($d \ll h$) to the right and then both the balls are released. Which way will the balls move immediately after this?

- (a) both balls remain static at their new positions.
 (b) ball on the surface moves towards right and ball above moves upwards.
 (c) both balls oscillate around their original positions.
 (d) ball on the surface moves towards left, ball above moves downwards.

Solution

Initially $\frac{kq^2}{h^2} = mg$. After the shift the separation is $r = \sqrt{h^2 + d^2}$ and the repulsion acts along the line joining them.

- Lower ball: horizontal component points away from the upper ball, i.e. to the **left**.
- Upper ball: vertical component $\frac{kq^2 h}{(h^2 + d^2)^{3/2}} < \frac{kq^2}{h^2} = mg$, so net force is **downward** (it also gets a small push to the right).

Answer: (d) lower ball left, upper ball down

Q13. Image of a Small Wire

Optics | *Thin lens, magnification*

A wire of length 0.01 mm is placed perpendicular to the axis of a thin convex lens, 30 cm from its center. If the focal length of the lens is 20 cm, what is the length of the image of the wire?

- (a) 1 mm (b) 0.03 mm
(c) 0.02 mm (d) 0.01 mm

Solution

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ with $u = -30$ cm: $\frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60} \Rightarrow v = 60$ cm.
 $|m| = v/|u| = 2$, so image length = $2 \times 0.01 = 0.02$ mm.

Answer: (c) 0.02 mm

Q14. Parameters of a 3×3 Hermitian Matrix

Mathematical Physics | *Matrices*

The number of independent real numbers that parameterize any (3×3) Hermitian matrix is

- (a) 6 (b) 8
(c) 3 (d) 9

Solution

Diagonal: 3 real numbers. Above-diagonal: 3 complex numbers = 6 real (below-diagonal fixed by $H_{ji} = H_{ij}^*$). Total = $n^2 = 9$.

Answer: (d) 9

Q15. Slow Loss of Solar Mass

Classical Mechanics | *Kepler problem, adiabatic invariants*

Suppose the mass of the Sun is reduced to half of its original value very slowly, e.g., over a billion years, what will be the effect of this on the Earth's orbit?

- (a) Remains elliptical, but the mean radius changes.
(b) The Earth flies away.
(c) Orbit remains closed but not elliptical.
(d) Remains elliptical with the same mean radius.

Solution

The force stays central, so $L = m\sqrt{GMa(1-e^2)}$ is conserved exactly. Because the change is slow compared with the orbital period, the radial action is an adiabatic invariant too, which keeps e fixed. Hence

$$a \propto \frac{1}{M} \Rightarrow a \rightarrow 2a.$$

At every instant the orbit is a Kepler ellipse in a $1/r$ potential, so it stays elliptical, but with a larger mean radius. (A *sudden* halving would unbind a circular orbit; a slow one does not.)

Answer: (a) elliptical; mean radius changes

Q16. Reversible Adiabatic Relation in P - T

Thermodynamics | *Adiabatic processes*

One mole of ideal gas with a constant C_V undergoes a reversible adiabatic expansion. Which one of the following equations is valid? $\left[\gamma = \frac{C_P}{C_V} \text{ for the gas} \right]$

- (a) $P^{1-\gamma}T^\gamma = \text{constant}$ (b) $P^{\gamma-1}T^\gamma = \text{constant}$
 (c) $V^\gamma T = \text{constant}$ (d) $VT^\gamma = \text{constant}$

Solution

$PV^\gamma = \text{const}$ with $V = RT/P$: $P(T/P)^\gamma = \text{const} \Rightarrow P^{1-\gamma}T^\gamma = \text{const}$.

Answer: (a) $P^{1-\gamma}T^\gamma = \text{const}$

Q17. Work in a Central Linear Force Field

Classical Mechanics | *Conservative forces*

A particle is moving under the force field given by $\vec{F} = k\vec{r}$, where k is a positive constant. The difference in work done (in arbitrary units) if the particle moves from point A(-1, 0, 0) to point B(1, 0, 0) following semi-circular paths in the clockwise and anti-clockwise directions on the X-Y plane will be

- (a) 0 (b) $\frac{1}{2}\pi k$
 (c) πk (d) $2\pi k$

Solution

$\vec{\nabla} \times (k\vec{r}) = 0$, so $\vec{F} = -\nabla U$ with $U = -\frac{1}{2}kr^2$. Work is path independent. (Indeed along either semicircle $\vec{F} \perp d\vec{r}$, so both works are zero.)

Answer: (a) 0

Q18. Smallest Non-Abelian Group

Mathematical Physics | Group theory

Consider the standard notation of discrete finite groups with $\mathbb{Z}_n$ corresponding to the rotation by $2\pi/n$ about a given axis, S_n corresponding to the permutation group of the set S having n elements, i.e. $S = \{1, 2, 3, \dots, n\}$, and the Dihedral group D_n corresponding to the reflection and rotation symmetries of a regular polygon with n number of sides. Which of the following is the smallest non-abelian group?

- (a) $\mathbb{Z}_3$ (b) D_4
(c) S_4 (d) S_3

Solution

All groups of order ≤ 5 are abelian (orders 1, 2, 3, 5 are prime; order 4 gives $\mathbb{Z}_4$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$). $S_3 \cong D_3$ has order 6 and is non-abelian, e.g. $(12)(13) \neq (13)(12)$. D_4 (order 8) and S_4 (order 24) are larger; $\mathbb{Z}_3$ is abelian.

Answer: (d) S_3

Q19. Expectation Value of $(A - a_1)(A - a_2)$

Quantum Mechanics | Operators and eigenstates

A quantum mechanical system is spanned by the eigenstates $|a_1\rangle$ and $|a_2\rangle$ of a Hermitian operator A with eigenvalues a_1 and a_2 respectively. If there is no degeneracy, what is the expectation value of the operator $(A - a_1)(A - a_2)$ in the state $\frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}}$?

- (a) $(a_2 - a_1)(a_1 - a_2)$ (b) $\frac{(a_2 - a_1)(a_1 - a_2)}{2}$
(c) 0 (d) 1

Solution

$(A - a_1)(A - a_2)|a_1\rangle = (a_1 - a_1)(\dots) = 0$ and $(A - a_1)(A - a_2)|a_2\rangle = 0$. The operator annihilates the whole 2-D space (Cayley–Hamilton), so the expectation value is 0.

Answer: (c) 0

Q20. State Functions of an Ideal Gas**Thermodynamics** | *State variables*

Which of the following thermodynamic variables is not a function of state for an ideal gas?

- (a) None of the others (b) Pressure
(c) Temperature (d) Entropy

Solution

P , T and S are all state functions (only heat and work are path dependent). Hence none of the listed variables fails.

Answer: (a) None of the others

Q21. Powers of the Fibonacci Matrix**Mathematical Physics** | *Diagonalisation*

Consider a 2×2 matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ which has eigenvalues $\lambda_1 = \frac{1+\sqrt{5}}{2}$ and $\lambda_2 = \frac{1-\sqrt{5}}{2}$. For any natural number n which of the following is correct?

- (a) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} - \lambda_2^{n-1} & \lambda_1^n - \lambda_2^n \\ \lambda_1^n - \lambda_2^n & \lambda_1^{n+1} - \lambda_2^{n+1} \end{bmatrix}$ (b) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} + \lambda_2^{n-1} & \lambda_1^n - \lambda_2^n \\ \lambda_1^n - \lambda_2^n & \lambda_1^{n+1} + \lambda_2^{n+1} \end{bmatrix}$
(c) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} - \lambda_2^{n-1} & \lambda_1^n + \lambda_2^n \\ \lambda_1^n + \lambda_2^n & \lambda_1^{n+1} - \lambda_2^{n+1} \end{bmatrix}$ (d) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} + \lambda_2^{n-1} & \lambda_1^n + \lambda_2^n \\ \lambda_1^n + \lambda_2^n & \lambda_1^{n+1} + \lambda_2^{n+1} \end{bmatrix}$

Solution

By induction $A^n = \begin{pmatrix} F_{n-1} & F_n \\ F_n & F_{n+1} \end{pmatrix}$ with Fibonacci numbers (A^1 : $F_0 = 0, F_1 = F_2 = 1$). Binet's formula gives $F_n = \frac{\lambda_1^n - \lambda_2^n}{\sqrt{5}}$. Check $n = 1$: $\frac{1}{\sqrt{5}}(\lambda_1^0 - \lambda_2^0) = 0$, $\frac{1}{\sqrt{5}}(\lambda_1 - \lambda_2) = 1$, $\frac{1}{\sqrt{5}}(\lambda_1^2 - \lambda_2^2) = \lambda_1 + \lambda_2 = 1$. ✓

Answer: (a) all entries with “−”

Q22. Complex Conjugate of a Stationary State

Quantum Mechanics | *Time-independent Schrödinger equation*

Consider the time-independent Schrödinger equation with a real potential and suppose $\psi(x)$ is a solution of this equation. Which of the following is true?

- (a) ψ^* is a solution of the same equation only if the potential is symmetric about $x = 0$.
- (b) ψ^* is a solution of the same equation only if the potential vanishes at infinity.
- (c) ψ^* is a solution of the same equation.
- (d) ψ^* is never a solution of the same equation.

Solution

Conjugate $-\frac{\hbar^2}{2m}\psi'' + V\psi = E\psi$. With V and E real, $-\frac{\hbar^2}{2m}\psi^{*''} + V\psi^* = E\psi^*$. So ψ^* solves the same equation (and hence real solutions can always be chosen).

Answer: (c) ψ^* is a solution

Q23. Vernier Constant

Experimental Physics | *Measuring instruments*

A slide calipers instrument has smallest main scale division of 0.4 mm and 40 vernier divisions match with 38 main scale divisions. The vernier constant of this instrument is

- (a) 0.05 mm
- (b) 0.02 mm
- (c) 0.1 mm
- (d) 0.01 mm

Solution

1 VSD = $\frac{38 \times 0.4}{40} = 0.38$ mm. VC = 1 MSD - 1 VSD = 0.40 - 0.38 = 0.02 mm.

Answer: (b) 0.02 mm

Q24. Two Bosons in Two Levels

Statistical Mechanics | *Quantum statistics, partition function*

Calculate the partition function for two indistinguishable bosonic particles at a temperature T , which can be distributed in two single-particle energy levels ϵ_1 and ϵ_2 . Consider $\beta = \frac{1}{k_B T}$.

- (a) $(e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2})^2$
- (b) $e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-\beta(\epsilon_1+\epsilon_2)}$
- (c) $\frac{1}{2!} (e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2})^2$
- (d) $e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-2\beta(\epsilon_1+\epsilon_2)}$

Solution

Allowed microstates (occupation numbers): (2, 0) with energy $2\epsilon_1$; (0, 2) with $2\epsilon_2$; (1, 1) with $\epsilon_1 + \epsilon_2$ — one state each.

$$Z = e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-\beta(\epsilon_1+\epsilon_2)}.$$

Answer: (b) $Z = e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-\beta(\epsilon_1+\epsilon_2)}$

Insight

Option (c) is the Maxwell–Boltzmann “corrected counting”; it wrongly gives the doubly occupied states weight $\frac{1}{2}$.

Q25. Debye T^3 Law

Solid State Physics | Lattice specific heat

If the lattice contribution to the C_V of a solid crystal at temperature $2K$ is found to be $0.5 \text{ mJ mole}^{-1} K^{-1}$, what will be the corresponding contribution at temperature $4K$?

- (a) $8 \text{ mJ mole}^{-1} K^{-1}$ (b) $1 \text{ mJ mole}^{-1} K^{-1}$
 (c) $4 \text{ mJ mole}^{-1} K^{-1}$ (d) $2 \text{ mJ mole}^{-1} K^{-1}$

Solution

At $T \ll \Theta_D$, $C_V \propto T^3$: $C_V(4 \text{ K}) = 0.5 \times 2^3 = 4 \text{ mJ mol}^{-1} K^{-1}$.

Answer: (c) $4 \text{ mJ mol}^{-1} K^{-1}$

Part B

3-Mark MCQ (Q1–Q15)

Q1. Divergence of $r^4 \vec{r}$

Mathematical Physics | Vector calculus

Evaluate $\vec{\nabla} \cdot (r^4 \vec{r})$, where $\vec{r}$ represents a three dimensional position vector.

- (a) $7r^4$ (b) $4r^4$
 (c) 0 (d) $5r^4$

Solution

$\vec{\nabla} \cdot (f \vec{r}) = 3f + r f'(r) = 3r^4 + 4r^4 = 7r^4$.
 (Check: $r^4 \vec{r} = r^5 \hat{r}$, $\frac{1}{r^2} \frac{d}{dr}(r^7) = 7r^4$.)

Answer: (a) $7r^4$

Q2. Doubling the Diode Current

Electronics | *p-n junction diode*

A silicon p-n junction diode operates at 27°C . The current I is doubled when the forward bias is increased. The increase in the forward bias is closest to:

[Assume $I \gg I_s$, where I_s is the reverse saturation current and the emission coefficient $\eta_{\text{Si}} = 2$.]

- (a) 54 mV (b) 18 mV
(c) 72 mV (d) 36 mV

Solution

$$I \simeq I_s e^{V/\eta V_T}, \text{ so } \Delta V = \eta V_T \ln \frac{I_2}{I_1} = 2 \times (25.9 \text{ mV}) \times \ln 2 \approx 35.8 \text{ mV}.$$

Answer: (d) 36 mV

Q3. Order-2 Elements of S_4

Mathematical Physics | *Group theory*

Consider the group S_4 corresponding to the permutations of the set S having four elements, say $S = \{1, 2, 3, 4\}$. How many non-identity self-inverse (i.e. order 2) elements does S_4 have?

- (a) 6 (b) 12
(c) 9 (d) 8

Solution

Order-2 permutations are products of disjoint transpositions:

- single transpositions (ab) : $\binom{4}{2} = 6$;
- double transpositions $(ab)(cd)$: $\frac{1}{2} \binom{4}{2} = 3$.

Total = 9.

Answer: (c) 9

Q4. Momentum Uncertainty in a Box

Quantum Mechanics | Infinite square well

For a particle in a one-dimensional box of width L , the uncertainty Δp in momentum in the n -th eigenstate of energy for large n is

- | | |
|---------------------------|----------------------------|
| (a) $\frac{2n\hbar}{L}$ | (b) $\frac{\hbar}{n\pi L}$ |
| (c) $\frac{n\pi\hbar}{L}$ | (d) $\frac{2n\pi\hbar}{L}$ |

Solution

$\langle p \rangle = 0$ (real wavefunction) and $\langle p^2 \rangle = 2mE_n = \left(\frac{n\pi\hbar}{L}\right)^2$. So $\Delta p = \frac{n\pi\hbar}{L}$ (exact for every n).

Answer: (c) $\frac{n\pi\hbar}{L}$

Q5. Moment of Inertia of a Non-uniform Disk

Classical Mechanics | Rigid body

Consider a circular disk of radius R and mass M in the X–Y plane, with a surface mass density $\sigma(r) = \sigma_0 e^{-r^2/a^2}$, where r is the distance from the center of the disk. What is the moment of inertia around the Z-axis through the center of the disk? [consider $R \gg a$]

- | | |
|-----------------------|-------------|
| (a) $\frac{1}{3}Ma^2$ | (b) $6Ma^2$ |
| (c) $\frac{1}{2}Ma^2$ | (d) Ma^2 |

Solution

With $R \gg a$ extend the integrals to ∞ and put $u = r^2/a^2$:

$$M = \int_0^\infty \sigma_0 e^{-r^2/a^2} 2\pi r dr = \pi\sigma_0 a^2, \quad I = \int_0^\infty \sigma_0 e^{-r^2/a^2} r^2 2\pi r dr = \pi\sigma_0 a^4.$$

Therefore $I = Ma^2$.

Answer: (d) Ma^2

Q6. Far Field of a Current Loop in its Own Plane

Magnetostatics | *Magnetic dipole field*

A circular loop of radius a , carrying a current I in an anticlockwise direction (when seen downwards from the positive Z axis), is placed on the X - Y plane centered at the origin. What is the magnetic field on the X - Y plane at $r \gg a$?

- (a) $\frac{\mu_0 I a^2}{4\pi r^3}$ in the positive Z direction
 (b) 0
 (c) $\frac{\mu_0 I a^2}{4 r^3}$ in the negative Z direction
 (d) $\frac{\mu_0 I a^2}{4 r^3} \hat{r}$

Solution

Dipole moment $\vec{m} = I\pi a^2 \hat{z}$. Dipole field: $\vec{B} = \frac{\mu_0}{4\pi r^3} [3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}]$. In the plane of the loop $\vec{m} \cdot \hat{r} = 0$:

$$\vec{B} = -\frac{\mu_0}{4\pi} \frac{I\pi a^2}{r^3} \hat{z} = \frac{\mu_0 I a^2}{4 r^3} (-\hat{z}).$$

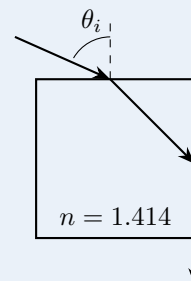
Answer: (c) $\frac{\mu_0 I a^2}{4 r^3}$ along $-Z$

Q7. Ray Grazing Down the Side of a Glass Cube

Optics | *Refraction, total internal reflection*

A ray of light is incident on a glass cube of refractive index 1.414 as shown in the figure. Find the angle of incidence θ_i , such that the ray grazes down the side of the glass cube.

- (a) 0
 (b) $\pi/3$
 (c) $\pi/2$
 (d) $\pi/4$



Solution

Top face: $\sin \theta_i = n \sin r$. The refracted ray meets the vertical side at angle of incidence $90^\circ - r$. For it to emerge grazing along the side, this must equal the critical angle:

$$\sin(90^\circ - r) = \cos r = \frac{1}{n} = \frac{1}{\sqrt{2}} \Rightarrow r = 45^\circ.$$

Then $\sin \theta_i = \sqrt{2} \sin 45^\circ = 1 \Rightarrow \theta_i = \pi/2$ (grazing incidence on the top face).

Answer: (c) $\pi/2$

Insight

For $n < \sqrt{2}$ any ray entering the top can escape from the side; $n = \sqrt{2}$ is exactly the borderline where only grazing incidence produces grazing emergence.

Q8. Ground-State Shift from an x^4 Perturbation

Quantum Mechanics | Perturbation theory

For a one-dimensional simple harmonic oscillator with mass m and angular frequency ω , consider a perturbation λx^4 in the Hamiltonian ($\lambda > 0$). What is the lowest order correction to the ground state energy?

[The position operator expressed in terms of the raising and lowering operators is $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^\dagger)$.]

(a) $\frac{3\lambda}{4} \left(\frac{\hbar}{m\omega}\right)^2$

(b) $\frac{5\lambda}{4} \left(\frac{\hbar}{m\omega}\right)^2$

(c) $\frac{3\lambda}{2} \left(\frac{\hbar}{m\omega}\right)^2$

(d) $\frac{5\lambda}{2} \left(\frac{\hbar}{m\omega}\right)^2$

Solution

$\langle 0|(a + a^\dagger)^4|0\rangle$: the non-zero orderings are $aaa^\dagger a^\dagger (= 2)$ and $aa^\dagger aa^\dagger (= 1)$, total 3. Hence

$$E_0^{(1)} = \lambda \left(\frac{\hbar}{2m\omega}\right)^2 \times 3 = \frac{3\lambda}{4} \left(\frac{\hbar}{m\omega}\right)^2.$$

Answer: (a) $\frac{3\lambda}{4}(\hbar/m\omega)^2$

Q9. Mean Energy of a 2-D Fermi Gas

Statistical Mechanics | Fermi gas, density of states

Consider a two-dimensional Fermi gas at 0 K with Fermi energy ϵ_F . The average energy per particle of this gas is:

(a) $\frac{\epsilon_F}{2}$

(b) $\frac{\epsilon_F}{4}$

(c) $\frac{3\epsilon_F}{5}$

(d) $\frac{\epsilon_F}{3}$

Solution

In 2-D, $g(\epsilon) = \text{const.}$ So $\langle \epsilon \rangle = \frac{\int_0^{\epsilon_F} \epsilon d\epsilon}{\int_0^{\epsilon_F} d\epsilon} = \frac{\epsilon_F}{2}$.

Answer: (a) $\epsilon_F/2$

Insight

In d dimensions $g \propto \epsilon^{d/2-1}$ and $\langle \epsilon \rangle = \frac{d}{d+2}\epsilon_F$: $\frac{1}{3}, \frac{1}{2}, \frac{3}{5}$ for $d = 1, 2, 3$.

Q10. Transverse Velocity Transformation

Special Relativity | Velocity addition

A particle is moving with velocity $v_x = v_y = v_z = c/2$ in frame S . The ratio of velocity component v_y to the velocity component $v_{y'}$ as measured in frame S' moving with velocity $c/2$ with respect to frame S along the common x direction is

- (a) $\sin(\pi/3)$ (b) $\cos(\pi/6)$
(c) $\cos(\pi/3)$ (d) $\sin(\pi/6)$

Solution

$$v_{y'} = \frac{v_y \sqrt{1 - u^2/c^2}}{1 - uv_x/c^2} = \frac{v_y(\sqrt{3}/2)}{1 - \frac{1}{4}} = \frac{2}{\sqrt{3}} v_y.$$

$$\frac{v_y}{v_{y'}} = \frac{\sqrt{3}}{2} = \cos \frac{\pi}{6}.$$

Answer: (b) $\cos(\pi/6)$

Q11. Charge Distribution of the Hydrogen Atom

Electrostatics | Poisson's equation, delta functions

The time averaged electrostatic potential of a neutral H-atom is given by

$$\Phi(\vec{r}) = \frac{q}{4\pi\epsilon_0} \frac{e^{-\alpha r}}{r} \left(1 + \frac{\alpha r}{2}\right)$$

The classical charge distribution corresponding to this is

- (a) $-\frac{q}{8\pi}\alpha^3 e^{-\alpha r} + q\delta^3(\vec{r})$ (b) $\frac{q}{8\pi}\alpha^3 e^{-\alpha r} \left(1 + \frac{\alpha r}{2}\right) - q\delta^3(\vec{r})$
(c) $-\frac{q}{8\pi}\alpha^3 e^{-\alpha r}$ (d) $qe^{-\alpha r} \left(1 + \frac{\alpha r}{2}\right)$

Solution

Write $\Phi = \frac{q}{4\pi\epsilon_0} \left(\frac{e^{-\alpha r}}{r} + \frac{\alpha}{2} e^{-\alpha r} \right)$ and use $\rho = -\epsilon_0 \nabla^2 \Phi$.

- $\nabla^2 \frac{e^{-\alpha r}}{r} = \alpha^2 \frac{e^{-\alpha r}}{r} - 4\pi\delta^3(\vec{r}).$
- $\nabla^2 e^{-\alpha r} = \frac{1}{r^2} \frac{d}{dr} (-\alpha r^2 e^{-\alpha r}) = \alpha^2 e^{-\alpha r} - \frac{2\alpha}{r} e^{-\alpha r}.$

$$\text{Sum: } \nabla^2(\dots) = \alpha^2 \frac{e^{-\alpha r}}{r} - 4\pi\delta^3 + \frac{\alpha^3}{2} e^{-\alpha r} - \alpha^2 \frac{e^{-\alpha r}}{r} = \frac{\alpha^3}{2} e^{-\alpha r} - 4\pi\delta^3(\vec{r}).$$

$$\rho = -\frac{q}{4\pi} \left[\frac{\alpha^3}{2} e^{-\alpha r} - 4\pi\delta^3(\vec{r}) \right] = q\delta^3(\vec{r}) - \frac{q\alpha^3}{8\pi} e^{-\alpha r}.$$

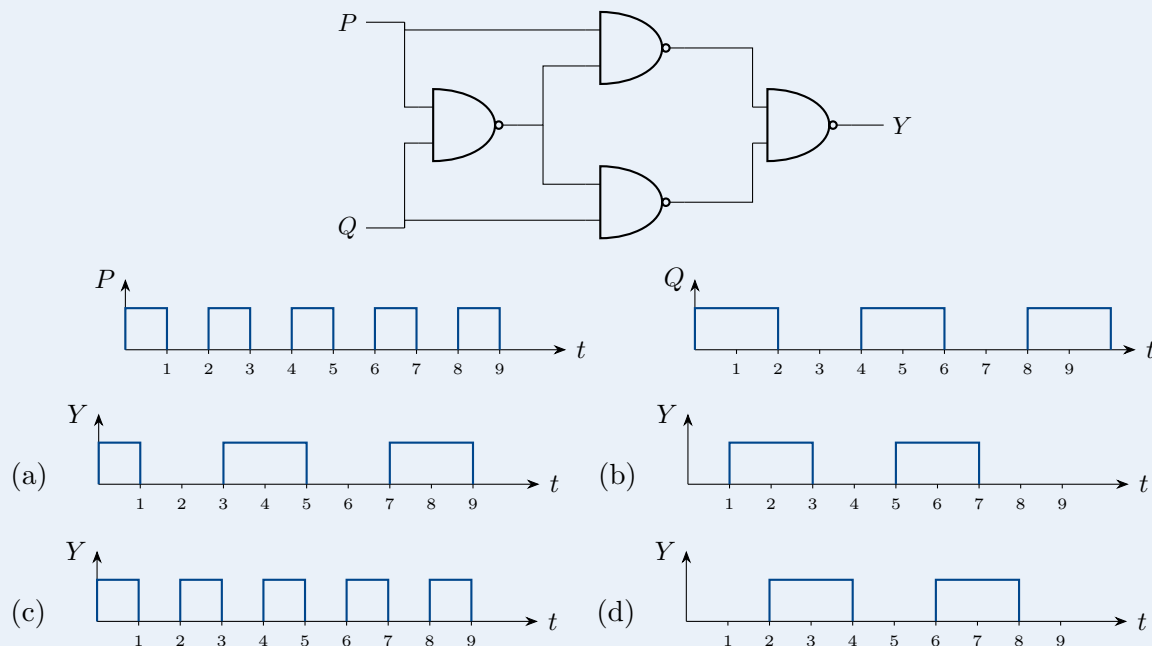
Check: $\int \frac{q\alpha^3}{8\pi} e^{-\alpha r} 4\pi r^2 dr = q$, so the atom is neutral. ✓

Answer: (a) $-\frac{q}{8\pi}\alpha^3 e^{-\alpha r} + q\delta^3(\vec{r})$

Q12. Output of a NAND-Gate Network

Electronics | Digital logic

For the circuit and the inputs P and Q shown, which of the following is the correct output Y ?



Solution

Let $X = \overline{PQ}$. The two middle gates give $\overline{PX}$ and $\overline{QX}$, and the last gate

$$Y = \overline{\overline{PX} \cdot \overline{QX}} = PX + QX = (P + Q) \overline{PQ} = P \oplus Q.$$

This is the standard four-NAND XOR. From the waveforms (P toggles every unit, Q every two units):

interval	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8
P	1	0	1	0	1	0	1	0
Q	1	1	0	0	1	1	0	0
Y	0	1	1	0	0	1	1	0

Y is high during 1–3 and 5–7: two pulses of double width, matching (b).

Answer: (b) $Y = P \oplus Q$

Q13. Doubling the Fermions in a Nucleus**Nuclear/Statistical Physics** | *Fermi gas model*

The volume of a nucleus, treated as a Fermi gas in three-dimensional space, is proportional to the number of fermions present in it. If the total number of fermions is changed from N to $2N$, the total energy of the system will

- (a) remain the same. (b) be doubled.
(c) be 4 times its original value. (d) be half of its original value.

Solution

$\epsilon_F = \frac{\hbar^2}{2m} \left(3\pi^2 \frac{N}{V} \right)^{2/3}$. With $V \propto N$, N/V is constant, so ϵ_F is unchanged. $E_{\text{tot}} = \frac{3}{5} N \epsilon_F \propto N$: it doubles.

Answer: (b) doubled**Q14. Stability Condition from Entropy****Thermodynamics** | *Concavity of entropy*

Given an isolated thermodynamic system with a total energy E , total volume V and total number of particles N , the condition for stable thermal equilibrium, in terms of its entropy S under small changes ΔE and ΔV , is given by

- (a) $-S(E + \Delta E, V + \Delta V, N) + S(E - \Delta E, V - \Delta V, N) - 2S(E, V, N) < 0$
(b) $S(E + \Delta E, V + \Delta V, N) + S(E - \Delta E, V - \Delta V, N) + 2S(E, V, N) < 0$
(c) $S(E + \Delta E, V + \Delta V, N) - S(E - \Delta E, V - \Delta V, N) - 2S(E, V, N) < 0$
(d) $S(E + \Delta E, V + \Delta V, N) + S(E - \Delta E, V - \Delta V, N) - 2S(E, V, N) < 0$

Solution

Split the system into two halves and transfer ΔE , ΔV from one to the other. Stability demands the total entropy decrease:

$$S(E + \Delta E, V + \Delta V) + S(E - \Delta E, V - \Delta V) < 2S(E, V),$$

i.e. S is concave (second-order Taylor term < 0).

Answer: (d) $S(+) + S(-) - 2S < 0$

Q15. Damped Oscillations of an Immersed Block

Classical Mechanics | *Damped oscillator*

A block, suspended from a massless spring, is fully immersed in a liquid contained in a reservoir. What is the time period of small oscillations of the block?

[Given: Mass of the block m , density of the block ρ_b , natural length of the spring L , spring constant k , acceleration due to gravity g , density of the liquid ρ_l , damping coefficient of the liquid i.e., damping per unit mass per unit velocity γ .]

(a) $2\pi\sqrt{\frac{m}{k}}$
 (c) $2\pi\sqrt{\frac{1}{k/m - \gamma^2/4}}$

(b) $2\pi\sqrt{\frac{1}{k/m + \gamma^2/4}}$
 (d) $2\pi\sqrt{\frac{L}{(1 - \rho_l/\rho_b)g}}$

Solution

Gravity and buoyancy are constant forces; they only shift the equilibrium. About equilibrium,

$$\ddot{x} + \gamma\dot{x} + \frac{k}{m}x = 0 \Rightarrow \omega_d = \sqrt{\frac{k}{m} - \frac{\gamma^2}{4}}, \quad T = \frac{2\pi}{\sqrt{k/m - \gamma^2/4}}.$$

Answer: (c) $2\pi / \sqrt{k/m - \gamma^2/4}$

Part C

3-Mark Numerical (Q1–Q10)

Q1. Contour Integral of $1/(1+z^2)$

Mathematical Physics | *Residue theorem*

What is the value of the integral

$$I = \frac{3}{2\pi} \oint_C \frac{dz}{1+z^2}$$

where the contour C is a circle of radius 2 centered at the origin?

Solution

Poles at $z = \pm i$, both inside $|z| = 2$. Residues: $\frac{1}{2i}$ at $z = i$ and $-\frac{1}{2i}$ at $z = -i$. Sum = 0, so $\oint = 0$ and $I = 0$.

Answer: $I = 0$

Q2. Average Power in a Series RC Circuit

Electronics | AC circuits

If a resistor of $10\text{ k}\Omega$ and a capacitor of $0.5\text{ }\mu\text{F}$ are connected in series across an AC supply of 220 V (rms) at 50 Hz , what is the average power (in mW, to the nearest integer) dissipated in the circuit?

Solution

$$X_C = \frac{1}{2\pi(50)(0.5 \times 10^{-6})} = 6366.2\text{ }\Omega, \quad |Z|^2 = R^2 + X_C^2 = 1.4053 \times 10^8\text{ }\Omega^2.$$

$$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2 R}{|Z|^2} = \frac{(220)^2(10^4)}{1.4053 \times 10^8} = 3.444\text{ W}.$$

Answer: $P \approx 3444\text{ mW}$

Q3. Width of the Central Maximum

Optics | Single-slit diffraction

The Fraunhofer diffraction intensity pattern for light of wavelength λ by a single slit of width a is given by

$$I = A_0^2 \left(\frac{\sin \beta}{\beta} \right)^2$$

where A_0 is the intensity of the central maximum and $\beta = \frac{\pi a \sin \theta}{\lambda}$, where θ is the angle with the incident beam. What is the angular separation in milli-radians, between the two first minima on two sides of the central beam, if $a = 1\text{ mm}$ and $\lambda = 5000\text{ }\text{\AA}$?

Solution

First minima at $\beta = \pm\pi \Rightarrow \sin \theta = \pm\lambda/a = \pm5 \times 10^{-4}$. Separation $= 2\lambda/a = 10^{-3}\text{ rad} = 1\text{ mrad}$.

Answer: 1 mrad

Q4. Entropy Change of a Heated Resistor

Thermodynamics | *Entropy*

A current of 10 A is maintained for 1 s in a resistor of resistance $25\ \Omega$, which is thermally insulated. The initial temperature of the resistor is 23°C . The resistor has a mass of 10 gm and a specific heat of $836\ \text{J kg}^{-1}\text{K}^{-1}$. What is the entropy change of the resistor, rounding off to the nearest whole number in units of J K^{-1} ?

Solution

$$Q = I^2 R t = 100 \times 25 \times 1 = 2500\ \text{J}; \quad mc = 0.01 \times 836 = 8.36\ \text{J/K}.$$

$$\Delta T = 2500/8.36 = 299.0\ \text{K}, \text{ so } T_1 = 296.15\ \text{K} \rightarrow T_2 = 595.2\ \text{K}.$$

$$\Delta S = mc \ln \frac{T_2}{T_1} = 8.36 \ln(2.0098) = 5.84\ \text{J/K}.$$

Answer: $\Delta S \approx 6\ \text{J K}^{-1}$

Q5. Isotope Shift of Deuterium

Atomic Physics | *Reduced-mass correction*

Given the mass of the proton $m_p \simeq 1836m_e$ and mass of the deuteron $m_d \simeq 3670m_e$, where m_e is the electron mass, find the fractional shift (in parts per million, to the nearest integer) of the ground state energy of the deuterium atom as compared to H-atom.

Solution

$$E_1 \propto \mu = \frac{m_e M}{m_e + M}.$$

$$\frac{E_D - E_H}{E_H} = \frac{\mu_D}{\mu_H} - 1 = \frac{3670}{3671} \cdot \frac{1837}{1836} - 1 = 2.721 \times 10^{-4}.$$

(Equivalently $\approx \frac{1}{1836} - \frac{1}{3670}$.) The deuterium level is deeper by about 272 ppm.

Answer: $\approx 272\ \text{ppm}$

Q6. Momentum of a Free Particle

Quantum Mechanics | *Free particle*

Suppose the wave function of a free particle in one dimension obeys $\frac{d^2\psi}{dx^2} = -4\psi$ in units where $\hbar = 1$. What is the magnitude of the momentum of the particle?

Solution

$$\psi \sim e^{\pm ikx} \text{ with } k^2 = 4 \Rightarrow k = 2; |p| = \hbar k = 2.$$

Answer: $|p| = 2$

Q7. Determinant from a Matrix Polynomial

Mathematical Physics | Eigenvalues

A 3×3 matrix M satisfies $M^2 - 3M + 2I = 0$. Find the determinant of the matrix M if its trace is 6.

Solution

Every eigenvalue satisfies $\lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda \in \{1, 2\}$. Three eigenvalues from $\{1, 2\}$ summing to 6 must be 2, 2, 2. So $\det M = 8$ (in fact $M = 2I$).

Answer: $\boxed{\det M = 8}$

Q8. Charged Pendulum above a Charged Sheet

Electrostatics | Effective gravity

A simple pendulum with effective length l and a bob of mass m has a time period T_1 . Suppose now that the bob is given an electric charge $+Q$. It is made to oscillate just above a two dimensional infinite sheet with surface charge density $+\sigma$, where $\frac{Q\sigma}{mg\epsilon_0} = \frac{3}{2}$, ϵ_0 being the permittivity of free space and g being the acceleration due to gravity. If the period of oscillation in this case is T_2 , determine $\frac{T_2}{T_1}$. [Neglect radiation from the charge.]

Solution

Sheet field $E = \frac{\sigma}{2\epsilon_0}$ (upward). Upward force $\frac{Q\sigma}{2\epsilon_0} = \frac{3}{4}mg$.

$g_{\text{eff}} = g - \frac{3}{4}g = \frac{g}{4}$, so $\frac{T_2}{T_1} = \sqrt{\frac{g}{g_{\text{eff}}}} = 2$.

Answer: $\boxed{T_2/T_1 = 2}$

Q9. Distance Travelled by a Muon

Special Relativity | Time dilation

The average lifetime of a muon in its rest frame is 2200 ns. What will be the average distance (in meters, to the nearest integer) travelled by it, when created with a velocity of $\frac{1}{3}c$, before it decays? Here c is the speed of light.

Solution

$\gamma = \frac{1}{\sqrt{1 - 1/9}} = \frac{3}{2\sqrt{2}} = 1.0607$.

$d = v\gamma\tau_0 = (10^8)(1.0607)(2.2 \times 10^{-6}) = 233.3 \text{ m}$ (with $c = 3 \times 10^8 \text{ m/s}$).

Answer: $\boxed{d \approx 233 \text{ m}}$

Q10. Source Temperature of a Carnot Engine

Thermodynamics | *Carnot efficiency*

A heat engine works between a high temperature source and a sink at 27°C . If the maximum efficiency possible for it to achieve is 50%, what is the temperature of the source in $^\circ\text{C}$?

Solution

$$\eta = 1 - \frac{T_2}{T_1} = 0.5 \Rightarrow T_1 = 2T_2 = 2 \times 300 = 600 \text{ K} = 327^\circ\text{C}.$$

Answer: 327°C